PHY100 — The Nature of the Physical World

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Lecture 4
Fields, Relativity

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RECAP NEWTON & COULOMB

$$F_G = G\frac{Mm}{d^2} \qquad\qquad F_e = k\frac{Qq}{d^2}$$

1) BOTH EQUATIONS HAVE A SIMILAR FORM

2) FORCE IS PROPORTIONAL TO THE PRODUCT OF THE QUANTITY THAT CAUSES THE FORCE: CHARGE FOR COULOMB, MASS FOR NEWTON.

3) BOTH FORCES DIMINISH AS d^2

4) BOTH "ACT AT A DISTANCE": THEY ARE LONG RANGE

5) F_e CAN BE REPULSIVE, BUT F_G IS ALWAYS ATTRACTIVE

6) BOTH ARE DIRECTED TOWARDS THE CENTER OF THE "SOURCES"

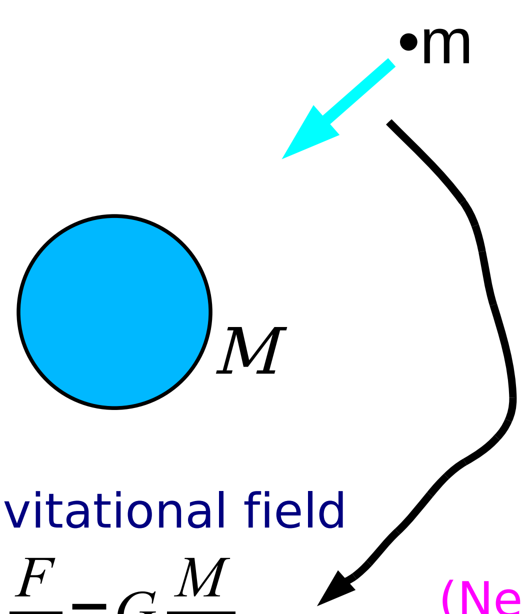
7) k IS MUCH LARGER THAN G. A UNIT OF CHARGE WILL ATTRACT A UNIT OF CHARGE WITH SIGNIFICANTLY MORE FORCE THAN A UNIT OF MASS ATTRACTS A UNIT OF MASS.

$\frac{F_e}{F_G} = \frac{q_1 q_2}{m_1 m_2} \cdot \frac{k}{G} = 4\times10^{42}$ (FOR ELECTRONS: $\frac{q}{m} \sim 2\times10^{11}\,\frac{C}{kg}$) COLOSSAL DIFFERENCE!

What is the essence of a force?

Gravitational Field

At each point in space, gravitational Force/mass (magnitude plus direction) that would be felt by a little test mass at that point



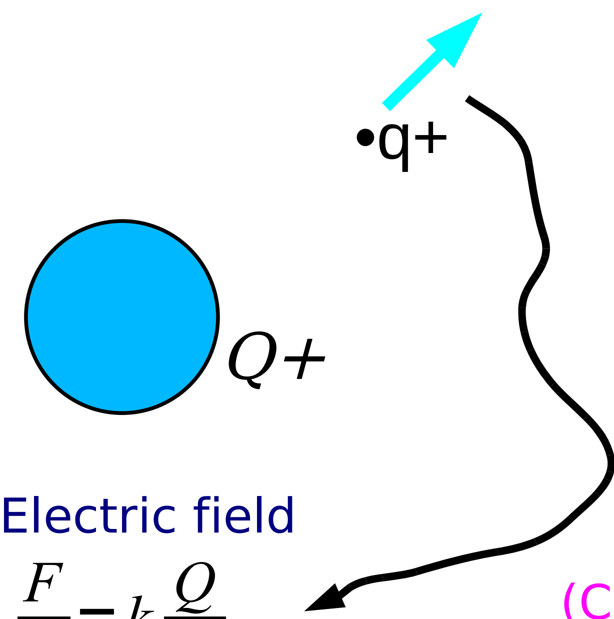
Gravitational field

$$\frac{F}{m} = G\frac{M}{d^2}$$

(Newton)

Electric Field

At each point in space, electric Force/charge (magnitude plus direction) that would be felt by a little test charge at that point



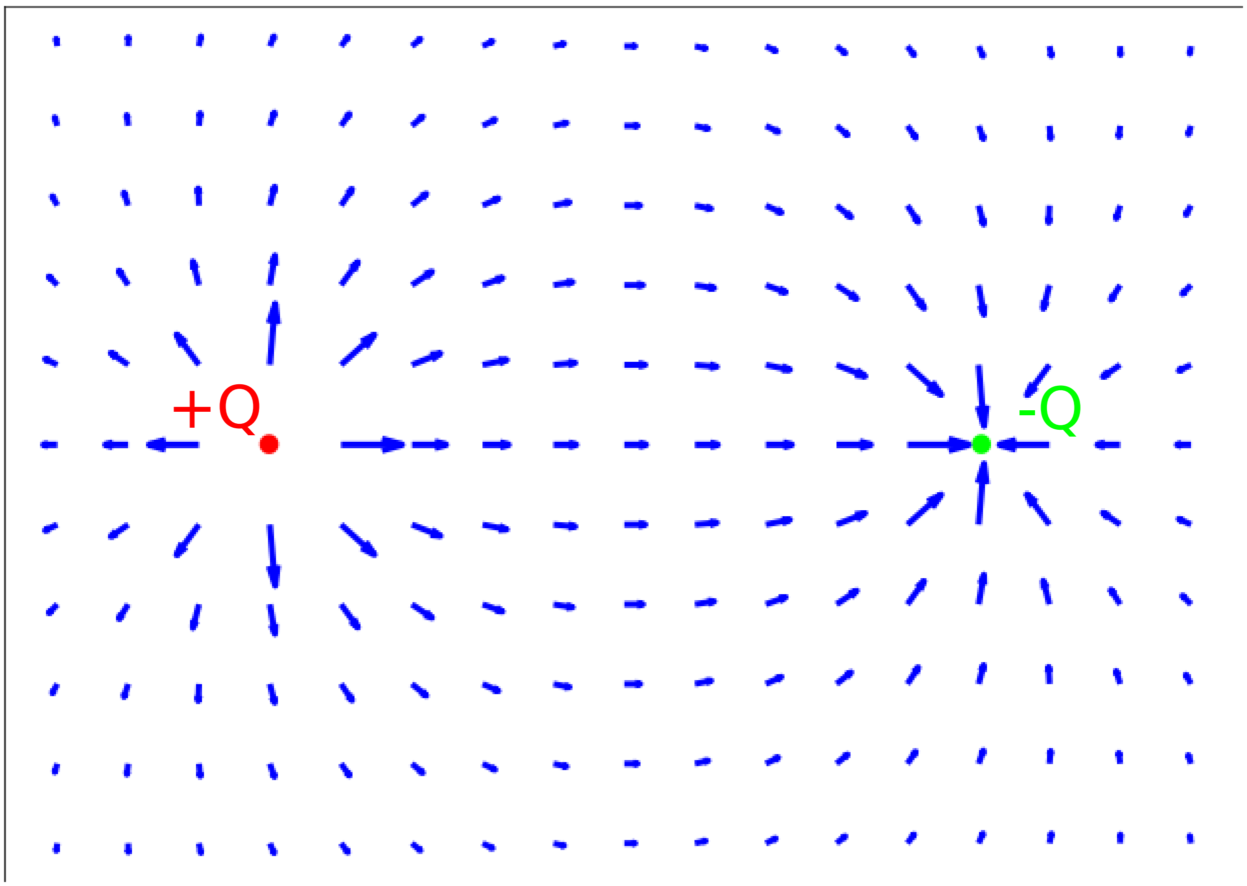
Electric field

$$\frac{F}{q} = k\frac{Q}{d^2}$$

(Coulomb)

Electric field between two charges

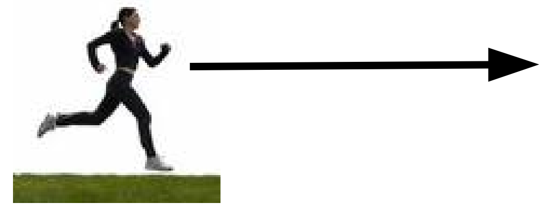
Demo of charges and fields



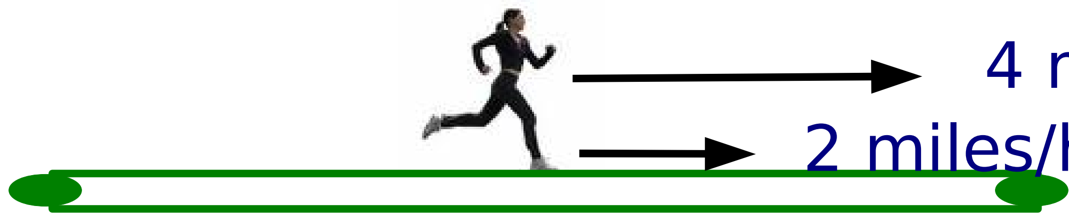
Lines of force: a visualization of the field
Remember the wind map

Galilean relativity

Velocities add: it's just common sense!



Speed with respect to you is 4 miles/hour



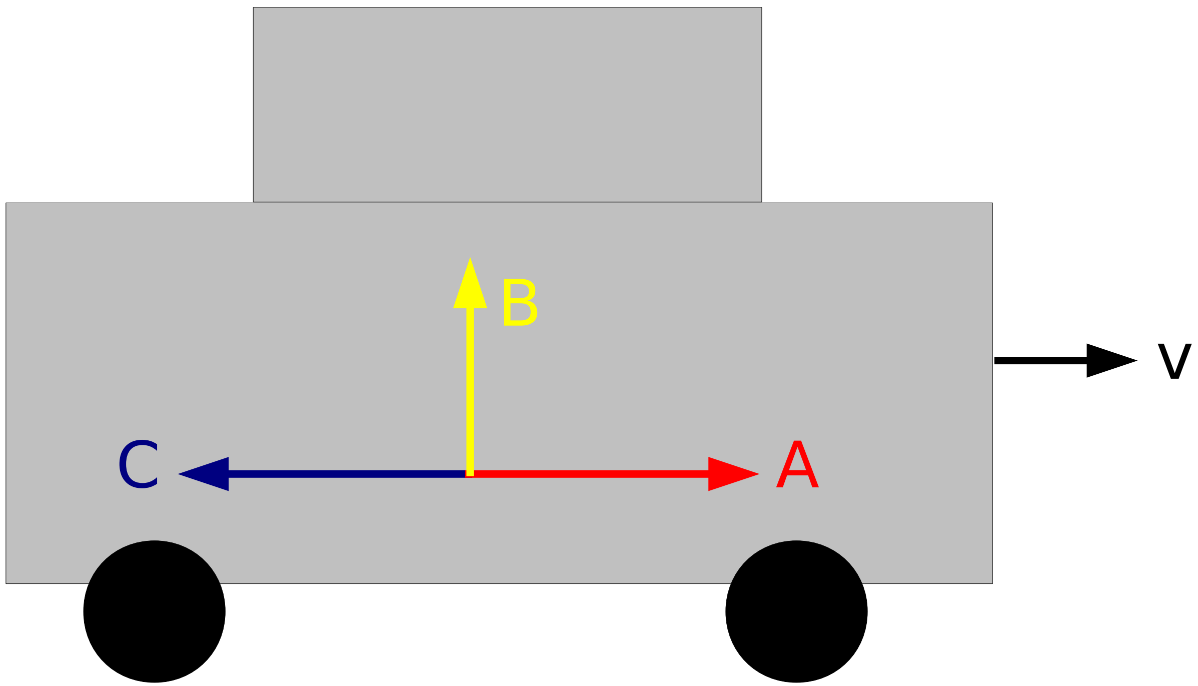
Speed with respect to you is 2+4=6 miles/hour

Implication: If you are in an airplane that moves at constant velocity, there is no way to tell if the airplane is at rest or moving. Everything looks the same on a plane in steady motion as it does on a plane at rest.

But you can tell when the plane is accelerating or turning.

Now with light

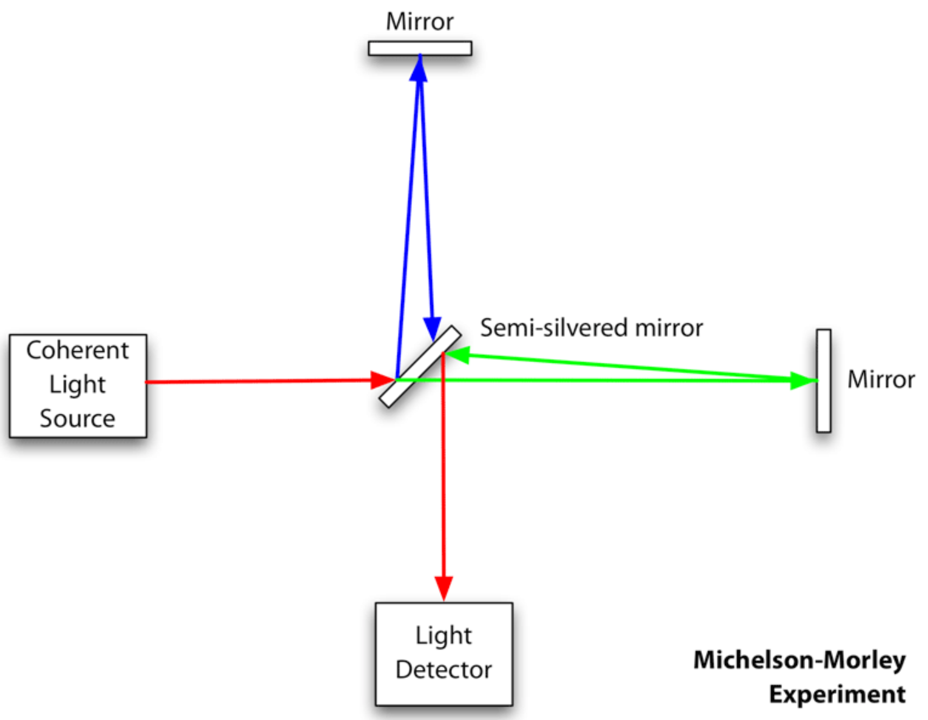
A car moves with velocity v and a passenger shines a flashlight in three directions.
The speed of light is greater for beam A, B or C?

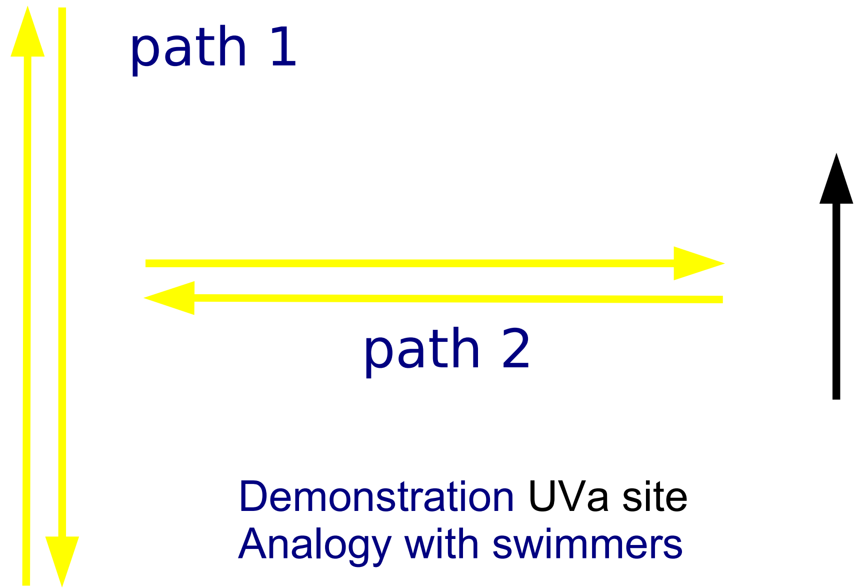


Experiment says:
the speed of light is the same in all directions!!

Michelson-Morley (1887)

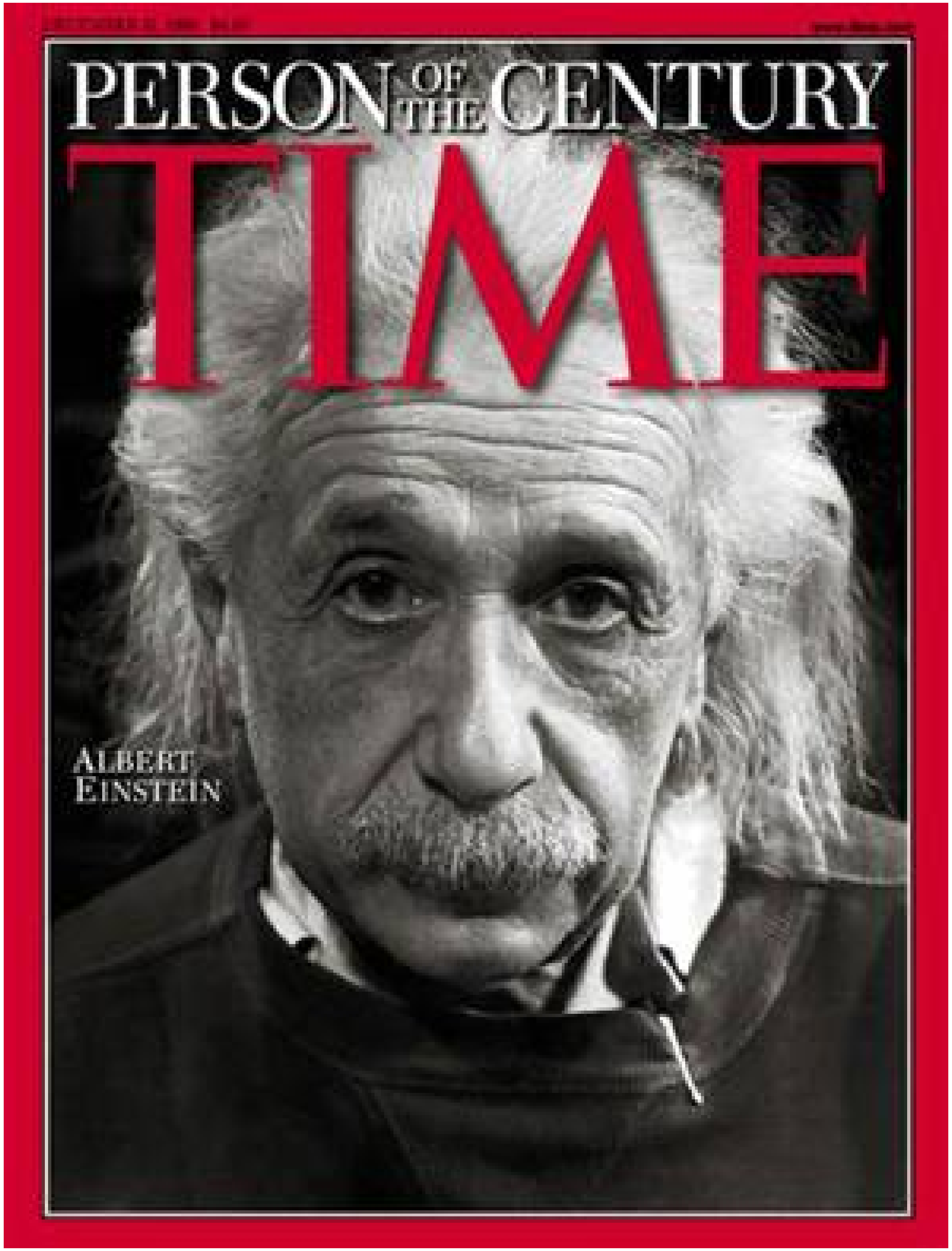
- Wanted to prove that light travels through “aether”: a pervasive substance that carries the light
- Idea was to measure the speed of light traveling upwind of this aether (when Earth glides along) and then downwind (in the opposite direction): the difference should be twice the wind speed.
- Not easy! The speed of light is much much faster than the aether wind (Earth speed in orbit ~20 miles/second ; speed of light ~200,000 miles/second) Need to measure 1 part in 10,000
- They devised a race for two beams of light:





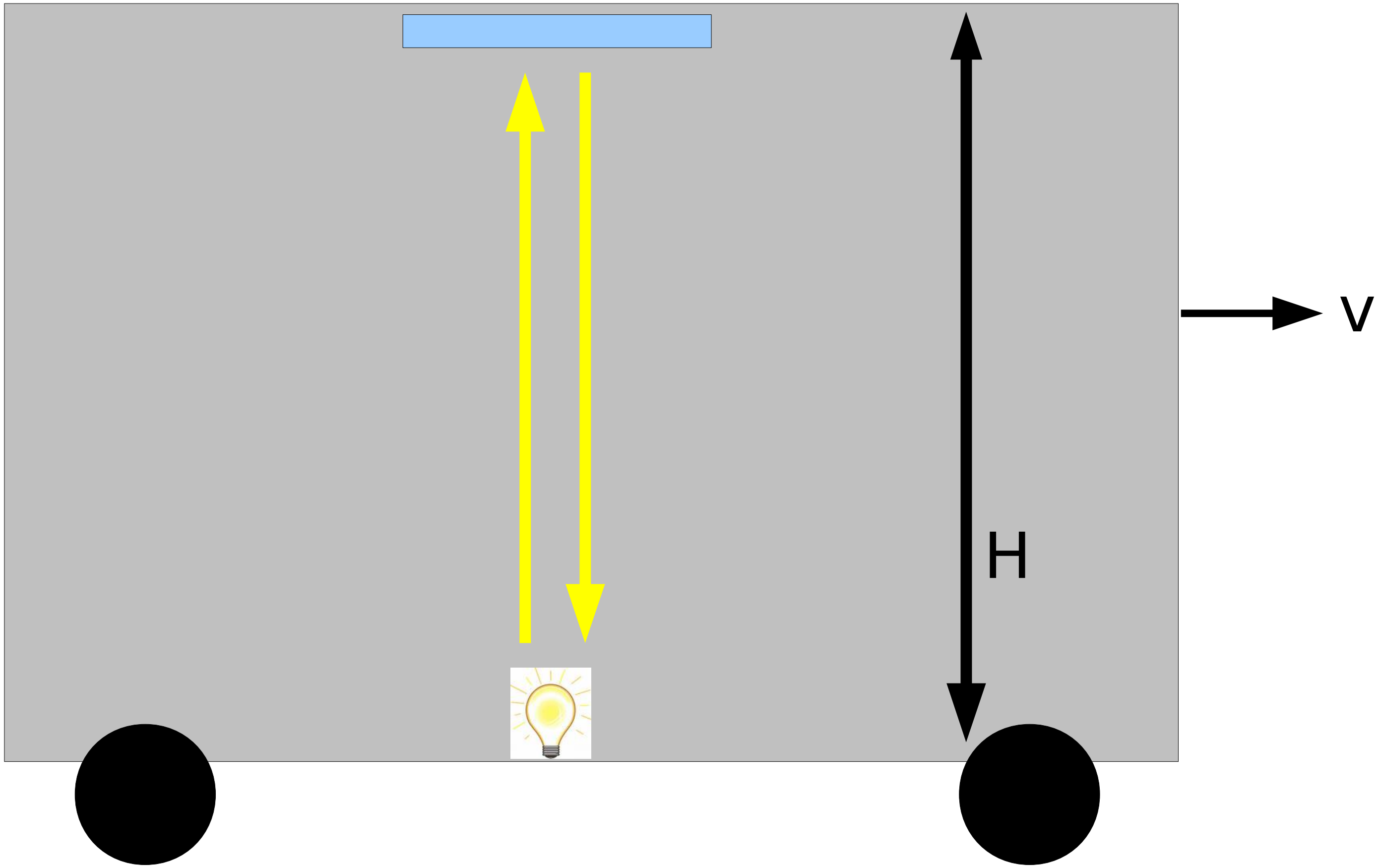
Demonstration UVa site
Analogy with swimmers

Enter our man Einstein!



Einstein thought experiment

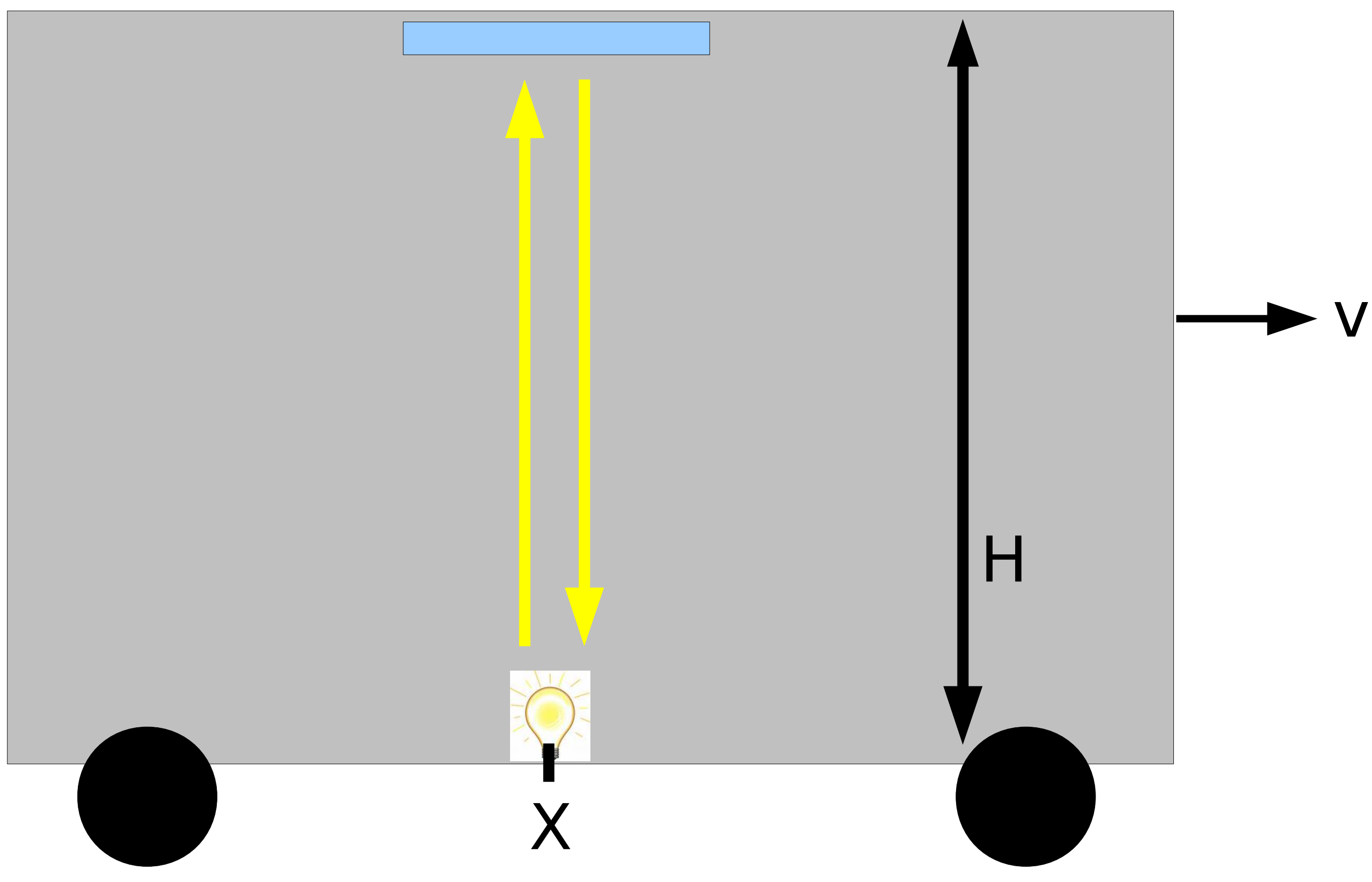
Consider a train moving at constant speed.
A beam of light is emitted from the floor of the wagon and bounces off a mirror on the ceiling and returns to the point on the floor where it was emitted



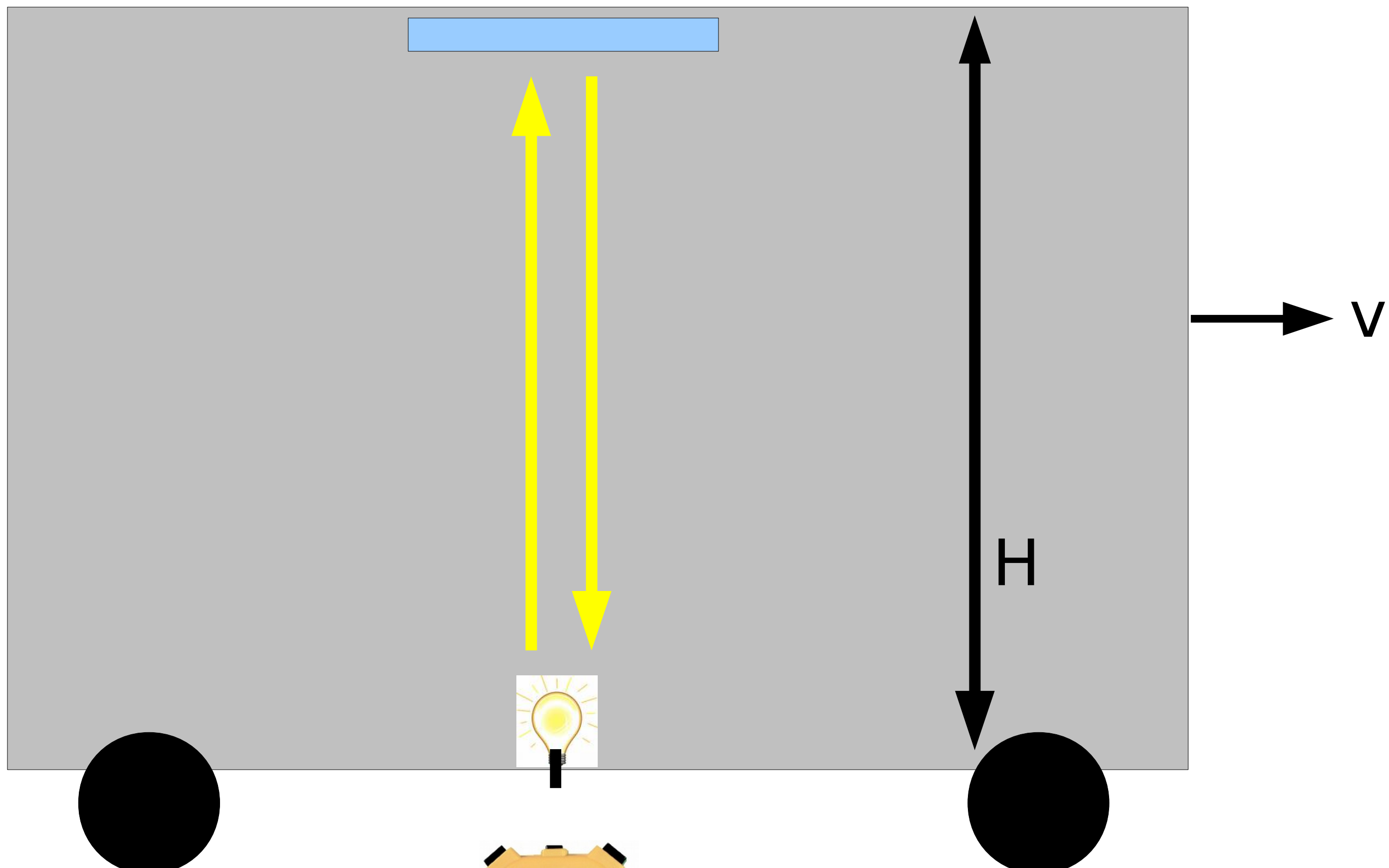
What do we see?

Fact: Light is emitted and detected in point X

This must be true no matter who makes the measurement!



Sam is on the train



Sam measures T_S

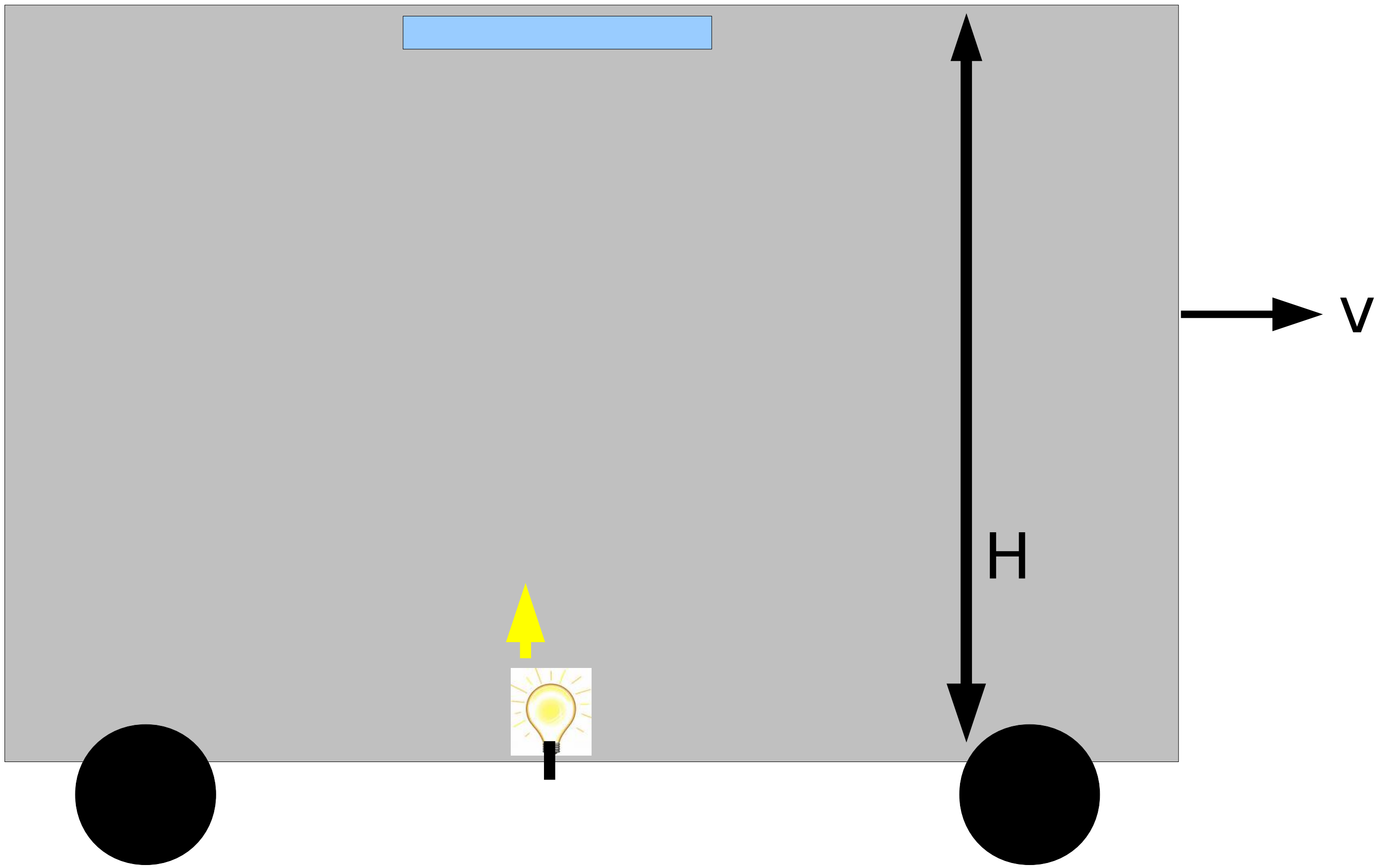
Velocity of light = c

c = distance / time

$c = 2H/T_S$

$T_S = 2H/c$

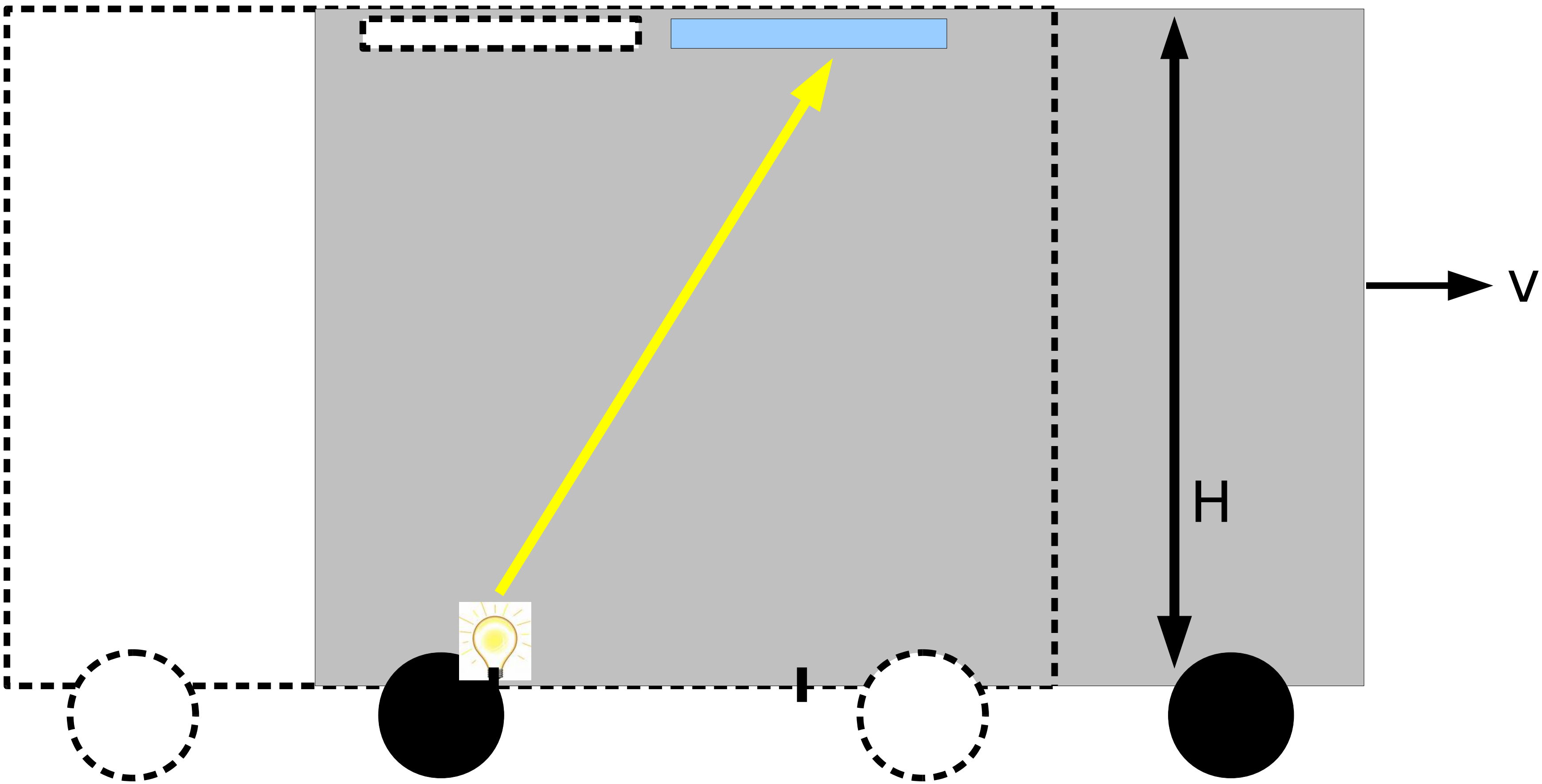
Alice watches the train pass from the platform



Alice starts her stopwatch

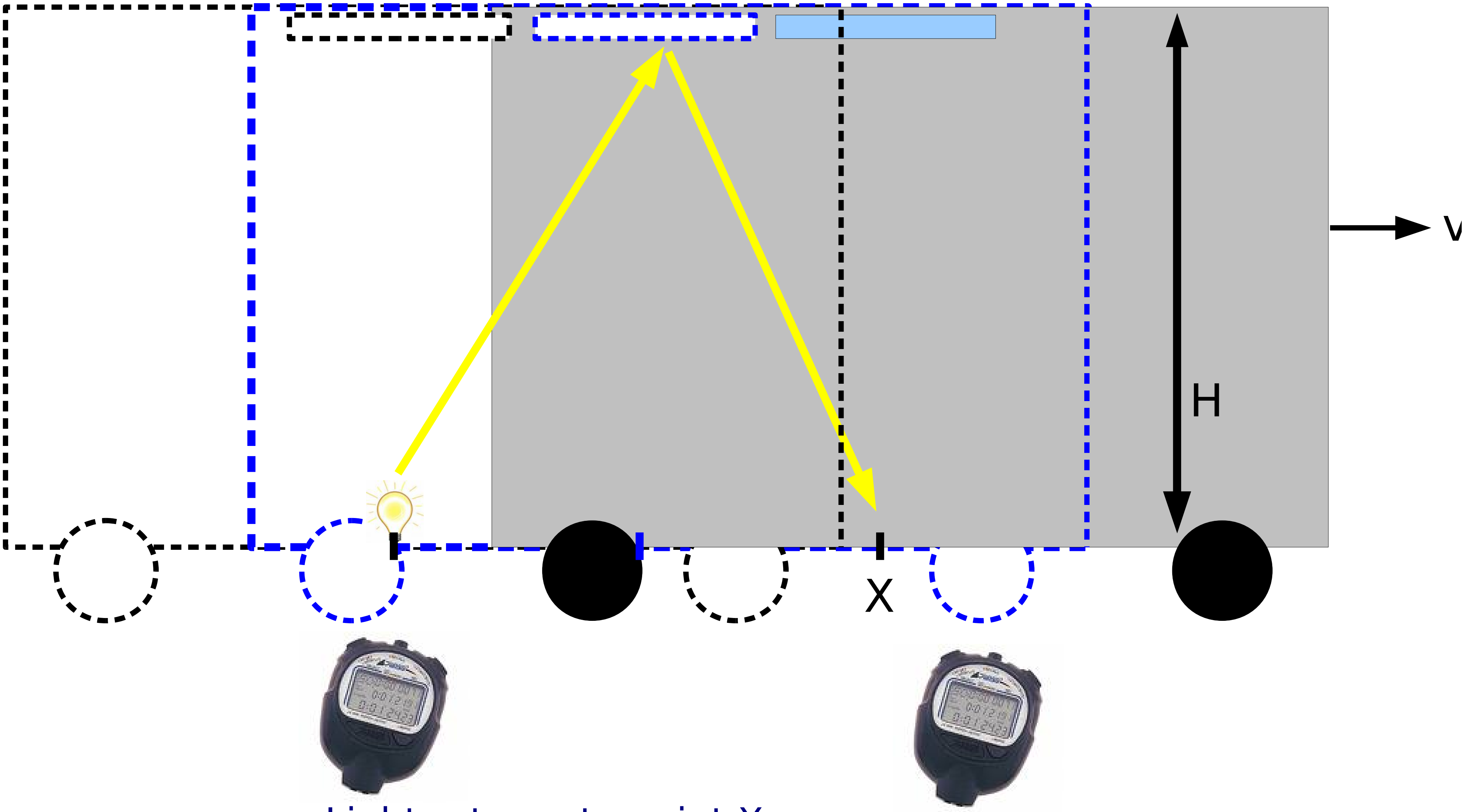
Light is emitted

Alice watches the light bounce



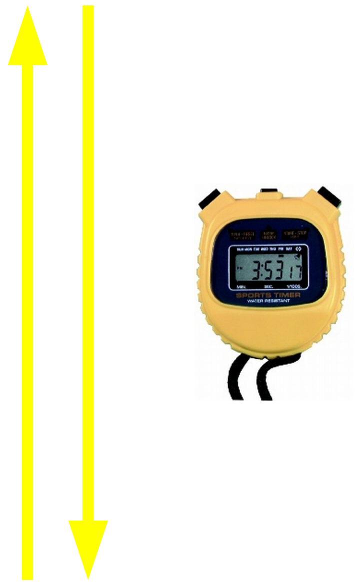
Light bounces on mirror

Alice measures stopwatch



Light returns to point X
Alice also sees the light return to X
Visual demo

Sam and Alice: comparison



Sam

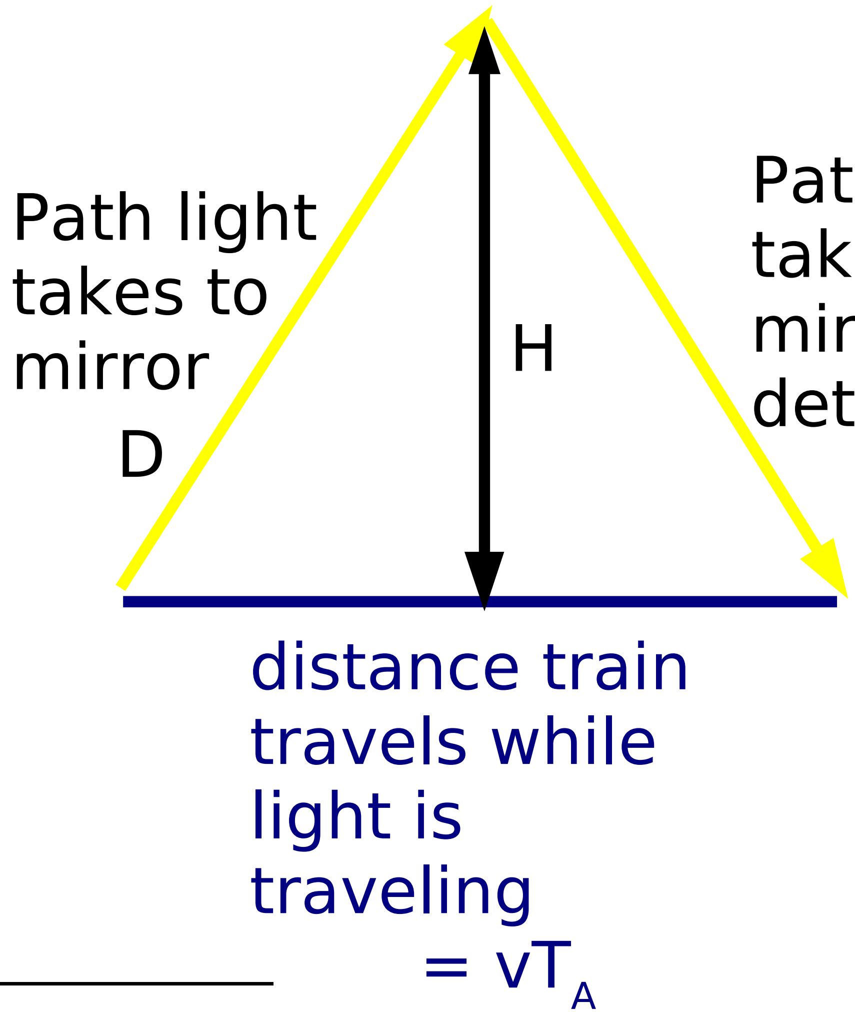
Alice

Alice sees the light traveling further

If light travels at constant speed, the same “event” must seem to take longer to Alice than Sam: $T_A > T_S$

Time is relative... not absolute!

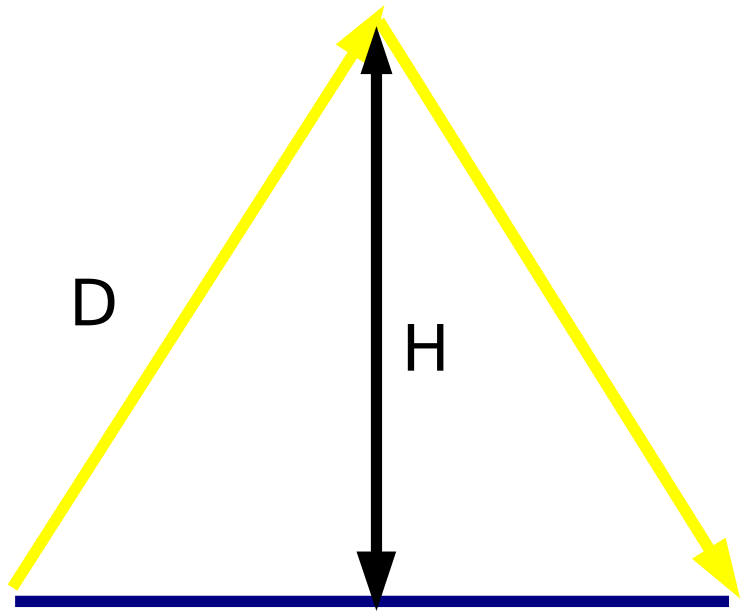
Alice's point of view



$$D = \sqrt{H^2 + \left(\frac{1}{2} vT_A\right)^2}$$

Makes use of Pythagoras theorem

Alice's point of view



c = distance/time = $2D/T_A$

$T_A = 2D/c$

A little algebra

Sam (on train)

$2H/T_S = c$

Alice (on ground)

$c = 2D/T_A$

$$c = \frac{2}{T_A}\sqrt{H^2 + (\frac{1}{2} vT_A)^2}$$

$$\frac{2H}{T_S} = \frac{2}{T_A}\sqrt{H^2 + (\frac{1}{2} vT_A)^2} \rightarrow \left(\frac{2H}{T_S}\right)^2 = \left(\frac{2H}{T_A}\right)^2 + \left(\frac{2}{T_A}\right)^2 \left(\frac{1}{2} vT_A\right)^2$$

$$\rightarrow \left(\frac{2H}{T_S}\right)^2 = \left(\frac{2H}{T_A}\right)^2 + v^2 \rightarrow \left(\frac{1}{T_S}\right)^2 = \left(\frac{1}{T_A}\right)^2 + \frac{v^2}{(2H)^2}$$

And finally...

$$\rightarrow \left(\frac{1}{T_S}\right)^2 = \left(\frac{1}{T_A}\right)^2 + \frac{v^2}{(2H)^2} \qquad \text{But} \quad 2H = cT_S$$

$$\left(\frac{1}{T_S}\right)^2 = \left(\frac{1}{T_A}\right)^2 + \frac{v^2}{(cT_S)^2} \qquad \rightarrow \quad c^2 = \frac{c^2 T_S^2}{T_A^2} + v^2$$

$$\boxed{T_A = \left[\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right] T_S}$$

And finally...

$$\rightarrow \quad \left(\frac{1}{T_S}\right)^2 = \left(\frac{1}{T_A}\right)^2 + \frac{v^2}{(2H)^2} \qquad \text{But} \quad 2H = cT_S$$

$$\left(\frac{1}{T_S}\right)^2 = \left(\frac{1}{T_A}\right)^2 + \frac{v^2}{(cT_S)^2} \qquad \rightarrow \quad c^2 = \frac{c^2 T_S^2}{T_A^2} + v^2$$

$$T_A = \left[\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right] T_S$$

This number is > 1
It becomes larger as v approaches c

And it all started because we imposed that c, the speed of light, is the same number for Sam and Alice!

Wow!

- Sam and Alice measure the time interval for the same event
- The only difference between Sam and Alice is that one is moving with respect to the other
- Yet, $T_{Alice} > T_{Sam}$
- Alice sees Sam's clock taking more time to tick: longer path → his clock runs slower
- The same event takes a different amount of time depending on your “reference frame”

Time is not absolute! It is relative!

Is this true?

- Yes! Experiments confirm it

Ground

Plane

Is this true?

- Yes! Experiments confirm it

Ground

Plane

Is this true?

- Yes! Experiments confirm it

Ground

Plane

Twin paradox

$$T_{Earth} = \left[\frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \right] T_{spaceship}$$

"proper time"

- A spaceship leaves Earth with Sam on it and moves with speed v=0.98c
- Alice remains on Earth
- When Sam gets back he has aged 50 years
- How much has Alice aged?

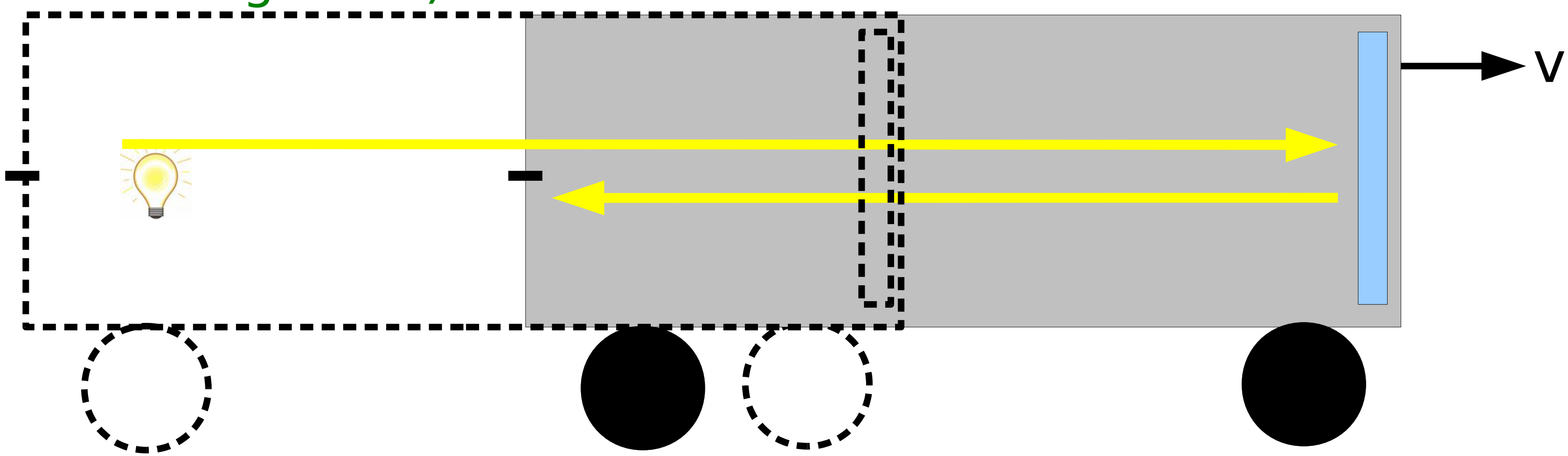
$$T_{Earth} = \left[\frac{1}{\sqrt{1 - \left(\frac{0.98c}{c}\right)^2}} \right] 50\text{years} = [5]50\text{years} = 250\text{years}$$

Length is relative too

- Time dilation ↔ length contraction
- Measure the length of the wagon, when you are on it (Sam):

$$L_A = \sqrt{1-\left(\frac{v}{c}\right)^2}\, L_S$$

- And now when the wagon moves past you (Alice, at rest in the ground):

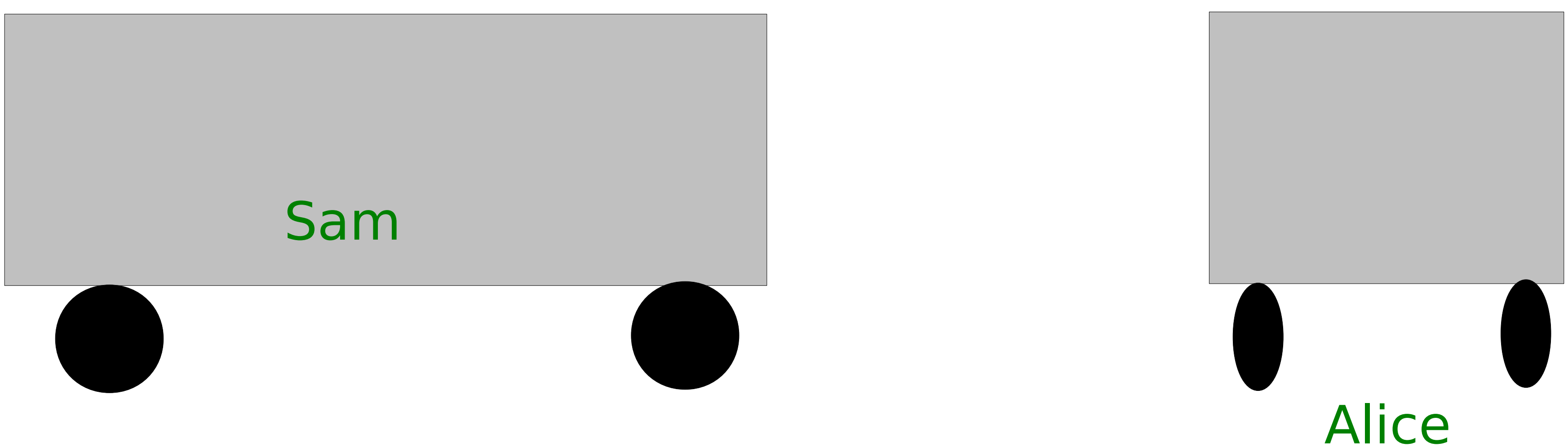


Length is relative too

- Time dilation ↔ length contraction
- For a measurement of the length of the wagon (along the same direction of motion of the wagon):

$$L_A = \sqrt{1-\left(\frac{v}{c}\right)^2}\,L_S$$

Smaller than 1



- Alice sees the wagon shorten in length (not in height): link

Einstein's special relativity

1) The Laws of Physics are the same in all Inertial Frames (Galileo)
2) Thus, any measurement of the speed of light in any inertial frame will always give 186,300 miles per second