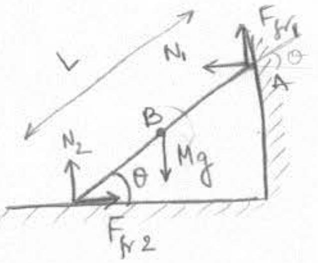
Problem

Stick is at rest



Force balance

$N_1 = F_{fr2}$ (horiz)

$Mg = N_2 + F_{fr1}$ (vertical)

Torque balance about center B (just choose a pt about which you want to balance Torque) All pt selection will be equivalent.

$$F_{fr1}\left(\frac{L}{2}\cos\theta\right) + N_1\left(\frac{L}{2}\sin\theta\right) + F_{fr2}\left(\frac{L}{2}\sin\theta\right) = N_2\left(\frac{L}{2}\cos\theta\right) \quad \text{— (i)}$$

Torque balance about A.

$$Mg\left(\frac{L}{2}\cos\theta\right) + F_{fr2}\left(L\sin\theta\right) = N_2\left(L\cos\theta\right) \quad \text{— (ii)}$$

Are these two equivalent? Yes!

start with (i)

$$(Mg - N_2)\left(\frac{L}{2}\cos\theta\right) + N_1\left(\frac{L}{2}\sin\theta\right) + F_{fr2}\left(\frac{L}{2}\sin\theta\right) = N_2\left(\frac{L}{2}\cos\theta\right)$$

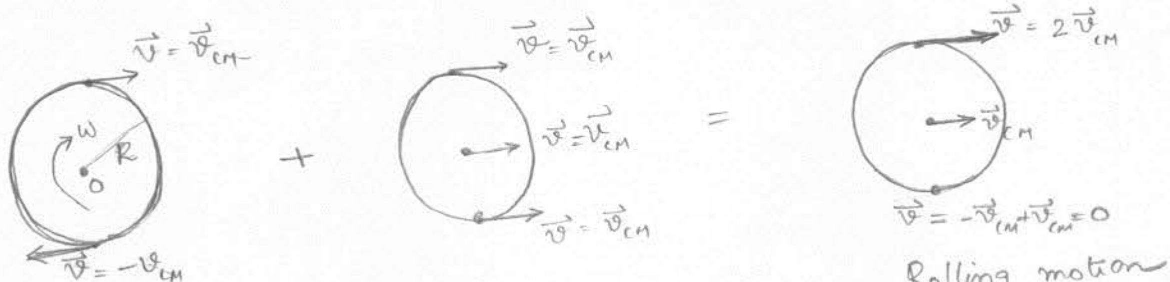
$$Mg\left(\frac{L}{2}\cos\theta\right) + F_{fr2}\left(\frac{L}{2}\sin\theta\right) + F_{fr2}\left(\frac{L}{2}\sin\theta\right) = \cancel{2}N_2\left(\frac{L}{\cancel{2}}\cos\theta\right)$$

use $N_1 = F_{fr2}$

$$Mg\left(\frac{L}{2}\cos\theta\right) + F_{fr2}\left(L\sin\theta\right) = N_2\left(L\cos\theta\right)$$

Same as (ii).

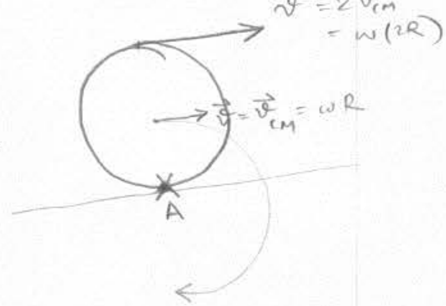
Rolling Motion



Pure rotation — Pure translation — Rolling motion

$$S = \theta R \Rightarrow v_{CM} = \omega R \text{ (smooth rolling motion)}$$

Rolling as pure rotation



It is like a pure rotation about the point of contact A with ω ang. velo.
(the pt A has zero velocity)

Kinetic energy of a rolling wheel of radius R and mass M.

first picture: $K = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I_{CM}\omega^2 = \frac{1}{2} M(\omega R)^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2$

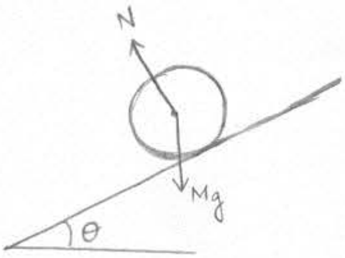
second picture: (pure rotation)

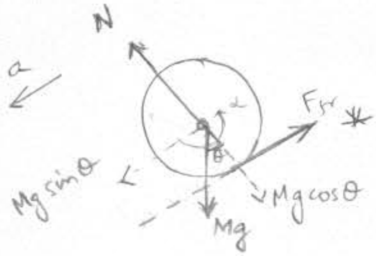
$$K = \frac{1}{2} I_A \omega^2 = \frac{1}{2}(I_{CM} + MR^2)\omega^2 = \frac{1}{2}(MR^2 + MR^2)\omega^2 = MR^2\omega^2$$

Both pictures give same K (as expected).

A rolling object has two types of KE, translational K.E. (motion of CM) and rotational KE (rotation about CM).

<u>Rolling Barrel</u>. (cylinder)





$N = Mg \cos\theta$

$Mg\sin\theta - F_{fr} = Ma$

$\tau = I\alpha = F_{fr} R$

$\alpha = \frac{a}{R}$ (pure rolling)

$$F_{fr} = \frac{I\alpha}{R} = \frac{1}{2}M\cancel{R^2}\frac{a}{\cancel{R^2}} = \frac{1}{2}Ma$$

$$F_{fr} = M(g\sin\theta - a)$$

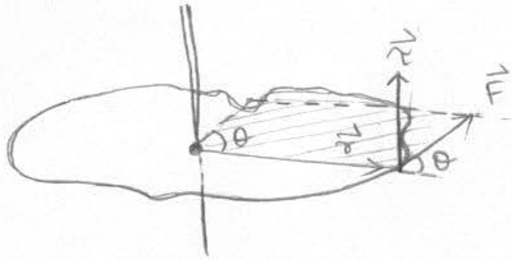
$$\frac{1}{2}Ma = Mg\sin\theta - Ma \Rightarrow \frac{3}{2}\cancel{M}a = \cancel{M}g\sin\theta \Rightarrow \boxed{a = \frac{2}{3}g\sin\theta}$$

and $F_{fr} = \frac{1}{2}M\,\frac{2}{3}g\sin\theta$

$$\boxed{F_{fr} = \frac{1}{3}Mg\sin\theta}$$

* Why did we choose the F_{fr} in upward direction?
If the body was to slide <u>down</u> the ramp, the frictional force had to work up the ramp.

Torque. geometrical picture.



$$\vec{\tau} = \vec{r} \times \vec{F}$$

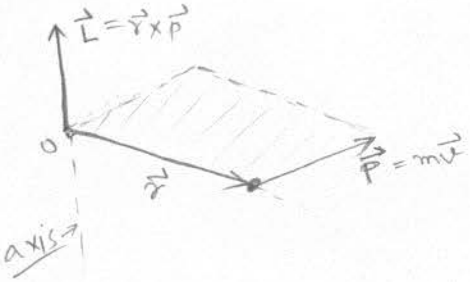
$\tau = rF\sin\theta$ (Area of shaded ||gm)

$\vec{\tau}$ is directed $\perp$ to the plane containing $\vec{r}$ & $\vec{F}$; which is the direction of the area vector of the ||gm (shaded)

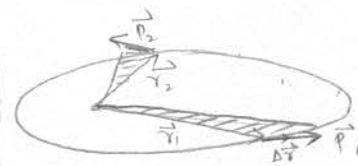
Angular momentum.

$$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

Right hand rule



Kepler's Second Law



$$\vec{L}_i = \vec{r}_1 \times \vec{p}_1$$

$$\vec{L}_f = \vec{r}_2 \times \vec{p}_2$$

$$\vec{L} = \text{const} \Rightarrow \vec{r}_1 \times \vec{p}_1 = \vec{r}_2 \times \vec{p}_2$$

$$\therefore \cancel{m}(\vec{r}_1 \times \vec{v}_1) = \cancel{m}(\vec{r}_2 \times \vec{p}_2)$$

$$\vec{r}_1 \times \frac{\Delta\vec{r}}{\Delta t} = \vec{r}_2 \times \frac{\Delta\vec{r}}{\Delta t}$$

$\left(\text{use } \vec{v} = \frac{\Delta\vec{r}}{\Delta t}\right)$

$\boxed{\Delta A_1 = \Delta A_2}$ Kepler's Second Law.

Newtons Second law

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \quad \text{(single particle)}$$

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

$$\left[\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \underbrace{\vec{v} \times m\vec{v}}_{0} + \vec{r} \times \vec{F} = \vec{\tau}_{net} \right]$$

Angular momentum of a system of particles

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \cdots = \sum_{i=1}^{n} \vec{L}_i$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \frac{d\vec{L}_i}{dt} = \sum_{i=1}^{n} \vec{\tau}_{net,i}$$

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} \quad \text{(system of particles)}$$

The net external torque $\vec{\tau}_{net}$ acting on a system of particles is equal to the rate of change of the system's angular momentum $\vec{L}$.

Angular momentum of a rigid rotating body

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow L = rmv \sin 90^\circ = m r \omega r = (mr^2)\omega$$

$$\boxed{L = I\omega}$$

Conservation of Angular Momentum

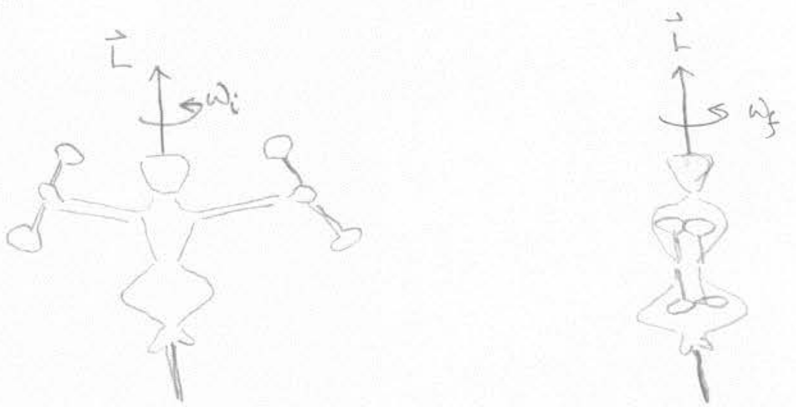
If there is no external torque on a system then $\vec{L}$ of the system will remain constant

$$\boxed{\vec{L}_i = \vec{L}_f} \Rightarrow I_i \omega_i = I_f \omega_f \qquad \frac{d\vec{L}}{dt} = \vec{\tau} = 0 \Rightarrow \vec{L} = \text{const}$$

Check; if net external torque along an axis is zero, then component of $\vec{L}$ that axis is zero.

Examples.

Spinning person



No external torque : $L_i = L_f$

$$I_i \omega_i = I_f \omega_f \qquad (\text{here } I_i > I_f)$$

$$\omega_i < \omega_f$$

By spreading out he reduces the angular velocity

Spacecraft orientation

$$L_i = L_f$$
$$0 = I_1 \omega_1 + I_2 \omega_2$$