Part I
Electrostatics

1: Charge and Coulomb's Law July 6, 2008

1.1 What is Electric Charge?

1.1.1 History

Before 1600CE, very little was known about electric properties of materials, or anything to do with electricity. At this time and before, people noticed that rubbing various materials would cause them to then interact in unusual ways. Depending on the materials, it was found that rubbing various objects together in different combinations could cause them to attract or repel one another in different combinations. Because amber responded strongly to this sort of treatment, the Greek word $\eta\lambda\epsilon\chi\tau\rho\omega\nu$(electron) became the root used to describe all of these behaviors.

What early investigator couldn't possibly have known was that the act of rubbing material causes an excess of tiny particles we call *electrons* to move from one object to the other. Electrons are so named for the simple reason that they are what carries *electric charge* in these situations; for now, we don't need to know anything about them *other* than that they carry electric charge and are exceedingly small (however one cares to measure size).

Demo

1.1.2 Gravitational Analogue

But I haven't answered the question. Electric charge is carried by electrons, great, but that doesn't tell us what charge *is*. To understand the answer to this question, first think about what might seem an unrelated question: What is mass? We have an intuitive sense of what mass means: its how much material an object has. Mass makes things heavy, or in the absence of gravity, hard to move around (inertia). So, a reasonable definition of mass is that it is the property which causes objects to have *weight* when in the presence of gravity (Ignore for the moment the apparently unrelated question of inertia. It isn't unrelated, but that's far outside the scope of this course.) You've seen this definition of mass before, although you probably didn't realize it at the time. You see it in the force law for Newtonian Gravitation:

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$

I define the terms below to refresh your memory and make sure we are on the same page with notation. The notation $[x] = units$ (used below) means that the units or dimensions of x are *units*. You should always think about units when you see an equation or solve a problem, so I'm emphasizing them here to help.

$\vec{F}_{12}$ — The force of Gravity between 2 objects. The arrow on top indicates that this is a vector quantity. The 12 subscript means this is the force *by* mass 1 *on* mass 2. (Before reading on, do you remember what direction this should be in?) $\left[\vec{F}_{12}\right] = N$

G — Newtons gravitational constant. This is a scalar we measure through experiment. Its value is $6.67 \times 10^{-11} N \cdot m^2/kg^2$. $[G] = N \cdot m^2/kg^2$

m_1, m_2 — Masses of the 2 interacting objects. Often this is written m, M because one mass is much larger than the other. For example, for objects falling to the earth, the system is so obviously lopsided that it is helpful to make it clear that the small m is the one falling, while M is the Earth. There is no reason why this always has to be the case, however, and the physics is actually clearer if we think of the two masses on equal footing. $[m_{1,2}] = kg$

r_{12}^2 Square of the distance between the objects. $r_{12} = r_{21} \left[r_{12}^2\right] = m^2$

$\hat{r}_{12}$ This is the unit vector giving direction to $\vec{F}_{12}$. In this case, it tells us that gravity pulls each mass towards the other. The $_{12}$ subscript indicates that the unit vector points *from* mass 1 *to* mass 2. However, since the overall sign of $\vec{F}_{12}$ is negative, the direction of the force is in the *opposite* direction from $\hat{r}_{12}$. Remember, the negative of a vector simply points in the opposite direction ($\hat{r}_{12} = -\hat{r}_{21}$). $[\hat{r}_{12}] = 1$ (this means that $\hat{r}$ is *unit-less* or *dimensionless*. It just points, *nothing* else)

So what have we learned? This should be review, but I want you to focus on the role played by the masses. In this equation, the masses tell us how *strong* the force of gravitation exerted by one object on another is. When the force of gravity is stronger, we say that the objects have more mass (again, ignore the idea of inertia for now: it is actually the same thing even tho we don't know exactly why).

1.1.3 Coulomb's Law

Careful studies of electric charge and the forces it exerts by Charles Coulomb in the 1700s revealed that electric charge behaves eerily similar to mass with a few very important differences. First, lets look at the force law he discovered:

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

All I've done is change a few letters from the gravitational equation above, and lost the overall minus sign. The form is exactly the same, with a few different definitions:

$\vec{F}_{12}$ Electrostatic force between 2 objects. Otherwise the same as in the gravitation case.

k Constant of proportionality. This gets expressed in different ways in different unit systems, but its always a constant in direct analogy with G from Newton's law. Be careful not to confuse this with Boltzmann's constant, which is also written k but has a completely different meaning. The front of your text lists Boltzmann's constant for k under fundamental constants. Often we write k as $\frac{1}{4\pi\epsilon_0}$. This is the exact same thing, but written in terms of the *permittivity of free space.* Don't worry if you see an equation one way in one place and another somewhere else: there is no difference beyond k being shorter to write. In the units most comparable to G, $k = 8.99 \times 10^9 N \cdot m^2/C^2$. $[k] = N \cdot m^2/C^2$

q_1, q_2 Electric charge. The larger each q, the more electric charge the objects possess and the stronger the electrical force between them. Here is the super-important difference from gravity: qs can be either positive or negative. Masses are always positive. Because both masses are always positive and the force law contains an overall minus sign, the force of gravitation is always *negative* (*attractive*). If one electric charge is negative and the other positive, the overall force is again negative and gives an attractive force. However, if both charges are the same sign, the overall force is *positive*, which means the objects *repel* each other! $[q_{1,2}] = C$

That last point about the sign of the charges doesn't sound like much, but it has huge implications. Because of that one little detail, electric phenomena are far more varied than their gravitation cousins. As a result, the first 6 chapters of our text follow almost exclusively from Coulomb's law. Newton's gravity, on the other hand, really only does one thing: makes stuff fall. Sometimes you get orbits because things move while they fall, but that really isn't a lot of variety.

Examples

- 2 unequal charges: compare forces on each.

- 3 unequal charges in a line.
- 3 equal charges in an equilateral triangle.

Coulomb's Law lets us calculate lots of interesting results in electrostatics, but before we continue down that road, lets take a step back. We've defined electric charge, but early science discovered some interesting properties of electric charges without ever knowing Coulomb's law, or what electric charge really is. Now that we've cheated to define charge, lets see how it behaves.

1.2 Basic Electrostatics

1.2.1 Charge conservation

Our daily experience gives us an intuitive sense that mass is conserved: mass can neither be created nor destroyed so we always have the same amount, no matter what we do to it. (While this isn't strictly true in exotic circumstances, a broader law of conservation still holds, so for now trust your intuition on this one.) It was realized early on that this property also holds true for electric charge. This realization is actually why we talk about positive and negative charges rather than, say, glass charges and fur charges. Whenever you somehow build up a static charge, by rubbing e.g. a cloth on a piece of plastic, you are building up two separate charges, one positive and one negative, such that they cancel one another out. If we think of electric charge as being composed of electrons, this just means we can't destroy or create the electrons, but charge conservation is in fact a much more general principal that applies any time you have a charge.

Charge conservation is a simple idea with profound consequences: keep it in mind.

1.2.2 Atoms: Electrons and Protons

I keep talking about electrons: what's their story? Electrons are light, fundamental particles with a negative electric charge, and a few other properties that we couldn't care less about at the moment. There are a few different units for measuring charge. One commonly used unit is the Coulomb. Since the Coulomb was developed to measure charges typical of a macroscopic system of glass rods, metal balls, and scraps of fur, the electron charge is quite tiny in these units: $-1.602 \times 10^{-19} C$. In another commonly used set of units, the charge of the electron is $-1e$. This is handy, because one of the neat things about electric charge is that it only comes in multiples of the electron charge. Nowhere will you find $.3e$, or $3.14e$ of an electron's charge. You may have heard of an exception with quarks: don't worry about it, it's a special case that isn't really an exception. If you've never heard of it, then great, it has nothing to do with us.

Matter, at least normal matter like we're used to, is made of atoms. Atoms are made of a bunch of electrons flying around a nucleus. The nucleus is made of neutrons (which we don't care about right now) and protons. The only properties of protons we care about right now are that they are much more massive than electrons, and have an electric charge of $+1e$, the exact opposite of electrons. So, a clump of protons attracts a cloud of electrons such that the charges cancel out. This is how atoms work: the Coulomb force between negatively charged electrons and positively charged protons makes them stick together.

But if that were exactly the case, then the ancients never would have discovered static electricity by rubbing stuff together, because everything would stay electrically neutral. In reality, when atoms are collected together in materials, they can lose a bunch of electrons as a group. This loss is where electric charge buildups come from. Exactly how these buildups behave depends on what kind of material they are in. There are two basic types of material: insulators and conductors.

1.2.3 Insulation and Conduction

Conductors are materials which allow for the free flow of electrons. If a charge is placed on a conductor, the charge freely spreads out evenly (on the surface–we'll see why later). If two conductors touch one another, the charge will flow from one to the other.

Insulators are the opposite: they don't accept charge flowing into, out of, or through them. If you place an insulator between two conductors, the insulator will prevent the charge from flowing between them (hence the name).

This behavior is what allowed early experiments to learn much of anything about how electric charges interact. Why is that? In order to come up with his equation, Coulomb needed to have some way of knowing what the charges he was dealing with were. How do you measure something for the very first time? There aren't standards to compare against, there aren't units, there's nothing.

Coulomb's strategy would have been something like the following. Take several identical conductors. We'll call them C_1, C_2, C_3, etc. Place a charge Q_0 on C_1 (We usually use the letters q and Q for charges). It doesn't matter how you do this, anything that works is fine to start, and you don't need to have any idea how much charge there is. Next, touch C_1 and C_2 together. This will cause the charge to spread evenly between the two. Now you know you've got two equal charges $Q(C_1) = Q(C_2) = Q_0/2$, and you can measure the force between them. Next, you can touch C_3 to C_2. Now you have 3 charged conductors: $Q(C_1) = Q_0/2$, $Q(C_2) = Q(C_3) = Q_0/4$ If you now measure the force between C_1 and C_2, you've got another piece of information about the force law. If you keep at it, it is possible to profile the force law pretty well: that's why Coulomb gets his name on the formula. You will in fact use a similar method to verify Coulomb's Law in your first lab.

Of course, there's a lot more to conduction and insulation: they are the basis of our electronics-permeated environment. Lights, power, TVs, computer... It all starts with conduction, insulation, and the intermediary set of materials known as semiconductors. We don't know enough to do much with them yet, but stay tuned.

1.2.4 Induction

There is one additional interesting and important consequence of conduction we need to (and can) cover now, however. Induction is a simple consequence of two pieces of information we've already covered. First: Coulomb's Law.

Coulomb's Law tells us that charges exert forces on one another from a distance: they needn't be in contact or even very close. Second: electrons move freely in a conductor.

Now, imagine that we have 2 conductors C_1, C_2 some distance apart. C_1 is charged with Q_1 while C_2 is neutral (uncharged). What happens when they are brought close together, but don't touch? The charge Q_1 on C_1 will exert a force on the electrons and protons in C_2. The protons are locked in place, so the force doesn't really do much to them, but the electrons also feel a force, and since they are in a conductor, they can move freely. So what happens? If Q_1 is negative, it will repel the electrons in C_2. These electrons will crowd up on the side of C_2 away from C_1. The protons will stay in place. The net result is a positive charge q_+ on the near side of C_2, and negative $q_- = -q_+$ on the far. A charge has been *induced*. The net induced charge is zero, as it must be, but the two neighboring charges of opposite sign will have real, measurable effects.

Examples

- 2 spheres: one charged one not.
- Cylinder and sphere.
- Sphere and ring.