Introduction to Condensed Matter

PHY 251/PHY 420

Prof. A. Badolato, Department of Physics and Astronomy, University of Rochester, USA

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(*Undergraduate students do not have to answer the questions marked with the symbol* ■.)

Intrinsic Carrier Concentration *(15 points)*

Indium arsenide (InAs) is a semiconductor crystalline compound made from the elements indium (In) and arsenide (As). It is used in a number of modern advanced technologies including construction of infrared photovoltaic photodiodes and diode lasers. Calculate the intrinsic carrier concentration in InAs at 300 K and 600 K. (Note that the sum of heavy and light hole has to be included.)

Shallow Donors in InSb *(15 points)*

Indium antimonide (InSb) is a semiconductor crystalline compound made from the elements indium (In) and antimony (Sb). It is a narrow-gap semiconductor material used in infrared detectors, including thermal imaging cameras, infrared homing missile guidance systems, and in infrared astronomy. InSb has energy gap $E_{\mathrm{g}} = 0.23eV$; dielectric constant $\epsilon = 18$; and electron effective mass $m_{\mathrm{e}} = 0.015\,m$. Calculate

(a) the shallow donor ionization energy;

(b) the radius of the ground state orbit;

(c) at what minimum donor concentration will an impurity band occur?

Semiconductor Heterostructures *(40 points)*

Semiconductor heterostructures offer the unique opportunity to manipulate the behavior of electrons and holes through *band engineering*. A first approach to construct energy band diagrams (i.e. band-edge potential profile) of the heterojunction between two semiconductor materials (A and B) is the Anderson's rule, Fig 1. This is based on the electron affinity (χ) of the materials, the energy required to take an electron from the bottom of the conduction band E_c to the vacuum, that is, to escape from the crystal.

Electron affinity is nearly independent of the position of the Fermi level. Anderson's rule states

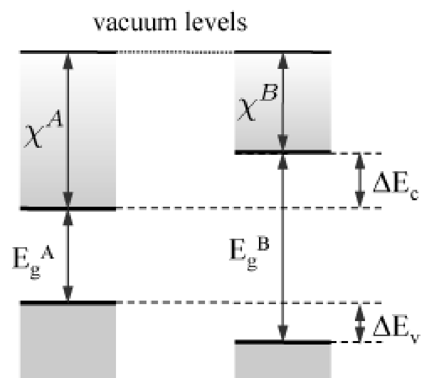

Figura 1: A/B heterojuction, energy band diagram.

that once the vacuum levels are aligned it is possible to use the electron affinity ($\chi^{A,B}$) and band gap ($E_g^{A,B}$) values for each semiconductor to calculate the conduction band (ΔE_c) and valence band (ΔE_v) offsets.

1.1) Estimate $\Delta E_{c,v}$ (at the Γ-point and T = 300 K) for the nearly lattice-matched systems, GaAs/AlAs, $In_{0.53}Ga_{0.47}As$/InP, and InAs/GaSb. (A possible source of data for semiconductor is http://www.ioffe.ru/SVA/NSM/.) How well do the results agree with the accepted values in litterature?

In many important devices, semiconductor are doped to enable a significant additional potential (e.g. p-n junction in inhomogeneous semiconductors) due to the large number of charge carries. At equilibrium the additional electrostatic potential $V_\rho(z)$ arising from the the spatial charge distribution (ρ) has to satisfies the Poisson's equation $\nabla^2 V_\rho = -\rho/\varepsilon_r\varepsilon_0$.

1.2) Sketch the band diagram for the heterojuction n-$Al_{0.33}Ga_{0.67}As$/GaAs (i.e. n-doped $Al_{0.33}Ga_{0.67}As$ and undoped GaAs) and show that this can trap a two-dimensional electron gas (2DEG) at the interface. Repeat for p-$Al_{0.33}Ga_{0.67}As$/n-GaAs and elaborate on the possible formation of a two-dimensional carrier gas.

1.3) Sketch the band diagram for both p-n and n-p generic heterojuctions of type II (or staggered alignment). Determine whether electrons or holes can be trapped at the interface.

1.4 ■) Consider an undoped InAs/GaSb heterojuction (type III or broken gap alignment). Sketch the band diagram at equilibrium. Next, suppose that a sequence of narrow InAs/GaSb layers is grown. Each layer of InAs now behaves as a quantum well for electrons, and the lowest state is raised above the bottom of the conduction band. The energy of the lowest state for holes in GaSb is raised too. For very thin layers the states will be raised so far in energy that they no longer overlap. Estimate the threshold thickness for alternating layers of equal thickness. (Treat the wells as infinitely deep.)

Static Magnetoconductivity Tensor *(30 points)*

Using the Drude model, show that in presence of a static electric (E) and magnetic field (H) the static current density flowing in a wire can be written in matrix form as

$$\begin{pmatrix} j_\mathrm{x} \\ j_\mathrm{y} \\ j_\mathrm{z} \end{pmatrix} = \frac{\sigma_0}{1+(\omega_\mathrm{c}\tau)^2} \begin{pmatrix} 1 & -\omega_\mathrm{c}\tau & 0 \\ \omega_\mathrm{c}\tau & 1 & 0 \\ 0 & 0 & 1+(\omega_\mathrm{c}\tau)^2 \end{pmatrix} \begin{pmatrix} E_\mathrm{x} \\ E_\mathrm{y} \\ E_\mathrm{z} \end{pmatrix}.$$

In the high magnetic field limit $\omega_\mathrm{c}\tau \gg 1$, show that $\sigma_\mathrm{yx} = nec/H = -\sigma_\mathrm{xy}$. In this limit $\sigma_\mathrm{xx} \simeq 0$, to order $1/\omega_\mathrm{c}\tau$. The quantity σ_yx is called the *Hall conductivity*.