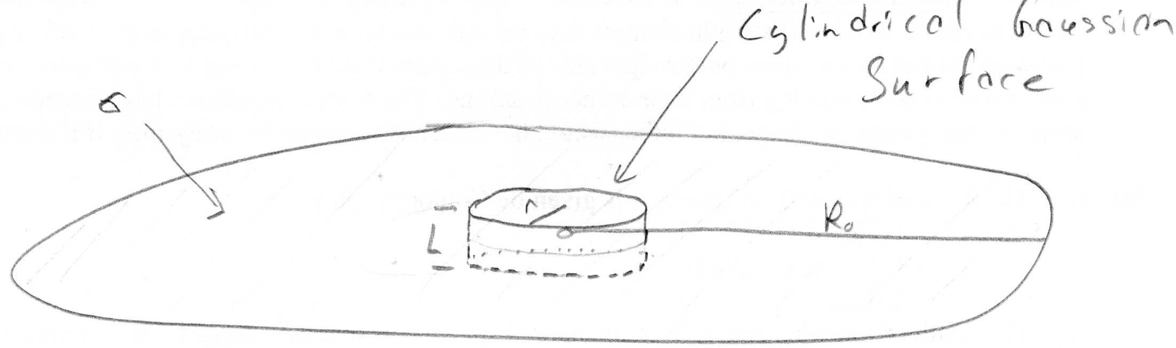
7-3-08

Uniformly Charged Disk



Again our surface is composed of 3 surfaces, the round wall + 2 end caps.

$$\oint_C \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\int_{caps} E \, dA_{caps} = \frac{1}{\epsilon_0} \int_D \sigma \, dA_D$$

$$E \int_{caps} dA_{caps} = \frac{\sigma}{\epsilon_0} \int dA_D$$

Area of a disk $= \pi r^2$

Caps are same size as enclosed disk, but there are 2.

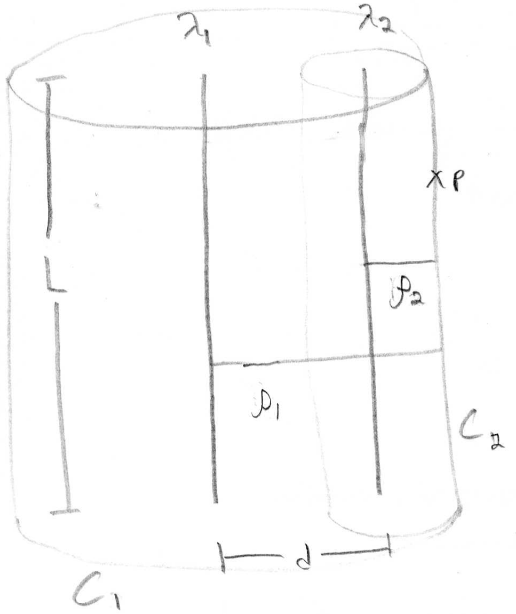
$$E(2\cancel{\pi}\cancel{r^2}) = \frac{\sigma}{\epsilon_0}(\cancel{\pi}\cancel{r^2})$$

$$E = \frac{\sigma}{2\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{z}}$$

So much easier!

2 line charges: Gauss's Law



We need 2 cylinders, one centered on each line.

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\oint_{C_1} \vec{E}_1 \cdot d\vec{A}_1 = \frac{Q_{enc,1}}{\epsilon_0}$$

C_1 is composed of 3 pieces. The top and bottom surfaces won't contribute because $\vec{E}_1$ is parallel to the surface ($\vec{E}_1 \cdot d\vec{A}_1 = 0$).

$$E_1 (2\pi \rho_1) \cancel{L} = \lambda_1 \cancel{L}$$

$$E_1 = \frac{\lambda_1}{2\pi\rho_1}$$

$$\boxed{\vec{E}_1 = \frac{\lambda_1}{2\pi\rho_1} \hat{\rho}_1}$$

$\vec{E}_2$ is the same, but in a coordinate system centered on line 2.

$$\boxed{\vec{E}_2 = \frac{\lambda_2}{2\pi\rho_2} \hat{\rho}_2}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{1}{2\pi}\left(\frac{\lambda_1}{\rho_1} \hat{\rho}_1 + \frac{\lambda_2}{\rho_2} \hat{\rho}_2 \right)$$

In order to do this, we would need to convert $\hat{\rho}_1$ and $\hat{\rho}_2$ to the same coordinate system. this is doable but ugly, involving a lot of trig functions. To simplify, pick a point P that is in the plane containing both lines of charge.

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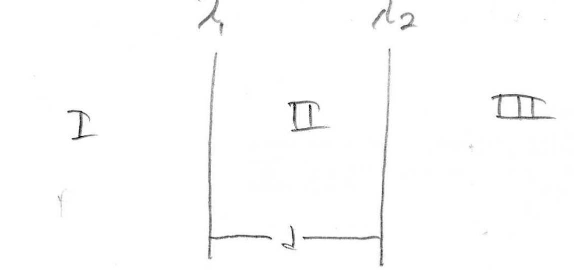
With this choice, we can convert to $\hat{x}, \hat{y}$ coordinates. There are 3 regions:



$$\left.\begin{array}{ll} \text{I}: & \hat{\rho}_1 = \hat{\rho}_2 = -\hat{x} \\ \text{II}: & \hat{\rho}_1 = \hat{x} \quad \hat{\rho}_2 = -\hat{x} \\ \text{III}: & \hat{\rho}_1 = \hat{\rho}_2 = \hat{x} \end{array}\right\} \text{ for points in regions}$$

So we have 3 expressions for $\vec{E}$, depending on region:

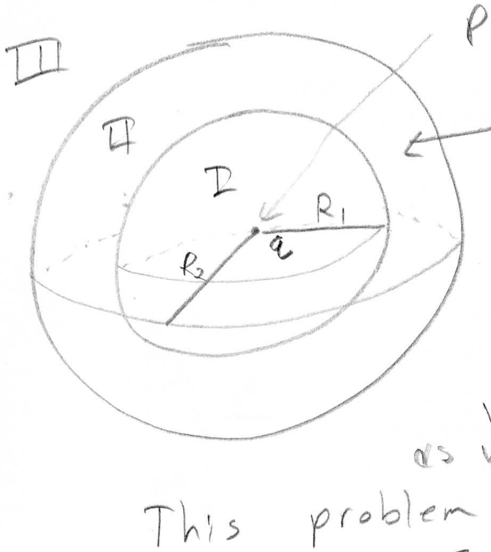
$$\vec{E}(\text{I}) = \frac{1}{2\pi}\left(\frac{\lambda_1}{\rho_1}(-\hat{x}) + \frac{\lambda_2}{\rho_2}(-\hat{x})\right)$$

$$\vec{E}(\text{I}) = -\frac{1}{2\pi}\left(\frac{\lambda_1}{\rho_1} + \frac{\lambda_2}{\rho_1 + d}\right)\hat{x} = \frac{1}{2\pi}\left(-\frac{\lambda_1}{\rho_1} - \frac{\lambda_2}{\rho_1 + d}\right)\hat{x}$$

$$\vec{E}(\text{II}) = \frac{1}{2\pi}\left(\frac{\lambda_1}{\rho_1} - \frac{\lambda_2}{d - \rho_1}\right)\hat{x} = \frac{1}{2\pi}\left(\frac{\lambda_1}{\rho_1} + \frac{\lambda_1}{\rho_1 - d}\right)\hat{x}$$

$$\vec{E}(\text{III}) = \frac{1}{2\pi}\left(\frac{\lambda_1}{\rho_1} + \frac{\lambda_2}{\rho_1 - d}\right)\hat{x} = \frac{1}{2\pi}\left(\frac{\lambda_1}{\rho_1} - \frac{\lambda_1}{\rho_1 - d}\right)\hat{x}$$

Compound Spherical System



Point charge q

Charged conducting shell (total charge Q)

Charge q is at the center of the charged conducting shell between R_1 and R_2.

We want $\vec{E}$ in all 3 regions I, II, III, as well as all surface charges.

This problem forces us to take more steps than usual. First, the central charge q will induce charges in the shell. Second, the charge in the shell (the net charge, seperate from the induced charge) will concentrate on the surfaces.

Region I, however, can be solved w/o any of this added effort. Gauss's law tells us that only $Q_{enclosed}$ contributes to $\vec{E}$, so:

$$\vec{E}(I) = k\frac{q}{r^2}\hat{r}$$

we can write this:

$$\boxed{\vec{E}(r < R_1) = k\frac{q}{r^2}\hat{r}}$$

Region II is simple. Because this is a conducting shell,

$$\boxed{\vec{E}(R_1 < r < R_2) = 0}$$

Gauss's law also gives the field in region III easily:

$$\vec{E}(r>R_2) = \vec{E}_q + \vec{E}_Q$$

$$\vec{E}_q = k\frac{q}{r^2}\hat{r}$$

$$\oint \vec{E}_Q \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$E_Q(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E_Q = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

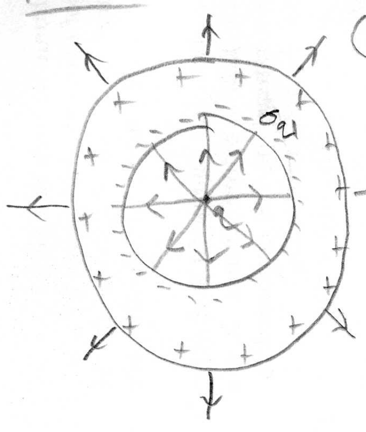
$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$\boxed{\vec{E}(r>R_2) = k\frac{q+Q}{r^2}\hat{r}}$$

Thats it for the fields, what about surface charges?

q will induce a total charge $-q$ on the inside surface. Why? Draw the field lines:

(draw for $q>0$)



All the field lines that start at q must end at σ_{q1}, the inside surface. However, all of these charges come from somewhere, so the outside has an equal (total) and opposite surface charge σ_{q2}.

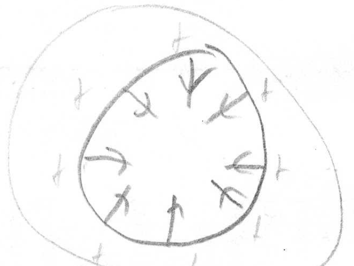
Thus, $\sigma_{q1} = \frac{q}{A_1}$

$$\boxed{\sigma_{q1} = \frac{q}{4\pi R_1^2}}$$

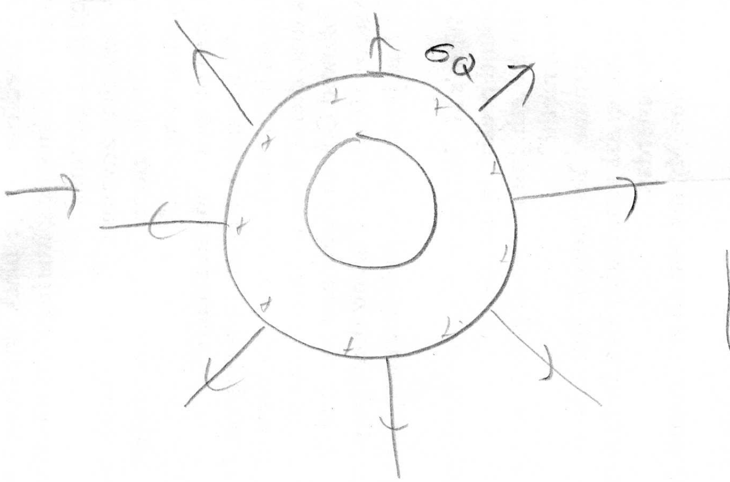
$$\boxed{\sigma_{q2} = -\frac{q}{4\pi R_2^2}}$$

What about the net charge Q on the shell? None of this can be on the inside surface. Why? Draw the field lines:

Again drawing lines for $q > 0$ case



No where for them to go!



$$\boxed{\sigma_Q = \frac{Q}{4\pi R_2}}$$

$$\sigma_1 = \sigma_{q_1}$$

$$\boxed{\sigma_1 = \frac{q}{4\pi R_1^2}}$$

$$\sigma_2 = \sigma_{q_2} + \sigma_Q$$

$$\boxed{\sigma_2 = \frac{Q-q}{4\pi R_2^2}}$$