Introduction to Condensed Matter

PHY 251 / PHY 420 / ECE 224 / ECE 424

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— MIDTERM EXAM, 11/16/2011 — Duration 1h 15 min.

(*Undergraduate students do not have to answer the questions marked with the symbol* ■.)

1 Free Electron Models *(20 points)*

(a) For the metal copper at very low temperatures ($T \simeq 0$ K), determine the Fermi energy, the Fermi speed, and the average energy of electrons. (The copper mass density is $\rho_{\rm m}$= 8.94 g·cm^{-3} and the atomic mass is $A = 63.5$ g.) At the same temperature, estimate the average energy of electrons if the electrons are treated as a classical ideal gas.

2 Harmonic Crystal Vibrations *(40 points)*

(a) Given an N-ion harmonic crystal, under what conditions will the specific heat value be $c_{\rm v} = 3Nk_{\rm B}/V$? Given a metal with Fermi temperature $T_{\rm F} \simeq 10^4$ K and the Debye temperature $T_{\rm D} \simeq 10^2$ K, estimate the ratio of the electronic and phonon specific heat ($c_{\rm v,\,phonons}/c_{\rm v,\,electrons}$) at $T = 1$ K. [Hint: $c_{\rm v,\,phonons} \propto (T/T_{\rm D})^3\, Nk_{\rm B}/V$.]

(b) Consider the elastic vibrations of a one-dimensional crystal with one atom in the primitive cell. Assuming nearest neighbors interactions, what is the ratio of the displacements of two successive planes for $k = \pi/a$? Demonstrate that the group velocity for $k = \pi/a$ is equal to zero.

(c) In GaAs the long wavelength ($k\,a << 1$) *optical mode* vibrations have frequency $\nu = 8$ THz. Assuming the one-dimensional model with nearest neighbors interactions and knowing that $M_{\rm Ga} \simeq 70\, m_{\rm p}$ and $M_{\rm As} \simeq 75\, m_{\rm p}$ (with $m_{\rm p} = 9.4 \times 10^8\,{\rm eV/c^2}$), calculate the force constant C of the "spring" between the atoms and the sound speed. (The unit cell size of the GaAs is $a = 0.565$ nm.)

3 Electronic Band Structures *(40 points)*

Electrons in a crystal are subject to the one-dimensional periodic potential

$$V(x) = V_0 + A\cos(\frac{4\pi x}{a}) + B\cos(\frac{6\pi x}{a}).$$

(a) Under what conditions will the nearly free-electron approximation work? Assuming that the conditions are satisfied, sketch the four lowest energy bands. Number the energy bands starting from one at the lowest band.

(b) Applying the nearly free-electron approximation at the four lowest energy bands, calculate (to the first-order) the energy gaps at $k = 0$ and $k = \pi/a$.

(c ■) Answer the same questions in (b) assuming a one-dimensional potential $V(x)$ composed by a periodic sequence of δ-like potential barriers

$$V(x) = \sum_n a\, V_0\, \delta(x - na),$$

where a is the lattice constant.