16: Light Waves and Interference

16.1 Wave vs. Particle Debate and Evidence

By this point we all "know" that light is a wave, and in particular we've discussed that it is an electromagnetic wave. However, we used the ray picture of light, dating back to Newton's particle theory of light, for geometrical optics and it worked just fine. Historically speaking, the nature of light has been one of the more confusing and controversial debates in physics and as we shall see when we start quantum phenomena, the WM wave picture isn't the end of the story.

The fundamental reason that it was hard to discern the wave nature of light boils down to the simple fact that visible light has really short wavelengths. All of the behaviors that distinguish a propagating wave from a flying stream of particles are dependent upon the wavelength of the wave in question, and since typical visible light is around $500nm = 5 \times 10^{-4}mm$ there aren't a lot of everyday experiences that make the wave nature obvious. This turns out to be the exact same reason that much of quantum mechanics seems alien and wasn't discovered earlier, but on a much more extreme scale, so keep the concept in mind.

The 3 main wavelike behaviors of light that can be used to identify it as such are diffraction, refraction, and interference. To understand how these work lets go back and discuss Huygens' principle, which explains much of generic wave behavior from water waves to sound to light and so forth.

16.2 Huygens' Principle

Given a single source of a wave in isolation, it is easy to draw where the wavefronts (or crests) will travel (assuming sufficient artistic ability, that is). The wave will just propagate outward at a uniform speed, producing perfect concentric evenly spaced circles. However, it is much more difficult to predict

1. What happens when this wave encounters an obstacle such, as a rock near the shore of the ocean?

2. How a pre-existing, non-circular, wavefront will propagate forward with time.

Huygens' Principle deals with this problem. We first look at the second scenario. Given such a wavefront, Huygens' principle simply tells us to treat *each point* on the wavefront as its own source. When we draw concentric circles (with a uniform radius equal to the wavelength) around each point of this wavefront, their tangents will form a new front out front of the original. In fact, two new fronts will be defined. One is where the wave is going, and one tells you where it likely came from some short time ago. In order to know which is which we need to know which direction the wave is traveling, tho we can make educated guesses based on surroundings.

Dealing with obstacle encounters can be more complicated, but is still solvable with Huygens' principle. When a wave encounters an obstacle, it can either be reflected, absorbed, or transmitted. If the obstacle absorbs the wave, our job is simplest. A full obstruction which absorbs the wave just makes the wave go away and there is nothing to be done. But if the obstruction is only partial, then it destroys the sections of wavefront which encounter the obstacle, leaving the rest to propagate. We use Huygens' principle on the remaining sections of wavefront to discover how the wave will propagate after having a section destroyed. This process will lead to diffraction.

When an object reflects rather than absorbs, we again use Huygens' principle, but now we must treat each point along the obstacle as a source point. To the extent that wavefronts impact the obstacle at different times, different points on the obstacle will be treated at sources at different moments. For a complex shape this can be impossible to draw by hand, but with a computer can be used to produce an extremely effective picture of scattering from a complex object. This is easier for simple cases, but we still aren't going to worry about it here as there are easier ways to understand reflection: Huygens' principle's strength lies elsewhere even tho it works everywhere.

The last option for a wave incident upon an obstacle is transmission. Here again Huygen's principle is effective. In this case the important thing to remember is that the wavelength of our wave may be different in the obstacle material than the original, and this will affect how we draw our wavefronts. Doing so properly will allow us to predict Snell's Law and accurately model refraction.

16.2.1 Diffraction

To understand diffraction, consider a barrier with a hole in it, and a series of parallel wavefronts approaching from one side. Assume that any section of wavefront which impacts the barrier is absorbed so that we can ignore reflection. The barrier leaves a single section of wavefront: from this will propagate whatever waves are going to make it through the barrier. When we use Huygens' principle to see what happens, we discover that the result depends on how wide the hole in our barrier is with respect to the wavelength of the light. If the hole is very large, we essentially get a set of parallel wavefronts cut from the original which just keeps going straight. However, for smaller holes, the wave starts to bend around the opening, spreading out slightly. And if the hole is roughly the size of the wavelength, it behaves like a single point source and half-circle wavefronts propagate evenly out.

Note the fact that even in the case of a large opening there will be *some* spread because the "sources" of the wavefront right next to the edge will produce concentric circles as their contribution to the whole, and those will propagate out to the sides as ever. The difference between the parallel component and these edges is the strength of the wavefront. A front coming from a single point along the previous front, as at the edge of a large gap, will be *extremely* weak compared to a front composed by adding up the contributions of a continuum of source points along the barrier gap. The fringes become relatively stronger as the proportion of source points near the edge of the gap gets larger and larger. Once the whole gap is about the size of a wavelength or a few, the entire gap is near the edge and that proportion becomes an equality and the "fringes" are just as strong as the forward propagating portion so we get a circular wavefront again.

16.2.2 Refraction

Newton's particle theory of light predicted that it would travel more quickly through water (higher index of refraction) than air. I'm not familiar with his theory, so I don't know how he avoided predicting that particulate light would eventually slow down and stop in dense semi-transparent material like water, but he was a smart guy so we can assume that it made sense. In any event, Huygens predicted the opposite based on his wave theory: light travels more slowing through water than air. Luckily for Newton he got tons of other stuff right so we remember him for that instead.

To see how Huygen's principle predicts Snell's law, draw a single wavefront just about to pass from air to water at an angle. Now, draw another wavefront one wavelength ahead of that one. Ah, but what is a wavelength ahead? That depends on where along the wavefront you are looking. At the end just touching the water, one wavelength ahead is the wavelength in water (we aren't actually worrying about exact distances here, we just care that λ is shorter in the water than the air). Further away, the wavefront can travel a whole λ_{air} without encountering air. Starting from the farthest edge, draw the new wavefront until it encounters the water. Now we just have the region between these 2 extremes to fill in.

Consider now that if the light travels some fraction of a wavelength through air, and some fraction through water, the 'effective' wavelength for that whole wavelength is just going to be the average of the 2 weighted based on how far through each it travels. We can connect the two extremes of our wavefront with a straight line. This leaves us with a single wavefront with a single kink in it. We can now repeat this process for the next wave front, and so on, until the whole thing is traveling straight through the water. We could work out the geometry associated with this picture (as your text does. Go ahead and read it, its short and easy algebra) and find that

$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$.

Since this is just like Snell's Law except with velocities rather than the index of refraction, and Snell's Law was a known experimental result, Huygens' theory *predicted* that the ratio of velocities was proportional to the ratio of indicies of refraction, and that light traveled more slowly in the material with a larger index. A measurement of the velocity of light in water could have settled the question of the nature of light at this point, but nobody could measure it yet, and in fact by the time such experiments were doable everyone was sold on the wave theory anyway. Its still a neat result, tho.

16.3 Interference

Interference is the most definitive hallmark of a wave. Even more-so than refraction and diffraction, its just impossible to find a way to make streams of particles interfere with each other. Only waves can exhibit this kind of behavior and so observing it in light was considered the end of the debate at the time.

Interference can be seen between two *coherent* sources of a wave, which just means that the wavefronts are in sync rather than randomly distributed. Coherent sources of light are actually exceedingly rare in nature, which is why we don't see interference everywhere we look. It is possible to create effectively coherent sources, however, in the double-slit experiment.

In the double slit experiment, a single source of light is shone on a barrier which has 2 narrow openings close to each other. Behind this barrier is a screen of some sort that allows us to see what the light looks like on the other side. The ray theory of light would of course predict that 2 sharp spots show up on the screen, while a naive application of our understanding of diffraction would predict a superposition of 2 fuzzy blobs. In reality, we see a series of bright and dark stripes as a result of interference between the wavefronts coming from each slit.

Interference is possible because a wave gives both positive and negative values for whatever is waving at different points along its propagation. In the case of light, this means that there are points with $\vec{E} = E_0\hat{z}$ and points with $\vec{E} = E_0\,(-\hat{z})$, with appropriate magnetic fields at each point. If two light waves encounter each other at a point, the principle of superposition tells us to add the fields together. If both waves have positive fields, the result is *constructive interference* and the result is a stronger, brighter wave than either alone. If, on the other hand, they have opposite signs, there results *destructive interference* and a weak or nonexistent wave results.

With coherent sources separated by some distance d, an interference pattern will appear on the screen. We can determine the location of bright and dark spots by calculating where both waves will have gone through an integer number of oscillations/wavelengths (meaning they will once again be in sync and add) or an odd-half-integer number of oscillations/wavelengths (meaning they will be perfectly out of sync and cancel).

Geometrically, this works out to mean that the difference in the length traveled by the two waves is

$d\sin\theta = m\lambda$ Constructive

$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ Destructive

where $m = 1, 2, 3...$ and we can use the same angle for both waves because the slits are very close together, making d small and the waves nearly parallel. This is the same approximation we made while calculating the electric potential far from an electric dipole.

16.3.1 Color Fringing

The fact that the wavelength appears in the formula for the location of the interference patterns means that different colors will have different patterns. This results in a rainbow pattern when white light (such as from the sun) is used as a source for the double slit experiment. This actually enabled the first measurement of the actual wavelength of visible light. This is also one reason why the clearest demonstrations of interference fringing tend to use monochromatic (single wavelength/color) light.