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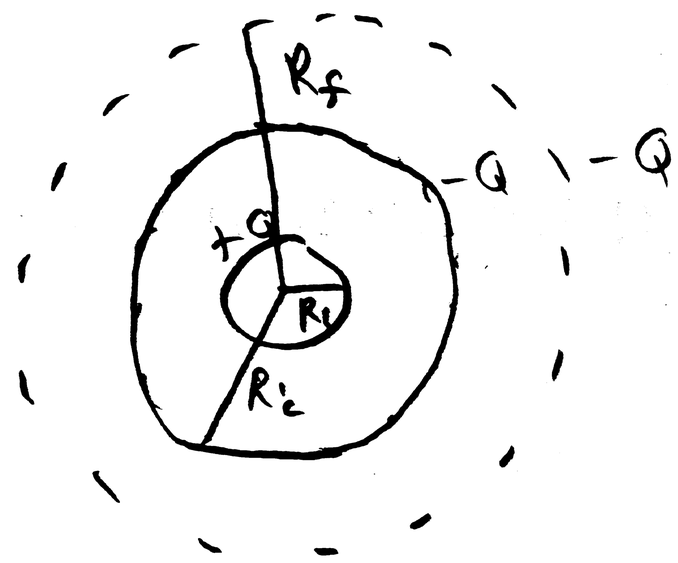
Inflate a Balloon Capacitor



According to our previous results,

$$C_i = \frac{1}{k}\left(\frac{R_i R_1}{R_i - R_1}\right)$$

$$C_f = \frac{1}{k}\left(\frac{R_f R_1}{R_f - R_1}\right)$$

Since the total charge remains the same, we can use

$$U = \frac{1}{2}\frac{Q^2}{C}$$

so that

$$U_i = \frac{Q^2}{2} k \frac{R_i - R_1}{R_i R_1}$$

$$U_f = \frac{Q^2}{2} k \frac{R_f - R_1}{R_f R_1}$$

The work done is the change in potential energy $U_{fi} = U_f - U_i$

$$U_{fi} = \frac{Q^2}{2} k \left[\frac{R_f - R_1}{R_f R_1} - \frac{R_i - R_1}{R_i R_1}\right]$$

$$= \frac{kQ^2}{2}\left[\frac{R_i(R_f - R_1) - R_f(R_i - R_1)}{R_f R_i R_1}\right]$$

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$$U_{fi} = \frac{kQ^2}{2}\left[\frac{R_i\cancel{R_f} - R_iR_1 - \cancel{R_f}R_i - R_fR_1}{R_fR_iR_1}\right]$$

$$= -\frac{kQ^2}{2}\left[\frac{\cancel{R_1}(R_i - R_f)}{R_fR_i\cancel{R_1}}\right]$$

$$\boxed{U_{fi} = \frac{kQ^2}{2}\,\frac{R_f - R_i}{R_fR_i}}$$

We can also express this as a ratio

$$\frac{U_f}{U_i} = \frac{\cancel{\frac{kQ^2}{2}}\,\frac{R_i - R_1}{R_iR_1}}{\cancel{\frac{kQ^2}{2}}\,\frac{R_f - R_1}{R_fR_1}} = \frac{R_i - R_1}{R_i\cancel{R_1}}\,\frac{R_f\cancel{R_1}}{R_f - R_1}$$

$$\boxed{\frac{U_f}{U_i} = \frac{R_f(R_i - R_1)}{R_i(R_f - R_1)}}$$

If $R_f = 2R_i$

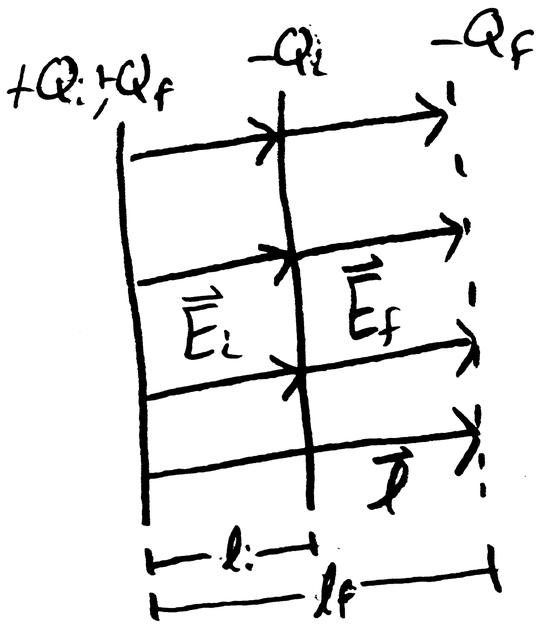
$$\frac{U_f}{U_i} = \frac{2\cancel{R_i}(R_i - R_1)}{\cancel{R_i}(2R_i - R_1)}$$

$$\boxed{\frac{U_f}{U_i} = 2\,\frac{R_i - R_1}{2R_i - R_1}}$$

$$U_{fi} = \frac{kQ^2}{2}\,\frac{2R_i - R_i}{2R_iR_i} = \frac{kQ^2}{2}\,\frac{\cancel{R_i}}{2R_i^{\cancel{2}}}$$

$$\boxed{U_{fi} = \frac{kQ^2}{4R_i}}$$

Pull Apart Capacitor 7-8-08



We can easily find the initial and final potential energies as before, but because charge is flowing, that doesn't tell us everything about the work done.

Because this time the voltage is held constant instead of Q, we use

$U = \frac{1}{2} C V^2$ rather than $U = \frac{1}{2} Q^2/C$

$$U_i = \frac{1}{2} C_i V^2 = \frac{1}{2}\left(\epsilon_0 \frac{A}{l_i}\right) V^2$$

$$U_f = \frac{1}{2}\left(\epsilon_0 \frac{A}{l_f}\right) V^2$$

So $$U_{fi} = \frac{\epsilon_0}{2} A V^2 \left(\frac{1}{l_f} - \frac{1}{l_i}\right) = \frac{\epsilon_0}{2} A V^2 \left(\frac{l_i - l_f}{l_i l_f}\right)$$

$$\boxed{U_{fi} = -\frac{\epsilon_0}{2} A V^2 \left(\frac{l_f - l_i}{l_f l_i}\right)}$$

So the capacitor now loses energy.

if e.g. $l_f = 3 l_i$

$$U_{fi} = -\frac{\epsilon_0}{\cancel{2}} A V^2 \left(\frac{\cancel{2}\, \cancel{l_i}}{3\, l_i^{\cancel{2}}}\right)$$

$$\boxed{U_{fi} = -\epsilon_0 \frac{A V^2}{3 l_i}}$$

$U = \frac{1}{2} C V^2$, so if V stays the same and C gets smaller, decreasing U makes sense.

If the capacitor loses energy, the battery must gain it. In order to figure out how much the battery gains, we have to calculate the work done directly,

$$W = \int \vec{F} \cdot d\vec{l} = Q \int \vec{E} \cdot d\vec{l}$$

In our case, $\vec{E} \cdot d\vec{l} = E\,dl$ because you can see from the diagram the are in the same direction.

$$W = \int_{l_i}^{l_f} QE\,dl$$

Remember that V_{+-} is from + to − so $V_{+-} > 0$

In a parallel plate capacitor, we've already found that $V_{-+} = -El \rightarrow E = \frac{V_{+-}}{l}$

However, remember that this is the result of the principal of super position, $\vec{E} = \vec{E}_+ + \vec{E}_-$. Each contributes equally to the total field, but one plate can only feel the force from the <u>other</u> plate's field, not its own.

So, $E = \frac{V_{+-}}{2l}$.

$$W = \frac{V_{+-}}{2} \int_{l_i}^{l_f} Q \frac{dl}{l}$$

Q changes over this integral, so we need to express it in terms of l and constants.

Since V_{+-} is constant and we know C as a function of l, we use

$$Q = C V_{+-} \qquad C = \epsilon_0 \frac{A}{l}$$

$$Q = \epsilon_0 \frac{V_{+-} A}{l}$$

$$W = \epsilon_0 \frac{A V_{+-}^2}{2} \int_{l_i}^{l_f} \frac{dl}{l^2}$$

$$= -\epsilon_0 \frac{A V_{+-}^2}{2} \left(\frac{1}{l_f} - \frac{1}{l_i} \right)$$

$$\boxed{W = \frac{\epsilon_0}{2} A V_{+-}^2 \left(\frac{l_f - l_i}{l_f l_i} \right)}$$

if $l_f = 3 l_i$,

$$W = \frac{\epsilon_0}{2} A V_{+-}^2 \left(\frac{3 l_i - l_i}{3 l_i^2} \right) = \frac{\epsilon_0}{\cancel{2}} A V_{+-}^2 \frac{\cancel{2} \cancel{l_i}}{3 l_i^{\cancel{2}}}$$

$$\boxed{W = \frac{\epsilon_0}{3} \frac{A V_{+-}^2}{l_i}}$$

In order to find the energy change of the battery, we combine the work done and change in stored energy.

Work is the total change in energy of the system

$$W = \Delta U_{Battery} + U_{fi}$$

$$\Delta U_{Batt} = W - U_{fi}$$

$$= \frac{\epsilon_0}{3}\frac{AV_{+-}^2}{l_i} - - \epsilon_0 \frac{AV_{+-}^2}{3l_i}$$

$$\boxed{\Delta U_{Batt} = \frac{2\epsilon_0}{3}\frac{AV_{+-}^2}{l_i}}$$

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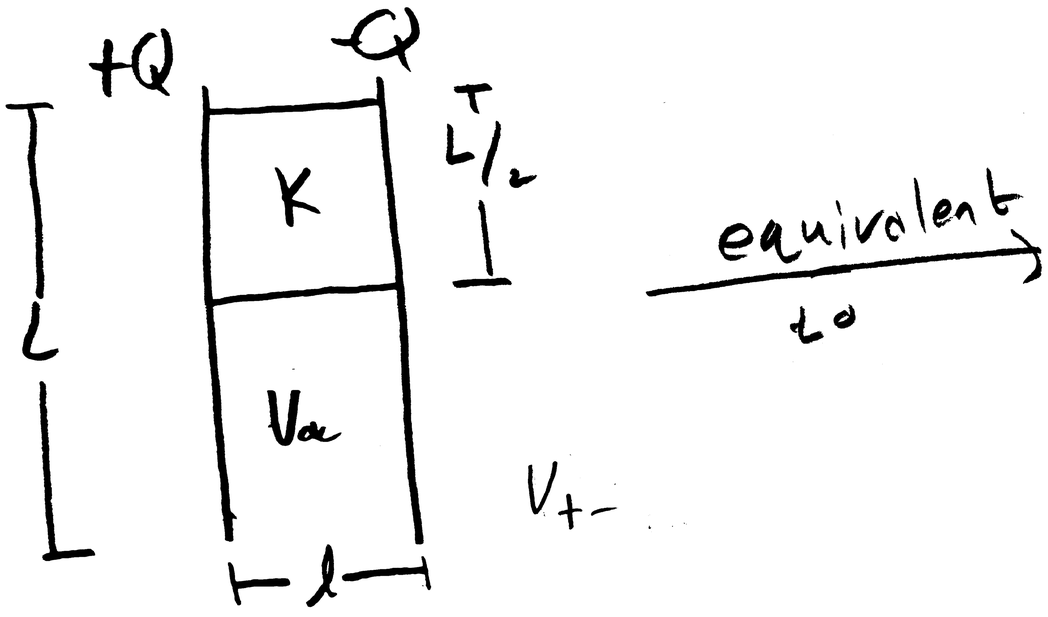
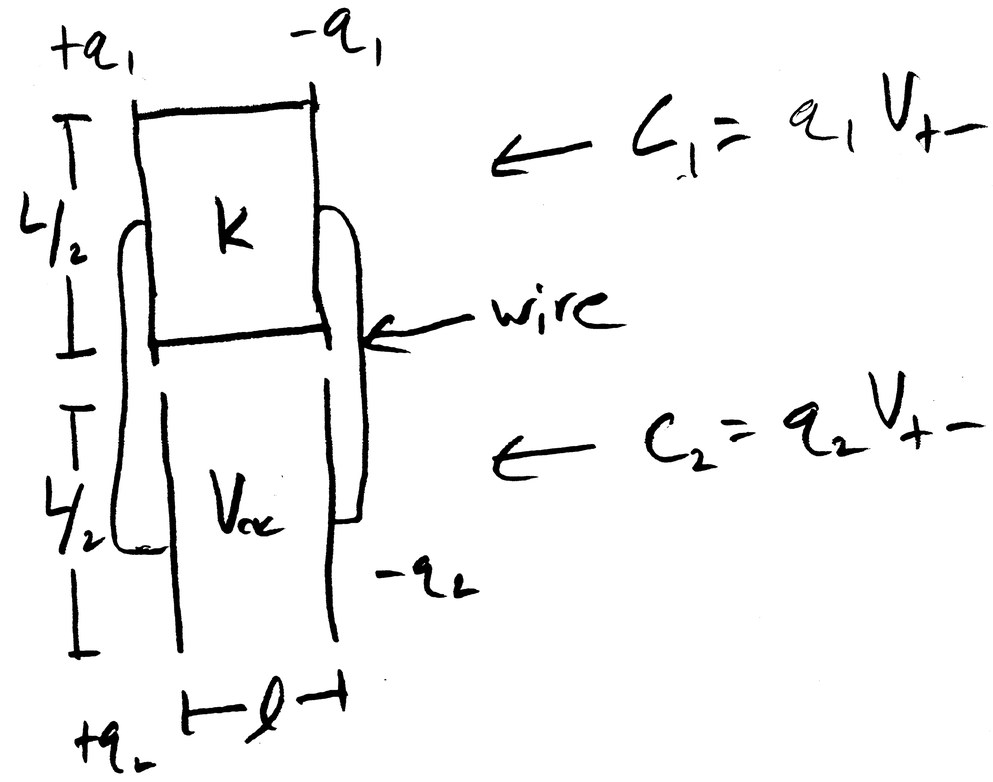
Split capacitor





$C = Q\,V_{+-}$

$$C = C_1 + C_2$$

C of a parallel plate capacitor is

$$C_0 = \epsilon_0 \frac{A}{l}, \quad C = K\epsilon_0 \frac{A}{l}$$

so

$$C_1 = K\epsilon_0 \frac{A/2}{l} \qquad C_2 = \epsilon_0 \frac{A/2}{l}$$

$$C_1 + C_2 = \frac{\epsilon_0 A}{2l}(K+1)$$

$$\boxed{C = \epsilon_0 \frac{A}{l} \frac{K+1}{2}}$$

If $K=1$ (vacuum) we get back $C = \epsilon_0 \frac{A}{l}$ ✓

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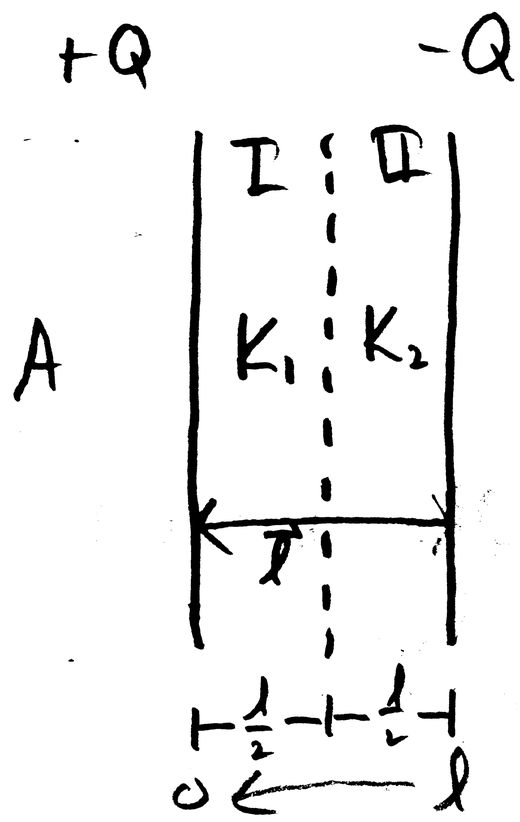
Divided Capacitor



What did we do to find the capacitance of the original parallel plate capacitor? We integrated $\vec{E}\cdot d\vec{l}$ to find V_{+-}. Do the same here.

$\vec{E}$ now has 2 different regions, however. Remember that with a dielectric instead of a vacuum, $\epsilon_0 \to \epsilon$. So,

$$\vec{E}_I = \frac{Q}{\epsilon_1}\frac{1}{A}\hat{x} \quad \text{where} \quad \epsilon_1 = K_1\epsilon_0$$

$$\vec{E}_{II} = \frac{Q}{\epsilon_2}\frac{1}{A}\hat{x} \qquad \epsilon_2 = K_2\epsilon_0$$

$$V_{+-} = -\int_l^0 \vec{E}\cdot d\vec{l} = -\int_l^{l/2}\vec{E}_{II}\cdot d\vec{l} - \int_{l/2}^0 \vec{E}_I\cdot d\vec{l}$$

$$= -\frac{Q}{A}\left[\frac{1}{\epsilon_2}\int_l^{l/2} dl + \frac{1}{\epsilon_1}\int_{l/2}^0 dl\right]$$

$$= -\frac{Q}{A}\left[\frac{1}{\epsilon_2}\left(\frac{l}{2}-l\right) + \frac{1}{\epsilon_1}\left(0-\frac{l}{2}\right)\right]$$

$$V_{+-} = \frac{Q}{2}\frac{l}{A}\left[\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}\right] = \frac{Q}{2}\frac{l}{A}\frac{\epsilon_1+\epsilon_2}{\epsilon_1\epsilon_2}$$

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$$C = \frac{Q}{V_{+-}}$$

$$\boxed{C = 2\frac{A}{l}\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}$$

$$= 2\frac{A}{l}\frac{K_1 K_2 \epsilon_0}{(K_1 + K_2)\cancel{\epsilon_0}}$$

$$\boxed{C = \epsilon_0 \frac{A}{l}\frac{2K_1 K_2}{K_1 + K_2}}$$

If $K_1 = K_2 = 1$

$$\rightarrow \quad C = \epsilon_0 \frac{A}{l}$$