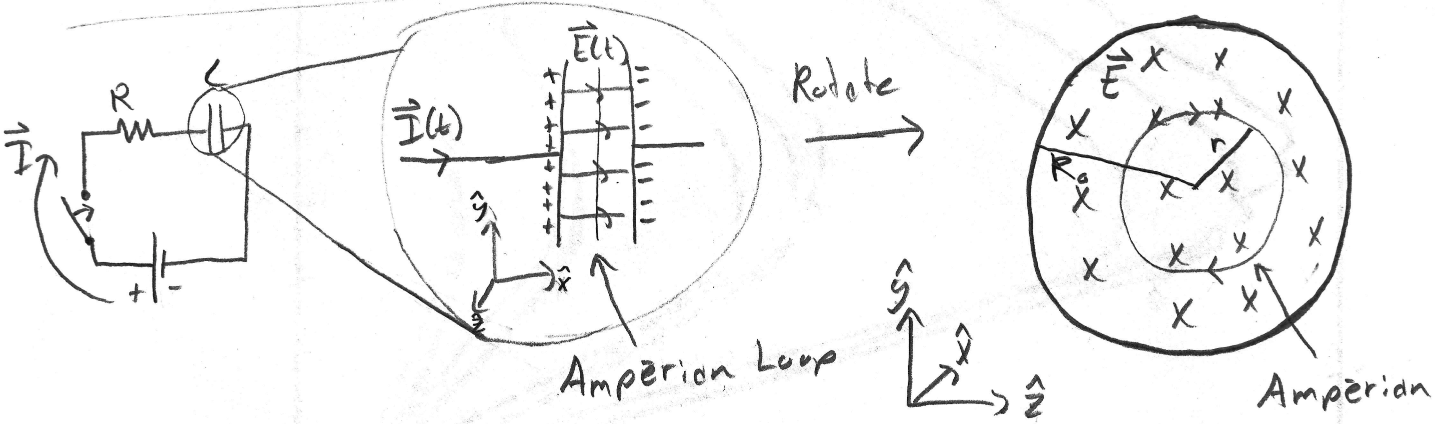
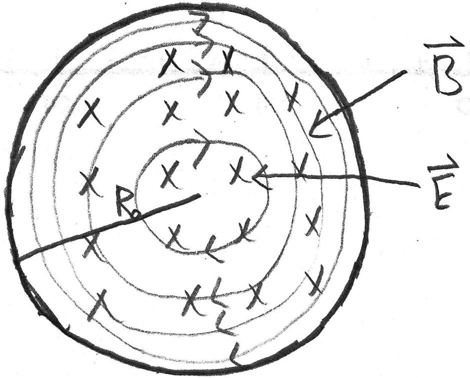


$\vec{I}$
R
+
-
$\vec{E}(t)$
$\vec{I}(t)$
$\hat{y}$
$\hat{x}$
$\hat{z}$
Amperian Loop
Rotate
$\hat{y}$
$\hat{x}$
$\hat{z}$
$\vec{E}$
r
R_0
Amperian

Will result:



$\vec{B}$
$\vec{E}$
R_0

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To find the induced $\vec{B}$ field, we'll use Ampère's Law w/ Maxwell's Correction.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Our loop is a circle at r, so

$$d\vec{l} = r\, d\theta \ \widehat{RHR}$$

Because this is still Ampère's Law, we find directions the same way. Just treat the change in flux like a current. If we start off uncharged and charge, we are increasing the charge, thus increasing the flux, so we act as tho there is a current into the page. Thus, we orient our loop clockwise.

Assume that $\vec{B}$ behaves as it would for enclosed current, meaning the field lines will be circular and B depends only on r, not angle around the loop.

$r < R_0$

$$B \oint r\, d\theta = \mu_0 \epsilon_0 \frac{d}{dt} E(t) A(r)$$

$$B(2\cancel{\pi} r) = \mu_0 \epsilon_0 (\cancel{\pi} r^2) \frac{dE(t)}{dt}$$

$$B(r<R_0) = \frac{\mu_0 \epsilon_0}{2} r \frac{dE(t)}{dt}$$

So far so good. What's $E(t)$? We need to go back and steal some of our old results to find out. First, we saw earlier that for a capacitor,

$$\frac{dQ}{dt} = \epsilon_0 A_0 \frac{dE}{dt}. \qquad A_0 = A(r=R) = \pi R_0^2$$

Also, back when we did RC circuits, we found that

$$Q(t) = CV_0\left(1 - e^{-t/RC}\right)$$

$$\frac{dQ(t)}{dt} = CV_0 \frac{d}{dt}\left(1 - e^{-t/RC}\right)$$

$$= \cancel{C}V_0\left(0 - \frac{-1}{R\cancel{C}} e^{-t/RC}\right)$$

$$= \frac{V_0}{R} e^{-t/RC}$$

$$= I_0 e^{-t/RC}$$

$$\rightarrow \frac{dE}{dt} = \frac{1}{\epsilon_0 A_0}\frac{dQ}{dt}$$

$$= \frac{I_0}{\epsilon_0 A_0} e^{-t/RC}$$

$$= \frac{1}{\pi \epsilon_0} \frac{I_0}{R_0^2} e^{-t/RC}$$

Now we can just throw these pieces together,

$$B(r<R_o) = \frac{\mu_o \epsilon_o}{2} r\left(\frac{1}{\pi \epsilon_o}\frac{I_o}{R_o^2} e^{-t/RC}\right)$$

$$\boxed{B(r<R_o) = \frac{\mu_o I_o}{2\pi}\frac{r}{R_o^2} e^{-t/RC}}$$

While it isn't important for you to be extremely comfertable with exponentials, you should know that e^{-x} is a function that gets small very quickly, while e^{x} gets large very quickly. In this case, we see that, the a magnetic field is induced, it dies off rapidly as the capacitor charges.

What about $r > R_o$?

For $r > R_o$, all of the flux is contained, not just part. So

$$B \oint r\, d\theta = \mu_o \epsilon_o \frac{d}{dt} E(t) A(r)$$

becomes

$$B \oint r\, d\theta = \mu_o \epsilon_o \frac{d}{dt} E(t) A(R_o)$$

So then

$$B(2\pi r) = \mu_0 \epsilon_0 \pi R_0^2 \frac{d}{dt} E(t)$$

$$B = \frac{\mu_0 \epsilon_0}{2} \frac{R_0^2}{r} \frac{d}{dt} E(t)$$

so, compared to the $r<R_0$ case,

$$r \rightarrow \frac{R_0^2}{r}.$$

So change this in the final

result:

$$B(r>R_0) = \frac{\mu_0 I_0}{2\pi} \frac{\cancel{R_0^2}}{r \cancel{R_0^2}} e^{-t/RC}$$

$$\boxed{B(r>R_0) = \frac{\mu_0}{2\pi} \frac{I_0}{r} e^{-t/RC}}$$

So:

Inside the capacitor, B grows with r because you enclose more and more flux as you go out. Outside, B starts to fall off w/ r because we are getting further away.

→ Compare to the B field in and around a thick wire:

$$B(r<R_0) = \frac{\mu_0 I}{2\pi} \frac{r}{R_0^2}$$

$$B(r>R_0) = \frac{\mu_0}{2\pi} \frac{I}{r}$$

Same, without the time dependant piece!

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What is the I_0 in the expression for B generated by the changine electric flux? It's just the current into the capacitor at $t=0$ (the peak)