13: Electromagnetic Waves

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13.1 Theoretical Basis

We have seen that changing fields will induce other fields. We have also seen that this can set up a sort of chain reaction that results in oscillating fields flowing outward from certain systems of charge and currents. This was a purely pictorial argument, which is fine, but it leaves us with a very vague picture of what electromagnetic waves look like. We aren't going to calculate or derive the exact forms for any EM waves, but I am going to sketch out some more specifics about what the waves look like and how they behave.

13.1.1 Maxwell's Equations in a Vacuum

As mentioned before, there are two broad regions of the fields of an EM wave: near field and far field. The near field is full of all sorts of complexities dealing with how exactly EM waves are generated and are of course important, but they aren't terribly interesting in a broad picture. We will therefore be focusing on the far field region. In the far field, we aren't near any charges or currents. In fact, we treat the far field region as a vacuum. Maxwell's equations simplify somewhat in this case, making it easier to see just the wave behavior.

$$\begin{aligned}
\oint \vec{E} \cdot d\vec{A} &= 0 \\
\oint \vec{B} \cdot d\vec{A} &= 0 \\
\oint \vec{E} \cdot d\vec{\ell} &= -\frac{d\Phi_B}{dt} \\
\oint \vec{B} \cdot d\vec{\ell} &= \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}
\end{aligned}$$

In a vacuum, Gauss's Law becomes "trivial" in the sense that it always gives you 0 for the field. The only place for fields to come from here is the variation of other fields, which makes sense since we are in a vacuum. All that is here are the fields that brought themselves. Because of the lack of sources, the fields in a vacuum form into plane waves, which are the simplest and most uniform.

Given the above equations, it is possible to directly calculate the existence and functional form of electromagnetic waves. Doing so is involved, hard, and not very enlightening for us. So instead I am just going to wave my hands and conjure the relevant results straight out of thin air.

13.1.2 Electric and Magnetic fields in EM Waves

EM Waves can take an enormous variety of forms, but the most basic are sinusoidal:

$\vec{E} = E_0 \sin(kx - \omega t)\,\hat{y}$

$\vec{B} = B_0 \sin(kx - \omega t)\,\hat{z}$

This particular form corresponds to a wave which is traveling in the positive $\hat{x}$ direction. Remember that $\vec{E}$, $\vec{B}$, and $\vec{v}$ must all be perpendicular.

k	wave number
ω	angular frequency
$f = \frac{\omega}{2\pi}$	frequency
$\lambda = \frac{2\pi}{k}$	wavelength
$v = f\lambda = \frac{\omega}{k}$	velocity

The text goes through a fairly straightforward calculation using Maxwell's equations to show that this velocity ends up being

$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$.

This was one of the most shocking of Maxwell's original results, because numerically this works out to be

$v = 3.00 \times 10^8 \frac{m}{s} = c.$ speed of light in vacuum

The speed of light had of course been measured by this time, but there was absolutely no indication that light had the slightest thing to do with electricity and magnetism. Yet here was a result out of the blue which suggested that light was merely the propagation of oscillating electromagnetic fields. Quite by accident Maxwell had discovered the answer to one of the longest standing questions in physics, the nature of light. Also as a result of this discovery, it was possible to identify a number of other forms of radiation (mostly undiscovered at the time) as EM waves as well, but of a different frequency.

13.2 Description, Nature of the Waves

13.2.1 EM Spectrum

Physicists had already decided that light acted like a wave (contrary to Newton's conclusions), but it was unknown what was "waving". It was not yet known in Maxwell's time that the speed of light was a constant regardless of how its source was moving (the observation that ultimately led to the theory of special relativity), but the fact that his equations gave a specific value for the speed of propagation of EM waves without reference to the movement of the emitter was another peculiar feature of these phenomena.

Work following Maxwell showed that there are many other kinds of EM waves, differing in their frequency (or equivalently their wavelength). Radio waves we the first deliberately produced EM waves, and it was their generation and measurement which finally verified this aspect of Maxwell's theory. The variety of possible EM waves is virtually unlimited, since we can just keep cranking up the frequency, but there are only a handful of types of EM waves or radiation that we name.

Resonant frequencies Added discussion of resonant frequencies in response to a question. I'll update the notes to include roughly the same stuff soon.

Blackbody radiation Ditto for resonant frequencies.

Radio waves Lowest frequency and thus longest wavelength. The term radio waves covers a wide range of applications and corresponds generally to frequencies from thousands of hertz to thousands of megahertz (million cycles per second). Radio waves are characterized by the ability of a conducting antenna to generate and respond to the oscillation of the fields directly. When an antenna picks up a radio wave, the fields from the wave are literally accelerating the electrons in side it back and forth. At some level all EM waves are created and detected this way, but it is generally a microscopic process by some special material that gets amplified for our use (like the rods and cones in our eyes, or CCDs in a digital camera). Modern antenna technology is extending well into the microwave range for direct detection, but the distinction still has some utility in helping understand what counts as a radio wave vs. microwave.

Microwaves The same kind of radiation you use to heat up your food can be used for all sorts of other things, too, like high bandwidth communication. We'll see later that you can always carry more information with a high frequency wave, making it clear why the trend in modern wireless technology has been towards microwave frequencies away from the longer radio wavelengths.

Infrared Literally this just means "under red", and infrared waves are in fact those EM waves whose frequency is just too low to be visible to the naked eye. All animals ability to sense light is based on the same basic principals, but slight variations allow for some species to see colors that correspond to what we refer to as infrared (and ultraviolet) while potentially leaving sections of the "visible" spectrum dark to them.

Visible Light Exactly what the name suggests. Visible light is special for reasons other than the fact that we can see it, however. We've evolved to be able to see it for good reason. One is that this happens to be one of the limited ranges of the EM spectrum which the atmosphere is somewhat opaque to, but generally transparent enough to navigate through. Additionally, our Sun emits most brightly in visible light (for some reasonable definitions of "brightly").

Ultraviolet Ultraviolet of course means above violet, and these are the frequencies which are just beyond human vision on the high end of things. Bees and other animals can see a little UV radiation, bringing out some contrasts that we can't begin to notice.

X-rays When X-rays were first detected, it was obviously quite unclear what they were. They are very high frequency, which gives them a great ability to penetrate solid objects. It is this feature which has made them famously useful for medical imaging. Higher frequencies, we will see later, correspond to higher energy. Greater energy gives x-rays more oomph to punch through opaque materials, but if a material is sufficiently dense (like bone), it will stop even the x-ray. This give x-rays their special ability to take pictures of the insides of things.

Gamma Rays This term covers all very high energy EM radiation from the "hardest" X-rays to the most insanely energetic waves produced by exotic astrophysical objects spread throughout the universe. Depending on their energy they can have a wide range of properties and uses, nut none so famous as X-rays and radio waves

13.2.2 The Various Speeds of Light

The speed of light is constant in a vacuum, but if it as passing through a material, then we can find it from the permittivity and permeability of that material: μ and ϵ. As a rule, light travels more slowly through materials than it does through a vacuum. This fact leaves open the possibility for what many people mistakenly believe is impossible: objects can travel through a material faster than light itself would. This does *not* mean that the object is traveling faster than c, the speed of light in a vacuum. It means that the speed of light in this particular medium is reduced, and the object has been accelerated close enough to c that this distinction becomes important. While it is beyond our current scope, it is interesting just to note that when this happens, there *is* an analog to the "sonic boom" that accompanies any super sonic object called Cerenkov Radiation.

People have been making efforts to measure the speed of light for a very long time. Generally, since c is so huge it has required either extremely long distances or incredibly accurate measures of time in order to nail the number down better than simply "really bloody fast". In the modern era, the speed of light is in fact *defined* to have the exact value

$2.99792458 \times 10^8 \frac{m}{s}$. exact speed of light

We can assign an exact value without perfect measurements because this number then defines the *meter* in terms of the speed of light and the second (which has an independent definition in terms of atomic physics). Any correction or refinement in the measurement of the speed of light will thus actually refine the meaning of a meter, not the speed of light itself.

13.2.3 Energy in EM Waves

We've shown previously that electric and magnetic fields contain energy. Usually this hasn't come up that often, and when it has (capacitors and inductors) there has been an obvious object when contained the fields (and thus energy) in question. Now, however, we have EM fields flying off into space and yet still the claim is that they have energy. In fact, we can see that EM waves *must* carry energy by looking at the special case of radio waves. Remember that radio waves are especially easy to study as an electromagnetic phenomena because we have the ability to directly create them by moving charges, and detect them as induced currents in an antenna. If you transmit a signal from one antenna to another this way, then, you are inducing a current far far away. Any antenna is going to have some resistance, so any current requires some energy to produce. The only place for that energy to come from, even if we hadn't already derived that it must be there, is from the fields of the radio wave!

Energy Density

OK, so using what we've already learned about the energy content of fields, lets find the total energy content of our EM waves:

$u_{total} = u = u_E + u_B = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$

One simple consequence of the fact that for EM waves $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ is that

$B = \frac{E}{c}$

so we can write the entire energy density for the waves in terms of one of the fields:

$u = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} \epsilon_0 \mu_0 E^2 = \epsilon_0 E^2$

or
$u = \frac{1}{\mu_0} B^2$
or
$u = \frac{1}{2} \left(\epsilon_0 E \left(cB \right) + \frac{1}{\mu_0} B \left(\frac{E}{c} \right) \right) = \sqrt{\frac{\epsilon_0}{\mu_0}} EB.$

Energy Flow

Because EM waves travel through space and have an energy density, they must carry and transmit that energy. This flow of energy has a direction and a rate, and so is a vector. The vector we define to describe this energy flow is called the Poynting vector. We have so far defined an energy density, energy per volume, and a velocity for EM waves. We are also working under the assumption of *plane waves* as the form of our EM waves. Since a plane wave will be incident upon a wide area of whatever it is interacting with, the useful measure of energy flow is an energy flow per unit of area. We thus define the Poynting vector to have the magnitude

$S = \frac{1}{A} \frac{dU}{dt}$

where the total energy flow is going to be given by

$\frac{dU}{dt} = u \frac{dV_{ol}}{dt} = u \left(A \frac{dx}{dt} \right) = uAv$

where the wave is traveling in the $\hat{x}$ direction. This gives us for S

$S = uv = \sqrt{\frac{\epsilon_0}{\mu_0}} EB \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\mu_0} EB$

Since we already know that $\vec{v}$ for an EM wave is proportional to $\vec{E} \times \vec{B}$, we can write the full Poynting vector as

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}.$

We can take the average of this instantaneous value, since we are usually more interested with energy buildup over time periods larger than the tiny fractions of a second characteristic of EM wave frequencies.

$\overline{S} = \frac{E_0 B_0}{2\mu_0} = \frac{E_{rms} B_{rms}}{\mu_0}$

13.3 Applications

13.3.1 Radiation Pressure

We of light as entirely immaterial and without substance. While this is of course true for the traditional meaning of material and substance, we have just learned that it carries energy. If something carries energy and is traveling through space, then it must necessarily have some momentum! The momentum of an EM wave is given by the form

$\Delta p = \frac{\Delta U}{c}$.

We can't use the more traditional form for momentum of $p = mv$ because, well, light doesn't have a mass, but the above is just as (more, in fact, but lets not get too ahead of ourselves) valid. Once we know the change in momentum of an object, we can find a force applied,

$F = \frac{\Delta p}{\Delta t} = \frac{1}{c}\frac{\Delta U}{\Delta t} \longrightarrow \frac{1}{c}\frac{dU}{dt}$.

This expression is valid so long as all of the energy of the incident wave finds its way into whatever it is impacting. If, instead, the wave is reflect back, we must make a modification. Just like in elastic vs. inelastic collisions in mechanics, we have to worry about conservation of momentum. If we start off with some net momentum $\vec{p}$, we must end with that same total. If the wave is reflected back rather than being absorbed, it now contributes $-\vec{p}$ to the total, meaning that it must have transferred $2\vec{p}$ to whatever it reflected off of, in order to keep the sum the same. So, for reflected waves Δp, F, and everything proportional to them will be doubled as compared to the case in which the wave is absorbed.

Since we are dealing with plane waves, we will actually be more interested in the force applied per unit area, otherwise known as the pressure $P = \frac{F}{A}$. Since we've already defined the Poynting vector as the energy transfer per unit area, $\frac{1}{A}\frac{dU}{dt}$ is already just

$\frac{1}{A}\frac{dU}{dt} = \overline{S}$

So to find P, we just divide by c.

$P = \frac{\overline{S}}{c}$ pressure due to absorbed EM wave

$P = 2\frac{\overline{S}}{c}$ pressure due to reflected EM wave

Example: Solar Sail

When I first heard the idea of a solar sail I thought it was a joke or some silly sci-if notion. The idea of a spacecraft being pushed through space by sunlight hitting large reflective surface, tacking and trimming like an old Man-of-War just seems preposterous at first blush. However, with a better understanding of light based on their nature as electromagnetic waves with energy and momentum, it starts to sound a little less crazy. The idea is that you throw up a fairly enormous reflective surface, attached to your craft. When sunlight hits your "solar sail", there is a transfer of momentum that pushes you forward. And in fact many of the tactics of traditional sailing can be used to accelerate in directions other than straight away from the sun. In order to get an idea how reasonable this idea is, we need to figure out how big the sail would have to be to provide some sort of reasonable thrust.

Thrust is just force, and force is pressure times area:

$F = PA = 2\frac{\overline{S}}{c}A$

At the Earth's orbit, the energy content of the sun's visible radiation (our sail only reflects the visible part of the spectrum in this example) is about $\overline{S} = 1000\frac{W}{m^2}$. If we assume a reasonably but not utterly preposterous size for the sail of $(1km)^2 = 1 \times 10^6 m^2$, this gives for the thrust

$F = 2\frac{1\times10^3 \frac{N\cdot m}{s\cdot m^2}}{3\times10^8 \frac{m}{s}} 1 \times 10^6 m^2 = \frac{2}{3} \times 10^1 N \simeq 7N$

OK, $7N$ sounds pretty tiny, and it certainly isn't going to get you out of earth's orbit. However, once your craft is essentially traveling freely through space with no gravitational forces to oppose it, this constant acceleration adds up over time. Since there is no fuel, your craft can start light and stay light (The weight of a rocket is almost entirely the rocket fuel itself. The vast vast majority of fuel is expended in order to launch the other fuel. This is one of many reasons why rocket science is difficult.). We could pick a mass and use basic ballistic motion from 113 to calculate how far a craft would travel, but rather than do the math I'll just quote the example the text uses: a $500kg$ craft (not too small, this is about $.55\,tons$) would be propelled about $4AU$ in a year with this thrust. Considering that it take rocket powered flight several years to reach most destinations in the solar system, this is entirely competitive! Of course, you have to use rockets to get out of the earth's gravitational well to start with....

13.3.2 Wireless Communication/Detections

In modern times we are constantly surrounded by a variety of man-made electromagnetic waves. Every broadcast and satellite TV station is present in the air around you all the time (or at least outside in the case of satellite). Every AM/FM station you can pick up is flowing around and into your body. The UR wireless network, any emergency response network, nearby ham radio hobbyist, etc. all produce EM waves that fill the space around us.

In order to pick out the specific signal you are interested in, we use the frequency. Each signal has associated with it a frequency range. There are different ways of encoding the information within that signal, but everything relies on one channel or station having a sufficiently different frequency that we can separate it out from its neighbors. The ability to do this actually stems from the principal of superposition! The total EM field around us is composed of the sum of all of the different signals being sent out. But if we construct our antenna such that it is "tuned" to a specific frequency, we can still pull out just that piece. Antenna design can get quite complicated, but the fundamental point is that we want it to be especially responsive (resonant) to a specific frequency, while as impervious as possible to others. A simple way to do this is to make an antenna which is the same length as the wavelength of signal you want to receive. Think about the example of a wave in a rope. Remember that we can set up a "standing wave" for a given length of rope by driving it up and down at just the right speed such that the end on the other side of the room doesn't move. This is the only frequency of wave we will find it easy to maintain in the rope if we were to then fix the end. In the same way, the ends of the antenna "fix" the fields and the only frequencies the antenna will respond to are those that can fit between the ends evenly.