Introduction to Condensed Matter

PHY 251/PHY 420

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— ASSIGNMENT 1, 09/26/2011 — Due 10/03/2011

(*Undergraduate students do not have to answer the questions marked with the symbol* ■.)

1 Classical Theory of Conduction *(20 points)*

The Drude model of metal is based on a regular three-dimensional, fixed array of ions in presence of a large number of electrons that as a classical ideal gas are free to move throughout the entire metal.

1.1) Calculate the root mean square speed, v_{rms}, of free electrons at $T = 300\,\mathrm{K}$ and in the absence of external fields.

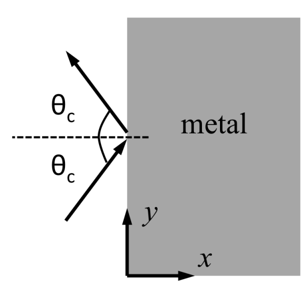
When a constant electric field is applied, a drift velocity (v_{d}) superimposes in the opposite direction of the field.

1.2) Compute the magnitude of the drift velocity and conductivity of electrons in a copper wire of radius 0.815 mm carrying a current of 1 A and with mean free path of 0.38 nm. (Assume one free electron per atom, $Z = 1$, and a mass density of 8.92 g/cm^3.)

1.3) Compute the electron conductivity assuming a mean free path of 0.38 nm.

2 X-Ray Metal Mirror *(20 points)*

X-rays which strike a metal surface at an angle of incidence to the normal greater than a critical angle θ_{c} are totally reflected. Consider a metal (see figure) that occupies the region $x > 0$ and X-rays that propagate in the $x - y$ plane (the plane of the picture) with their polarization vector pointing the $+z$-direction (coming out of the picture). Assume that the metal contains n free electrons per unit volume and apply the Drude model.



2.1) Calculate θ_c as a function of the frequency ω of the X-rays.

2.2) Experiments on gold show that X-rays with an energy of 1 keV have a critical reflection angle of 3.72 degrees. Calculate the plasma frequency ω_p using the result in 2.1) and comment on the striking discrepancy between this result and the fact that ω_p for gold actually lies in the visible range.

3 Wave Attenuation at Low Frequency *(24 points)*

A plane electromagnetic wave of frequency $\omega/2\pi$ and electric field amplitude E_0 is normally incident on the flat surface of a semi-infinite metal of conductivity σ. Use the Drude model and assume the magnetic permeability of the metal $\mu = 1$ and the frequency regime $\omega\tau \ll 1$, i.e., the displacement current inside the metal can be neglected.

3.1) Using Maxwell's equations, derive the skin depth, or characteristic penetration depth, of the field (δ) and evaluate it for copper at $\omega = 60\,\text{Hz}$ and $\omega = 100\,\text{MHz}$.

3.2 ■) Derive expressions for the transversal components of the electric and magnetic fields inside the metal. What is the ratio of the magnetic field amplitude to the electric field amplitude inside the metal?

3.3) For sea water ($\sigma = 4.5 \times 10^{10}\,\text{s}^{-1}$ in cgs units) and using radio waves of long wavelength ($\omega = 0.5\,\text{MHz}$) calculate the characteristic penetration depth (δ) and the intensity attenuation at 10 m. Which frequency range would you use to communicate with a deep submerged object like a submarine?

4 Fermi Energy of Gold *(12 points)*

Electrons in a piece of gold metal can be assumed to behave like an ideal Fermi gas and follow the Sommerfeld theory of metals. Gold metal in the solid state has a mass density of 19.30 g/cm^3. Assume that each gold atom donates one electron to the Fermi gas. Assume the system is in the ground-state ($T = 0\,\text{K}$).

4.1) Compute the Fermi speed, Fermi energy (in eV), the Fermi temperature.

5 Two-Dimensional and One-Dimensional Ideal Fermi Gas *(24 points)*

Nowadays, it is experimentally possible to confine electrons in thin layers forming two-dimensional (2D) systems or in thin wires forming one-dimensional (1D) systems. In this problem we want to study few properties of non-interacting electrons (i.e. ideal gas of spin 1/2 fermions) in 2D and 1D. Assume the electrons to be confined in 2D within a square of area $A = L^2$ and in 1D within a line of length L.

5.1) Express the Fermi wave vector (k_F), the Fermi energy (ε_F), and the total energy per unit of area of the system as a function of the electron density ($n_{2D} = N/A$ or $n_{1D} = N/L$).

5.2) Calculate the density of states of the system $g_{2D}(\varepsilon)$ and $g_{1D}(\varepsilon)$.