Introduction to Condensed Matter

PHY 251 / PHY 420 / ECE 224 / ECE 424

Prof. A. Badolato, Department of Physics and Astronomy, University of Rochester, USA

— FINAL EXAM, 12/18/2011 —

Duration 2h 30 min for undergraduate students and 2h 10 min for graduate students

---

Conductors *(12 points)*

A conductor at room temperature has plasma frequency $\omega_{\mathrm{p}} = 1.8 \times 10^{15}\ \mathrm{s}^{-1}$ and electron relaxation time $\tau = 2.83 \times 10^{-15}\ \mathrm{s}$.

(a) Calculate the electrical conductivity. Take $\mu = 1$ and convert the result to units $(\Omega\,\mathrm{cm})^{-1}$.

(b) From the crystal and chemical structure it is calculated that the conduction electron concentration is $4.7 \times 10^{21}\ \mathrm{cm}^{-3}$. Calculate the electron effective mass.

Lattice Vibrations *(12 points)*

(a) Calculate the Debye frequency for GaAs where the sound velocity is $5.6 \times 10^{5}\ \mathrm{cm \cdot s^{-1}}$ and the volume of the unit cell is $4.4 \times 10^{-23}\ \mathrm{cm}^3$.

(b) The optical phonon energy of GaAs is 36 meV at the center zone. What is the occupation probability of this optical phonons at 77 K and 300 K?

Semiconductors Heterostructures *(16 points)*

Draw a schematic of the band diagram for the following heterostructures:

(a) p-I-n consisting of three layers in the following sequence: p-doped GaAs, undoped insulator I, and n-doped GaAs.

(b) n-$Al_{0.33}Ga_{0.67}As$/GaAs/n-$Al_{0.33}Ga_{0.67}As$ (i.e., n-doped $Al_{0.33}Ga_{0.67}As$ followed by undoped GaAs and n-doped $Al_{0.33}Ga_{0.67}As$).

(c) $Al_{0.33}Ga_{0.67}As$/GaAs/$Al_{0.33}Ga_{0.67}As$.

(d) Under which condition on the temperature electrons confined in a heterostructure quantum well will experience two-dimensional behavior?

(e) The crystal electron effective mass in semiconductors approximately ranges from $0.01\,m$ to $1.00\,m$. What is the corresponding range of de Broglie wavelengths?

Tight Binding Approximation *(30 points)*

Consider a *one-dimensional molecule* whose quantum wells are δ-functions of strength S separated by a distance a, giving $V(x) = -S\left[\delta\left(x+\frac{1}{2}a\right)+\delta\left(x-\frac{1}{2}a\right)\right]$. The pair of wells has an even and an odd bound state. The wave function of a single such well at the origin is proportional to $\exp\left[-\kappa\,|x|\right]$ with $\hbar^2\kappa = mS$ and energy $-\hbar^2\kappa^2 = 2mE$.

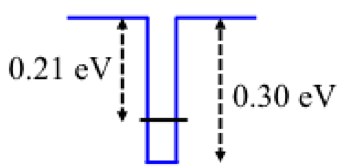
(a) Normalize this wave function and show that the parameters of the tight binding model of the two wells are

$$t = S\kappa \exp\left[-\kappa a\right]; \quad s = (1+\kappa a)\exp\left[-\kappa a\right]; \quad c = S\kappa\exp\left[-2\kappa a\right];$$

which are t the the overlap (or transfer) integral ($t = -\int \varphi_R^* \hat{V}_R \varphi_L dx$), c the crystal field ($c = -\int \varphi_L^* \hat{V}_R \varphi_L dx$), and s the non-orthogonality factor ($s = \int \varphi_R^* \varphi_L dx$).

(b) Show that the even and odd states are split by $2t$ in energy and that their mean is raised by $S\kappa^2 a\exp\left[-2\kappa a\right]$.

A quantum well 5 nm wide and 0.30 eV deep is formed in the GaAs of a AlGaAs/GaAs/AlGaAs heterostructure. As showed in figure, the lowest state of an electron in this well has a binding energy of 0.21 eV.



(c) Estimate the energy splitting when two such wells are separated by a 5 nm barrier (taking the effective mass as $0.067\,m$ everywhere). Do not aim for great accuracy but make drastic yet meaningful approximations to get first a simple expression for t, the overlap integral, and thus to get the order of magnitude of the splitting. (A good approximation is to treat the wells as δ-functions like in (a).)

Ionization of Donors in Semiconductors *(30 points)*

Electrons in *n-type* semiconductors can be thermally excited in conduction band from the valance band and from donor levels. If the main contribution comes from donors, the equilibrium concentration of conduction electrons (n) is given by

$$n \simeq \frac{2N_\mathrm{d}}{1+\sqrt{1+4\frac{N_\mathrm{d}}{n_0}e^{E_\mathrm{d}/k_B T}}},$$

with N_d the concentration of donors, E_d the donor ionization energy, and

$$n_0 \equiv 2 \left(\frac{m^* kT}{2\pi\hbar^2} \right)^{3/2} .$$

(a) What is n in the *freeze-out range* (i.e., when large number of donors are still not ionized)? What is n in the *saturation range*? Write explicitly the temperature condition for the two ranges.

(b) A semiconductor crystal has a donor concentration of $N_\mathrm{d} = 10^{13}$ cm^{-3} with an ionization energy (E_d) of $1\,\mathrm{meV}$ and an effective mass $m^* = 0.01\,m$. Estimate the concentration of conduction electrons at $T = 4$ K and the value of the Hall coefficient in CGS units. (Assume that the main contribution comes from donors and *freeze-out range*.)