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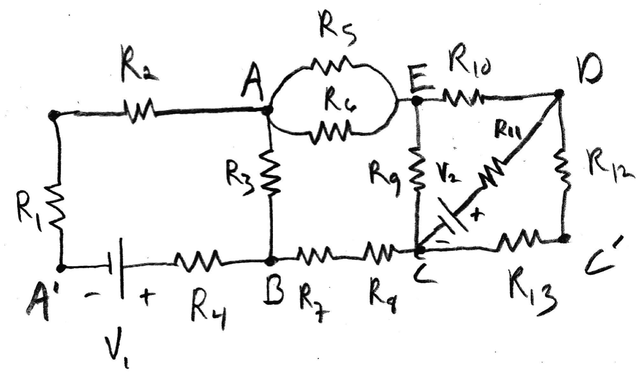
$\vec{I}_1$



For this, I'm using step 0; reduce using addition rules for resistors!

Given: V_1, V_2, all R_L.

0) Add resistors.

R_1 + R_2 combine in series

$R_A = R_1 + R_2$

R_5 + R_6 in parallel

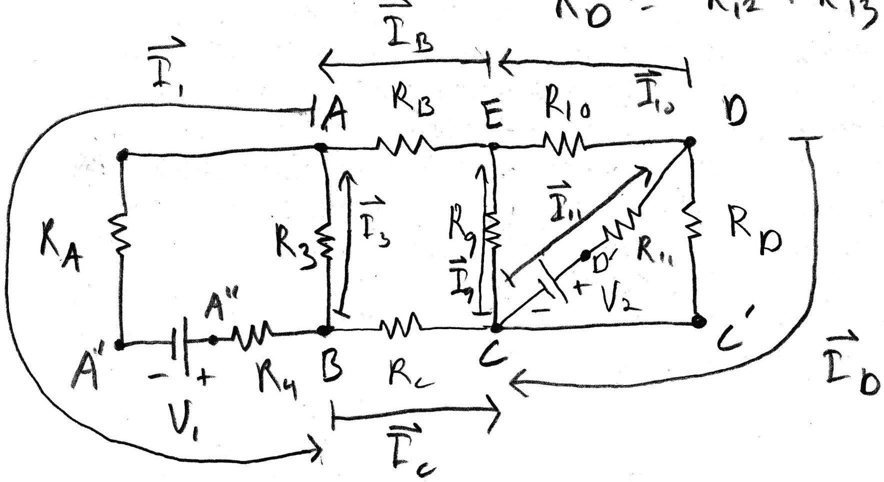
$$R_B = \frac{1}{1/R_5 + 1/R_6}$$

R_7 + R_8 in series

$R_C = R_7 + R_8$

R_{12} + R_{13} in series

$R_D = R_{12} + R_{13}$



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1) Label currents. I've named currents for one of the resistors they pass through. There are 8 of them.

2) Identify unknowns. All the currents:

$I_1, I_3, I_B, I_c, I_9, I_{14}, I_{11}, I_D$

3) Apply junction rule. Notice that I've labeled slightly fewer points this time. My gut reading of the diagram tells me that using all 5 A, B, C, D, E will be useful. Why? There are 8 unknown currents and only 5 equations, so the whole thing still has a lot of freedom.

A:
$$
\begin{aligned}
I(BA'A) &= -I_1 \\
+I(EA) &= I_B \\
+I(BA) &= I_3 \\
\hline
&\quad 0
\end{aligned}
$$

eq. 1

$$I_1 = I_B + I_3$$

B:
$$
\begin{aligned}
I(AA'B) &= I_1 \\
+I(AB) &= -I_3 \\
+I(CB) &= -I_c \\
\hline
&\quad 0
\end{aligned}
$$

$$I_1 = I_c + I_3$$

eq. 2

$$\boxed{I_B = I_c}$$

use I_B for both now.

C:
$$
\begin{aligned}
I(BC) &= I_B \\
+I(EC) &= -I_9 \\
+I(DC'C) &= I_D \\
+I(DC) &= -I_{11} \\
\hline
&\quad 0
\end{aligned}
$$

eq. 3

$$I_9 + I_{11} = I_B + I_D$$

D: $I(CC'D) = -I_D$
$+I(CD) = I_{11}$
$+I(ED) = -I_{10}$
$\overline{0}$

eq. 4
$$I_{11} = I_D + I_{10}$$

E: $I(DE) = I_{10}$
$+I(CE) = I_9$
$+I(AE) = -I_B$
$\overline{0}$

eq. 5
$$I_B = I_9 + I_{10}$$

4) Apply Loop Rule. We need 3 more equations. One choice of 3 that covers everything is:

① AA'A"BCC'DEA, ② BC, D, E, A, B, ③ CDEC

Loop AA'A"BCC'DEA:

$-AA'$:	$I_1 R_A$
$+A'A''$:	V_1
$-A''B$:	$I_1 R_4$
$-BC$:	$I_B R_C$
$-CC'$:	0
$-C'D$:	$-I_D R_D$
$-DE$:	$I_{10} R_{10}$
$-EA$:	$I_B R_B$
	0

eq. 6
$$V_1 + I_D R_D = I_1(R_A + R_4) + I_B(R_B + R_C) + I_{10} R_{10}$$

Loop BCD'DEAB:

$-BC$:	$I_B R_C$
$+CD'$:	V_2
$-D'D$:	$I_{11} R_{11}$
$-DE$:	$I_{10} R_{10}$
$-EA$:	$I_B R_B$
$-AB$:	$-I_3 R_3$
	0

eq. 7
$$V_2 + I_3 R_3 = I_B(R_B + R_C) + I_{10} R_{10} + I_{11} R_{11}$$

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Loop

CD'DEC

$$\left.\begin{array}{ll} CD': & V_2 \\ -D'D: & I_{11}R_{11} \\ -DE: & I_{10}R_{10} \\ -EC: & -I_9 R_9 \\ \hline & 0 \end{array}\right\} \quad V_2 + I_9 R_9 = I_{10}R_{10} + I_{11}R_{11}$$

eq. 8

5) Solve equations. First, simplify the currents down a bit.

eq. 5 → eq 3 $\quad \cancel{I_9} + I_{10} + I_D = \cancel{I_9} + I_{11}$

$$I_{11} = I_D + I_{10}$$

This is the same as eq. 4!

This means we have a reduncancy in our equations, so we are 1 short! Let's push on for now, and see if we can later identify a loop to help us. In the mean time, I am retiring eq. 3.

eq5. → eq 1. $\quad I_1 = I_9 + I_{10} + I_3$

So to sumarize what we know from junctions:

$$\boxed{\begin{array}{l} I_1 = I_9 + 10_{10} + I_3 \\ I_B = I_C = I_9 + I_{10} \\ I_{11} = I_D + I_{10} \end{array}}$$

This means if I find I_3, I_9, I_{10}, I_D then we get I_1, I_B, I_C, and I_{11}.

Now use loop eqs.

eq. 8 → eq. 7

$$I_{10}\cancel{R_{10}} + I_{11}\cancel{R_{11}} - I_9 R_9 = I_B (R_B + R_c) + \cancel{I_{10}R_{10}} + \cancel{I_{11}R_{11}} - I_3 R_3$$

$$I_B = \frac{I_3 R_3 - I_9 R_9}{R_B + R_c}$$

We can continue on like this but you've gotten the idea. This example is much more involved than any I will give, but demonstrates the techniques.