Vortex Line Ordering in the Driven 3-D Vortex Glass

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Outline

• The problem: driven vortex lines with random point pins

• The model to simulate: frustrated XY model with RSJ dynamics

• Previous results

• Our results
  importance of correlations parallel to the applied field

• Conclusions
Driven vortex lines with random point pinning

For *strong pinning*, such that the vortex lattice is *disordered in equilibrium*, how do the vortex lines order when in a driven steady state moving at large velocity?

- Koshelev and Vinokur, *PRL* 73, 3580 (1994)
  
  motion averages disorder $\Rightarrow$ shaking temperature $\Rightarrow$ ordered driven state

- Giamarchi and Le Doussal, *PRL* 76, 3408 (1996)
  
  transverse periodicity $\Rightarrow$ elastically coupled channels $\Rightarrow$ moving Bragg glass

- Balents, Marchetti and Radzihovsky, *PRL* 78, 751 (1997); *PRB* 57, 7705 (1998)
  
  longitudinal random force remains $\Rightarrow$ liquid channels $\Rightarrow$ moving smectic


*We simulate 3D vortex lines at finite $T > 0$.***
3D Frustrated XY Model

\[ \mathcal{H}[\theta_i] = - \sum_{\text{bonds } i \mu} J_{i\mu} \cos(\theta_i - \theta_{i+\mu} - A_{i\mu}) \]

kinetic energy of flowing supercurrents on a *discretized cubic grid*

uniform magnetic field along \( z \) direction; magnetic field is quenched: \( \lambda \to \infty \)

vortex line density \( f = 1/12 \)

*uniform couplings* between \( xy \) planes \( \parallel \) magnetic field

\[ J_{iz} = J_z \]

*random uncorrelated couplings* within \( xy \) planes

\[ J_{i\perp} = J_{\perp} (1 + p \epsilon_{i\mu}) \quad \langle \epsilon_{i\mu} \rangle = 0 \quad \langle \epsilon_{i\mu}^2 \rangle = 1 \]

weakly coupled \( xy \) planes

\[ J_z = J_{\perp} / 40 \]
**Equilibrium Phase Diagram** (from Monte Carlo simulations)

![Equilibrium Phase Diagram](image)

- At low temperature, $p < p_c$, the system is ordered vortex lattice.
- At critical pressure $p = p_c$, the system undergoes a phase transition.
- For $p > p_c$, the system is disordered vortex glass.

We will be investigating *driven steady states* for $p > p_c$.
Driven Steady State Phase Diagram
(from Resistively-Shunted-Junction Dynamics)

\[
I_{total} = \frac{V_{i\mu}}{R} + I_{i\mu}^c \sin(\theta_i - \theta_{i+\mu} - A_{i\mu}) + I_{noise}
\]

Units

- current density: \( I_0 = \frac{2eJ_\perp}{\hbar} \equiv 1 \)
- voltage/length: \( V_0 = RI_0 = \frac{\hbar}{2e\tau} \equiv 1 \)
- time: \( \tau = \frac{\hbar}{2eRI_0} \equiv 1 \)
- temperature: \( T_0 = J_\perp/k_B \equiv 1 \)

apply:
- current density \( I_x \)

response:
- voltage/length \( V_x \)
- vortex line drift \( v_y \)
Previous Simulations

*Domínguez, Grønbech-Jensen and Bishop* - PRL 78, 2644 (1997)

\[ f = \frac{1}{6}, \ 12 \leq L \leq 24, \ J_z = J_\perp, \ \text{weak disorder ??} \]

claim moving Bragg glass - algebraic correlations
vortex lines very dense, system sizes small, lines stiff

*Chen and Hu* - PRL 90, 117005 (2003)

\[ f = \frac{1}{20}, \ L = 40, \ J_z = J_\perp/40, \ \text{weak disorder} \ p \sim 1/2 \ p_c \]

claim moving Bragg glass at large drives with 1\textsuperscript{st} order transition to smectic
single system size, single disorder realization


\[ f = \frac{1}{20}, \ L = 40, \ J_z = J_\perp/40, \ \text{strong disorder} \ p \sim 3/2 \ p_c \]

claim moving Bragg glass at large drives with 1\textsuperscript{st} order transition to smectic
single system size, single disorder realization

We re-examine the nature of the moving state for strong disorder,
\[ p > p_c, \] using finite size analysis and averaging over many disorders
Quantities to Measure

structural

\[ S(k) = \frac{1}{fL_xL_yL_z} \sum_{\mathbf{R},\mathbf{r}} e^{i\mathbf{k} \cdot \mathbf{r}} \langle n_z(\mathbf{R} + \mathbf{r})n_z(\mathbf{R}) \rangle \]

\[ C(x, y, z) = \frac{1}{L_xL_yL_z} \sum_{\mathbf{k}} S(\mathbf{k})e^{-i\mathbf{k} \cdot \mathbf{r}} \]

\[ \tilde{C}(x, k_y, z) = \frac{1}{L_xL_z} \sum_{k_x, k_z} S(\mathbf{k})e^{-i(k_xx + k_yy)} \]

dynamic

use measured voltage drops to infer vortex line displacements
Driven Steady State Phase Diagram \( p = 0.15 > p_c \approx 0.14 \)

- Graph showing the relationship between \( L = 48 \), \( p = 0 \), \( p = 0.15 \), and \( I_x, V_x \)
- Graph depicting \( I = 0.48, L = 48 \), \( p = 0.15 \), and \( E \) vs. \( T \)
- Graph illustrating \( T = 0.05, L = 48 \), \( p = 0.15 \), and \( E \) vs. \( I \)
- Diagram showing vortex line motion \( v_y \)
- Graph of \( \ln S(k_\perp, k_z = 0) \)
Disordered state above 1st order melting $T_m$

When we increase the system size, the height of the peaks in $S(k)$ along the $k_x$ axis do NOT increase $\Rightarrow$ only short ranged translational order.

$\Rightarrow$ disordered state is anisotropic liquid
Ordered state below 1st order melting $T_m$

Bragg peak at $K_{10}$ ⇒ vortex motion is in periodically spaced channels
peak at $K_{11}$ sharp in $k_y$ direction ⇒ vortex lines periodic within each channel
peak at $K_{11}$ broad in $k_x$ direction ⇒ short range correlations between channels

⇒ ordered state is a smectic
Correlations between smectic channels

short ranged translational correlations between smectic channels
Correlations within a smectic channel

Correlations within channel are either long ranged, or decay algebraically with a slow power law $\sim 1/5$
Correlations along the magnetic field

\[ \xi_z \sim 9 \]

As vortex lines thread the system along \( z \), they wander in the direction of motion \( y \) a distance of order the inter-vortex spacing. Such wanderings important for decoupling of smectic planes.
Channels diffuse with respect to one another. Channels may have slightly different average velocities. Such effects lead to the short range correlations along $x$. 

Dynamics

center of mass displacement of vortex lines in each channel

$y_0(t)$ $y_3(t)$ $y_6(t)$ $y_9(t)$
Conclusions

For strong disorder $p > p_c$ (equilibrium is vortex glass)

- Driven system orders above a lower critical driving force
- Driven system melts above an upper critical force due to thermal vortex rings
- 1st order-like melting of driven *smectic* to driven *anisotropic liquid*
- Smectic channels have periodic (algebraic?) ordering in direction parallel to motion, short range order parallel to applied field; channels decouple (short range transverse order)
- Importance of vortex line wandering along field direction for decoupling of smectic channels
- Moving Bragg glass at lower temperature? or finite size effect?
Dynamics and correlations along the field direction $z$

See group of strongly correlated channels moving together.

Smectic channels that move together are channels in which vortex lines do not wander much as they travel along the field direction $z$.

Need lots of line diffusion along $z$ to decouple smectic channels.

As $L_z$ increases, all channels decouple.

Only a few decoupled channels are needed to destroy correlations along $x$. 
Behavior elsewhere in ordered driven state

So far analysis was for $I=0.48$, $T=0.09$ just below peak in $T_m(I)$

Many random realizations have short ranged correlations along $x$.
These are realizations where some channels have strong wandering along $z$.

Many random realizations have longer correlations along $x$; $\xi_x \sim L$
These are realizations where all channels have “straight” lines along $z$.

More ordered state at low $T$? Or finite size effect?
Digression: thermally excited vortex rings

The melting of the ordered state upon increasing current $I$ is due to the proliferation of thermally excited vortex rings from superfluid $^4$He.

$$F(R) \sim R \ln R - IR^2$$

Ring expands when $R > R_c \sim 1/I$.

Rings proliferate when $F(R_c) \sim T \sim 1/I$. 
RSJ details

twisted boundary conditions
\[ \theta_{i+L\mu} - \theta_i = L \Delta \mu \]

voltage/length
\[ V_\mu = \frac{\hbar}{2e} \frac{d\Delta \mu}{dt} \]

new variable with pbc
\[ \tilde{\theta}_i = \theta_i - r_i \cdot \Delta \quad \tilde{\theta}_{i+L\mu} - \tilde{\theta}_i = 0 \]

stochastic equations of motion
\[
\tau \frac{d\tilde{\theta}_i}{dt} = - \sum_{j\mu} G_{ij} \left[ \frac{J_{j\mu}}{J_\perp} \sin(\tilde{\theta}_j - \tilde{\theta}_{j+\mu} - A_{j\mu} - \Delta \mu) + \tilde{n}_{j\mu} \right]
\]
\[
\langle \tilde{n}_{i\mu}(t) \tilde{n}_{j\nu}(t') \rangle = \left( \frac{2k_B T}{J_\perp} \right) \tau \delta_{ij} \delta_{\mu\nu} \delta(t - t')
\]
\[
\tau \frac{d\Delta \mu}{dt} = \frac{1}{L^3} \sum_{i\mu} \frac{J_{i\mu}}{J_\perp} \sin(\tilde{\theta}_i - \tilde{\theta}_{i+\mu} - A_{i\mu} - \Delta \mu) + \xi_i - \frac{I_{i\mu}}{I_0}
\]
\[
\langle \xi_i(t) \xi_j(t') \rangle = \left( \frac{2k_B T}{J_\perp} \right) \frac{\tau}{L^3} \delta(t - t')
\]
Previous results of Chen and Hu $p \sim 1/2 p_c$ weak disorder

**FIG. 4.** Dynamical phase diagram in temperature-current plane. Solid lines with open cycles: 1st order (dynamical) melting from moving BrG to smectic. Dashed lines: continuous phase transition from the moving smectic to liquid.

We will more carefully examine the phases on either side of the 1st order transition

**FIG. 2.** The vortex structure factors just below and above $T_m$ for (a) $I = 0.5$, (b) $I = 0.3$, and (c) $I = 0.05$.

a, b - “moving Bragg glass”

algebraic correlations both

transverse and parallel to motion

a', b' - “moving smectic”

We will more carefully examine the phases on either side of the 1st order transition