1) [60 points]

Consider a hollow spherical shell of radius $R$, with a fixed electrostatic potential on its surface given by $\phi(R, \theta) = \phi_0 \cos^2 \theta$. Here $\theta$ is the usual polar angle in spherical coordinates, and the reference point is taken so that $\phi(r \to \infty) = 0$.

\[ \phi(R, \theta) = \phi_0 \cos^2 \theta \]

a) Find the electrostatic potential $\phi(r)$ both inside and outside the shell. [30 points]

b) What is the surface charge density $\sigma(\theta)$ on the surface of the sphere? [15 points]

c) What is the total amount of charge on the sphere? If we put the origin at the center of the sphere, does the charge distribution have an electric dipole moment? an electric quadrupole moment? [15 points]

Hint: the first few Legendre polynomials are,

\[ P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = (1/2)(3x^2 - 1), \quad P_3(x) = (1/2)(5x^3 - 3x) \]

2) [40 points]

Consider a point charge $q$ a distance $d$ in front of a plane conducting slab of thickness $w$, as shown below. The slab has a fixed net charge $Q$ on it.

a) Find the electric field on both sides of the slab. [25 points]

b) Find the force between the charge $q$ and the slab. [15 points]

For this problem, assume that the side area of the slab $A$ is finite, so that the average surface charge $Q/2A$ is finite; however you make work the problem out ignoring edge effects, i.e. assuming the plane is effectively infinite.