1 Define the creation-annihilation operators by
\[ a^\dagger |n > = \sqrt{(n+1)} |n+1 > \quad \text{for} \quad n = 0, 1, \cdots, \]
\[ a |n > = \sqrt{n} |n-1 > \quad \text{for} \quad n = 1, 2, \cdots, \]
and \[ a |0 > = 0. \] Here the collection of vectors \(|n >, n = 0, 1, \cdots\) is an orthonormal basis.

1.1 For any complex number \(z\), find an eigenvector for \(a\) with eigenvalue \(z\), as a linear combination \(\sum_{n=0}^{\infty} c_n(z) |n >\).

1.2 Find the length of the eigenvector by evaluating the sum \(\sum_{0}^{\infty} |c_n(z)|^2\) and use that to find an eigenvector of unit length.

2 Consider the matrix
\[ A = \begin{pmatrix} 1 & -i & 0 \\ i & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \]

2.1 What are its eigenvalues and eigenvectors?

2.2 Find its resolvent \(R(z) = (A - z)^{-1}\) and verify that the positions of its poles are the eigenvalues of \(A\).