1) [50 points total] You must explain your answer completely for all parts!

A box is partitioned by a wall into two parts, the right side and the left side. Each side is filled with an identical classical ideal gas of non-interacting, non-relativisitic, indistinguishable point particles of mass $m$. The gas on each side of the box has the same number of particles $N$. The volume of the left side of the box is $V_1$ and the volume of the right side of the box is $V_2$. The wall separating the two sides of the box is adiabatic (does not conduct heat), immoveable, and non-porous. Initially the gas on the left side is in equilibrium at temperature $T_1$, while the gas on the right side is in equilibrium at temperature $T_2$. The wall is now removed and each gas is free to fill the entire volume. The system then comes into its new state of equilibrium.

a) [5 pts] Is the final entropy of the total system larger or smaller than the initial total entropy?

b) [10 pts] What is the final temperature $T_f$ of the gas?

c) [15 pts] What is the final total pressure $p_f$ of the gas? Express $p_f$ only in terms of $T_1$ and $T_2$ and the initial pressures of the two sides $p_1$ and $p_2$. If initially we had $T_1 = T_2$, is $p_f$ greater or smaller than the average initial pressure $\bar{p} = (p_1 + p_2)/2$?

d) [20 pts] Compute the change in total entropy $\Delta S$ that results from removing the wall. Express your answer in terms of the variables $T_1$, $T_2$, $V_1$, $V_2$ and $N$ only. Show that $\Delta S = 0$ if $V_1 = V_2$ and $T_1 = T_2$.

2) [50 points total]

Consider a classical ideal gas of $N$ non-interacting, non-relativisitic, indistinguishable atoms of mass $m$, confined to a box of volume $V$ and in equilibrium at temperature $T$. Each atom $i$ has a net spin $s_i$ which can be in one of three possible states, $s_i = -1, 0, +1$. The magnetic moment produced by this spin interacts with an external magnetic field $h = h\hat{z}$ giving a contribution to the atom’s energy, $\epsilon_i = -\mu h s_i$.

a) [15 pts] Find the canonical partition function $Q_N(T, V, h)$.

b) [5 pts] Find the pressure $p$ as a function of $T, V, N$ and $h$.

c) [10 pts] Find the average total energy $\langle E \rangle$ of the gas as a function of $T, V, N$ and $h$.

d) [10 pts] Find the average total magnetization $\langle M \rangle = \mu \sum_{i=1}^{N} \langle s_i \rangle$ of the gas as a function of $T, V, N$ and $h$.

e) [5 pts] Find the chemical potential $\mu$ of the gas as a function of $T, V, N$ and $h$. 