Hall effect (1879) - determines the sign of the charges that carry the electric current in a metal.

Electron motion in combined static electric and magnetic fields.

Electric field $E_x$ applied in $\hat{x}$ direction produces flowing electric current $j_x$ in $\hat{x}$ direction. Magnetic field $H$ in $\hat{z}$ direction exerts Lorentz force $\hat{F}_L = \hat{v} \times \hat{H}$ on the moving charges carrying the current $j$. For $\hat{v}$ in $\hat{x}$ direction and $\hat{H}$ in $\hat{z}$ direction, the Lorentz force $\hat{F}_L$ is in the $-\hat{y}$ direction. $\hat{F}_L$ deflects the charge carriers to the side wall of the wire (the shaded wall in the figure) where they build up and create a surface charge density. The surface charge density produces an electric field $E_y$ in $\hat{y}$ direction. For a steady state situation, the force from $E_y$ will exactly cancel out the Lorentz force $\hat{F}_L$.

If $W$ is the width of the wire, then measuring the "Hall voltage" $V_y = E_y \cdot W$ allows one to determine the sign of the charges that carry the electric current.
If current is carried by negative charges $-q$, then

\[ \vec{j}_x = -q m \vec{v}_x \Rightarrow v_x < 0 \]

$E_y$ deflects the mobile negative charges carrying the current and negative charges build up on shaded surface.

Neutralization of system ⇒ absence of negative charge, i.e., positive charge builds up on opposite surface.

The electric field $E_y$ is in $-\hat{y}$ direction and Hall voltage is negative.

If current is carried by positive charges $+q$, then

\[ \vec{j}_x = q m \vec{v}_x \quad v_x > 0 \]

$E_y$ deflects the mobile positive charges carrying the current and positive charge builds up on the shaded $E_y$.

The electric field $E_y$ is in the $+\hat{y}$ direction and the Hall voltage is positive.
For most (but not all) metals one finds a negative Hall voltage. This established that it was negatively charged electrons that carry the electric current in most metals.

**Quantities to measure**

**Hall coefficient**  \( R_H = \frac{E_y}{j_x H} \)

since we expect force from \( E_y \) to exactly balance out Lorentz force \( F_L \) in steady state, we expect \( R_H \) to be independent of \( H \)

**Magnetoresistance**  \( \rho(\gamma) = \frac{E_x}{j_x} \)

We can compute both \( R_H \) and \( \rho \) using the Drude model.

\[
\frac{dp}{dt} = -e \left( E + \frac{p \times H}{mc} \right) - \frac{p}{\tau} = 0 \text{ in steady state}
\]

for \( x \) and \( y \) components

\[
0 = -eEx - \frac{eH}{mc} p_y - \frac{p_x}{\tau}
\]

\[
0 = -eEy + \frac{eH}{mc} p_x - \frac{p_y}{\tau}
\]

\( \omega_c = \frac{eH}{mc} \) cyclotron frequency = angular frequency of a charged particle in circular motion in uniform \( H \)
(1) \[ eE_x = -w_e p_y - \frac{p_y}{c} \]

(2) \[ eE_y = w_e p_x - \frac{p_y}{c} \]

In steady state, current flows only in \( x \) direction. No current flows out the side walls of the wire ⇒ \( p_y = 0 \).

with \( p_y = 0 \),

\[ p_x = -eE_x t \]

\[ j_x = -meu_x = -me \frac{p_x}{m} = \frac{me^2 c}{m} E_x \]

\[ \frac{E_x}{j_x} = \frac{m}{me^2 c} = \frac{1}{\sigma} \]

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Magnetoresistance \( \sigma(H) = \frac{1}{\sigma} = \frac{m}{me^2 c} \)

same as ordinary d.c. resistance \( \sigma \) when \( H = 0 \).

In Drude model, \( \sigma(H) \) is independent of \( H \).

Agreed with early measurements by Drude.

more modern results however do find \( \sigma \) can vary with \( H \).

(2) \[ E_y = \frac{we^2}{e} p_x = -\frac{we^2}{e} E_x \]

Hall coefficient \( R_H = \frac{E_y}{d_x H} = \frac{(\frac{we^2}{e} p_x)}{(-me \frac{p_x}{m})} = -\frac{we^2}{me^2 H} \)

Use \( \frac{we}{mc} = R_H = -\frac{(eH/mc)^2}{me^2 H} = -\frac{1}{me^2 mc} \)
\[ R_H = -\frac{1}{me} \] \hspace{1cm} \text{Hall coefficient independent of } H

But also, \( R_H \) is independent of our phenomenological parameter \( T \), the relaxation time.

\( R_H \) is something we can directly test against experiments, since it only depends on the electron density \( n \), which can be easily calculated.

In practice, \( R_H \) is found to depend on \( H \) and \( T \) and also on sample preparation. But at low \( T \), high \( H \) (\( \sim 10^4 \text{G} \)), and very pure samples, \( R_H \) is found to approach a constant value, often very close to the Drude value.

<table>
<thead>
<tr>
<th>Metal</th>
<th>Charge</th>
<th>(-\frac{1}{R_H \text{mcc}})</th>
<th>((=) for Drude)</th>
</tr>
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<tbody>
<tr>
<td>Li</td>
<td>1</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>K</td>
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<td></td>
</tr>
<tr>
<td>Rb</td>
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</tr>
<tr>
<td>Cs</td>
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</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>Ag</td>
<td>1</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Au</td>
<td>1</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
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<td>-0.2</td>
<td></td>
</tr>
<tr>
<td>Mg</td>
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<td></td>
</tr>
<tr>
<td>In</td>
<td>3</td>
<td>-0.3</td>
<td></td>
</tr>
<tr>
<td>Al</td>
<td>3</td>
<td>-0.3</td>
<td></td>
</tr>
</tbody>
</table>

Drude prediction very good for alkali metals which have single \( s \) shell electron as valence electron

Sign is negative! \( \Rightarrow \) current is carried by objects with positive sign
a.c. electric conductivity

\[ E(t) = \text{Re} \left[ E_0 e^{-i\omega t} \right] \]  
harmonic oscillating electric field

\[ \frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} - e\vec{E}(t) \]  
Dufay's eq. of motion

assume solution is also harmonic oscillation

\[ \vec{p}(t) = \text{Re} \left[ P_0 e^{-i\omega t} \right] \]

\[ -i\omega \vec{p}_0 = -\frac{\vec{p}_0}{\tau} - e\vec{E}_0 \]

\[ (\frac{1}{\tau} - i\omega) \vec{p}_0 = -e\vec{E}_0 \]

\[ \vec{p}_0 = -\frac{e}{\frac{1}{\tau} - i\omega} \vec{E}_0 = -\frac{e\tau}{1 - i\omega} \vec{E}_0 \]

current

\[ \vec{j}(t) = \text{Re} \left[ j_0 e^{-i\omega t} \right] \]

\[ \vec{j} = -en \vec{V} \]

\[ \vec{j}_0 = -en \frac{\vec{p}_0}{m} \]

\[ \vec{j}_0 = -\frac{me^2}{m(1 - i\omega\tau)} \vec{E}_0 \]

a.c. conductivity

\[ \vec{j}_0 = \sigma(\omega) \vec{E}_0 \]

\[ \Rightarrow \sigma(\omega) = \frac{me^2}{m(1 - i\omega\tau)} = \frac{\sigma_{dc}}{1 - i\omega\tau} \]
where \( \sigma_{dc} = \frac{me^2c}{m} \) is dc. Drude conductivity.

As \( \omega \to 0 \), \( \sigma(\omega) \to \sigma_{dc} \)

As \( \omega \to \infty \), \( \sigma(\omega) \to \frac{me^2c}{i\omega mc} \) and \( \sigma(\omega) \) for \( \omega \gg 1 \), i.e. \( \omega \gg \frac{1}{c} \), oscillation is fast compared to collision rate, so \( \sigma(\omega) \) becomes independent of \( \omega \).

Electromagnetic wave propagation in a metal

Approx 1) In cgs units, for a propagating electromagnetic plane wave \( |E| = |H| \).

So for the forces the EM wave exerts on a conduction electron

\[
\frac{|F_{mag}|}{|F_{elec}|} = \frac{e|\vec{H} \times \vec{H}|}{e|E|} \approx 1 \quad \omega \ll 1
\]

So we will ignore the force that the \( \vec{H} \) component of the wave exerts on the electron.

Approx 2) When wavelength \( \lambda \) of EM wave is much larger than mean free path \( l \) of collisions, \( \lambda \gg l \), the electric field that an electron sees over the time between collisions is roughly uniform in space. Good for waves in visible spectrum where \( \lambda \approx 5000 \text{ Å} \), \( l \approx 10 \text{ Å} \).

(1) + (2) \( \Rightarrow \) we can use the above computed a.c. conductivity \( \sigma(\omega) \) to find the relation...