Exam 1 (February 18, 2003)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (20 pts):

The 1-dimensional motion of a particle is described by the velocity-time graph given below. Draw the appropriate qualitative position-time and acceleration-time graphs for this particle. Assume the particle starts at x=0 at t=0.
Problem 2 (20 pts, no partial credit):

The sketch below represents the trajectory of a golf ball going from point A to point E on the surface of the earth.

What is the direction of the acceleration at point B?

a) It is up and to the right.
b) It is down and to the left.
c) It is straight up.
d) It is straight down.
e) It is zero.

What would your answer to this question be if the golf ball were hit on the moon instead of the earth? IT WOULD NOT CHANGE.

Back on earth, at point C the velocity of the ball is

a) a maximum and is directed to the right.
b) Directed to the left.
c) A maximum.
d) A minimum and is directed to the right

e) Zero.

Problem 3 (20 pts):

Briefly explain why you feel pressed back into your seat when sitting in an airplane that is taking off. You can use a diagram and refer to it if you wish.

When taking off an airplane accelerates. Since you must go with the airplane you must accelerate as well.

\[ \Sigma F_y = N - Mg > 0 \] during taxi on runway

\[ \Sigma F_x = N_{Back} = Ma \] plane

because a plane you feel a normal force against your back (which is the force accelerating your body). This is what one feels as being "pressed" back into your seat.
Problem 4 (20 pts):

A steel ball of mass $M=1.2$ kg is tethered to the end of a massless cable. The ball is attached to a spinning axle that causes the ball and cable to rotate in a vertical circle as shown below. The ball moves in a circle of radius $0.6$ m centered at a height of $2.0$ m above a flat surface. The rate of rotation increases VERY SLOWLY until the cable snaps. The cable snaps when the tension in the cable reaches $100$ N. Newtons. Relative to the position of the axle over the floor ($x=0$ in the sketch), where does the ball hit the floor?

At top of circle

\[ T = \frac{mv^2}{R} \]

\[ F = \frac{mv^2}{R} = T + Mg \]

At bottom of circle

\[ T = \frac{mv^2}{R} - Mg \]

So tension is always greatest at the bottom. That means the bottom is where the cable breaks. After that it is projectile motion!

\[ V^2 = \frac{R(T-Mg)}{M} \]

\[ V^2 = \frac{0.6(100 - 0.2)(9.8)}{1.2} \]

\[ V = 6.64 \text{ m/s} \]

\[ V = \frac{6.64 \text{ m/s}}{2} \]

At $x=0$

\[ X_F = VT \]

\[ T \text{ in Air} = T \]

\[ Y = y_0 + v_0yT + \frac{1}{2}aT^2 \]

\[ 0 = 1.4 - 9.8T^2 \]

\[ T = 0.53 \text{ seconds} \]

\[ X_F = (6.64 \text{ m/s})(0.535) \]

\[ X_F = 3.52 \text{ meters} \]

Ball hits floor 3.52 meters to right of $x=0$. 

\[ x = 3.52 \]
Problem 5 (20 pts):

Find \( m_s \) such that \( M_3 \) does not slip.

Consider the arrangement of masses shown below. Let \( M_1 = 3 \text{ kg} \) and \( M_2 = 6 \text{ kg} \) and \( M_3 = 4 \text{ kg} \). The pulley is massless and frictionless and the string is massless. The table surface is frictionless. The coefficient of static friction between the surfaces of \( M_1 \) and \( M_3 \) is \( \mu_s \). The coefficient of kinetic friction between the surfaces of \( M_1 \) and \( M_3 \) is \( \mu_k \). Assume the system is held initially and then let go, so it starts from rest. Find the acceleration of each mass and the tension in the string.

Find \( m_s \) such that mass 3 does not slip as the system moves.

\[
\begin{align*}
M_2 g - T &= M_2 a \\
M_3 g - N_1 &= M_3 a \\
N_1 - M_3 g &= M_3 a_y = 0
\end{align*}
\]

\[
\frac{M_2 g - T}{M_2} = \frac{M_3 g - N_1}{M_3}
\]

\[
T - \mu_s M_3 g = M_3 a
\]

\[
T = (M_1 + M_3) \mu_s g
\]

\[
T = (3 + 4) \frac{6}{13} 9.8
\]

\[
T = 31.7 \text{ N}
\]