Electrical Potential Difference and Electrical Potential Energy

July 8, 2008

First off: electrical potential (often called electrical potential difference for clarity) is *not the same thing* as electrical potential energy! The name is annoyingly misleading but unlikely ever to change, so we have to live with it. Make sure you always know which is being referred to in any description or equation. Potential energy will generally be represented by the symbol U (or U_a or some such), while potential difference (or electrical potential, or just plain old potential when maximum confusion is being sought) is typically represented by V (or V_a...). This is clear enough in print, but when handwritten can get pretty difficult. If ever my board work is unclear, please complain.

This opportunity for confusion is made worse by the fact that potential energy and potential difference are in fact related. We start with potential energy.

4.1 Definitions

4.1.1 Electrical Potential Energy

Electrical potential energy, as in the case of e.g. gravitational potential energy, is measured as a difference between two points. An absolute potential energy has no meaning. The difference in potential energy between two points a, b (in general, not just electrical) is the negative of the work done to move from a to b.

$U_b - U_a = -W_{ba}$ $\qquad [U] = [W] = J \ (J = joules)$

where W_{ba} is the work required to move from a to b.

$W_{ba} = \vec{F} \cdot \vec{d} = q\vec{E} \cdot \vec{d}$

and so

$U_{ba} = U_b - U_a = -q\vec{E} \cdot \vec{d}$

where $\vec{d}$ is the vector separating b from a, $\vec{d} = \vec{b} - \vec{a}$. Your text gives scalar expressions for these, but this can be misleading if the force isn't pointing directly along the direction of motion. An example of this form mechanics was sliding a block along a surface by pulling a rope, which may have had a vertical component. Keeping the dot product reminds us that it is the distance *along the direction of applied force* which gives the work done.

4.1.2 Electrical Potential Difference

In the same way that the electric field is initially defined as a trivial modification of the Coulomb force, $\vec{E} = \frac{\vec{F}}{q}$, the potential difference is defined simply by

$$V_a = \frac{U_a}{q} \text{ or } V_{ba} = \frac{U_{ba}}{q}. \qquad [V_a] = \frac{J}{C} = V \ (V = Volt)$$

Sometimes we write ΔV instead of V_{ba} and ΔU instead of U_{ba}. The delta "change in" notation is more general in that it doesn't refer to specific points. Because of this, we use it when we are making generic statements about potential difference or potential energy, while in a problem with specific points we are more likely to use the specific notation (especially if there is the possibility of confusion as to which points we are comparing). Combining these two definitions gives us a few expressions for ΔV:

$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}. \qquad [V_{ba}] = V \ (V \textit{ called voltage here})$$

Like in the case of defining $\vec{E}$ in terms of $\vec{F}$, the q in this case is the charge of a test charge, not the charge producing the potential difference.

Let me re-iterate that speaking of the electrical potential at a single point has no meaning without a difference. Anytime you see something like V_a (electrical potential) all by itself, there is an *understood 0 point*. That 0 point may be the ground, the point at infinity, or anywhere else. It should be defined somehow by the context. Really, V_a by itself is shorthand for $V_{a0} = V_a - V_0$ (electrical potential difference) where 0 refers to the ground point. I will try and keep this explicit when I do examples, feel free to object if I use a notation which is unclear.

Note from this definition that the electrical potential is higher near positive charges and lower near negative charges. This follows because a positive charge will feel a force away from the positive charge. This means that the work done is positive leaving a positive charge (work done increases as you move away). Since potential difference is the opposite of work done, potential difference will go *down* going from a positive to negative charge.

Note that potential difference is a scalar value, despite the fact that it measures a change between two specific points. The analogy to think of is elevation. There is a difference in height between any two locations which is just a number (with units). This analogy is actually almost perfect: electric potential corresponds to elevation, while electrical potential energy corresponds to gravitational potential energy. A mass that is high on a hill has a large gravitational potential energy while a charge at a region of high electric potential has a large potential energy. Electrical potential is a scalar field just like a bunch of hills. Think of it that way in what follows and you'll have a better intuitive feel for the electrical potential. Of course, you also have to remember that for a negative charge all of the hills are reversed! Negative charges feel the opposite force, so everything from there gets reversed too.

Another way of putting the above definitions together gives:

$\Delta U = U_b - U_a = q\left(V_b - V_a\right) = qV_{ba}$.

4.2 Electrical Potential and the Electrical Field

From mechanics, we remember that

$$U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{\ell}$$

If we divide both sides by the charge of a test charge,

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}$$

It shouldn't be surprising that the potential is defined from the field rather than from the force, since both were defined by factoring out the test charge from the more familiar quantity (U and $\vec{F}$).

The simplest example of electric potential difference is that of two parallel charged plates, as seen in Fig[4.1]. The electric field is constant between the plates, so integrating from one to the other is trivial. Start at the positive plate and integrate to the negative, thus finding the potential difference $V_- - V_+$.

$V_{-+} = V_- - V_+ = -\int_+^- \vec{E} \cdot d\vec{\ell} = -Ed.$ $\qquad [V] = \frac{N}{C}m = \frac{J}{C}$

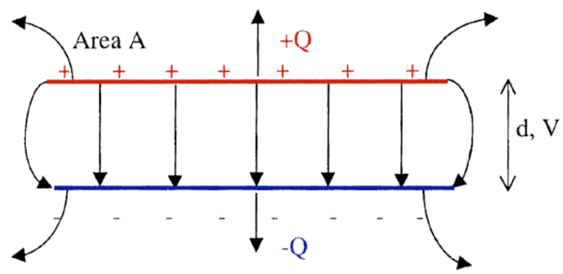


Figure 4.1: Parallel Charged Plates

The dot product is positive because both $\vec{E}$ and $d\vec{\ell}$ point in the same direction, from $+$ to $-$. This number is negative because V_+ is, as we argued earlier, larger than V_-.

It is worth noting explicitly even tho it is clear from the definition that $V_{ab} = -V_{ba}$ and $U_{ab} = -U_{ba}$.

4.3 Principal of Superposition

The law of superposition applies to electrical potential as well as to the electrical field:

$V = V_1 + V_2 + V_3 + \cdots$

where each V_i is the potential due to a separate charge distribution. It follows that

$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \cdots$

This means that we can use the expression for the electric potential of a point charge (see first example for derivation) to build up the potential of arbitrary charge distributions just as we can use Coulomb's law to find field expressions for arbitrary charge distributions.

$$V_a = \sum_{i=1}^{n} V_i = k \sum_{i=1}^{n} \frac{q_i}{r_{ia}}$$

We can also produce an integral from this:

$$V_a = \int dV_a = k \int \frac{dq}{r}$$

There are 2 differences between this expression and that we use for $\vec{E}$. First, the potential drops off as $\frac{1}{r}$ rather than $\frac{1}{r^2}$. This is significant, but doesn't make a huge difference in calculation. The fact that V is a scalar, however, can make these integrals much much easier to evaluate than their field counterparts.

4.3.1 Examples

Point charge

The electrical potential of a point charge illustrates the importance of taking a difference rather than evaluating at a given point. To see this, start with the electric field:

$\vec{E} = k\frac{Q}{r^2}\hat{r}$

so then

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell} = -kQ\int_a^b \frac{dr}{r^2} = -kQ\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Looks reasonable enough, right? Well, what if we decided that we wanted to set the reference point b as the origin, the location of the point charge itself? This is a natural enough thing to ask, since it feels similar to what we do when we write the electric field with reference to the origin where the charge lies. However, if $r_b = 0$,

$$V_{0r} = -kQ\left(\frac{1}{0} - \frac{1}{r}\right) \Rightarrow kQ\left(\frac{1}{r} - \infty\right) = \infty$$

This tells us nothing! This doesn't depend on anything, its just ∞ everywhere. This illustrates the important point that you have to measure electric potential difference from somewhere with a finite potential, *not* from a point of finite charge. Its still OK to measure from a point with a charge density such as ρ or σ because there is an infinitesimal charge at any given point and this problem doesn't rear its head. We'll see this in a later example.

Lightening rods

There was a question in one of the workshops about why lightening is more likely to strike taller objects such as trees and houses. I gave 2 reasons but neglected a third, which depends upon electric potential. As discussed in workshop (and the text), air is normally an insulator. This is what allows large charge differences to build up in the clouds relative to the ground.

However, every insulator has a limit to how large a field it can withstand and air is no different. At some point, which turns out to be about $3 \times 10^6 V/m$, the air molecules start to become ionized by the field. When this process starts, those ions are accelerated rapidly through the rest of the air, knocking electrons off from neighboring molecules and creating more and more ions. This cascade of flowing charge, driven by a strong electrical field separating clouds from sky, is lightening. The thunderclap arises because the superheated ionized gas left by the bolt rapidly cools once the charge differential has been equalized. Once this happens, the pressure of the column of ionized air connecting cloud and ground drops and the surrounding air rushes back in. When the air from all sides meets, there is a concussive shock wave which expands outward. That is the thunderclap. It actually turns out there is still some controversy on how exactly the shock wave is produced (it may be from the initial heating rather than abrupt collapse, for instance), but this is at least the basic root cause.

The reasons lightening is more likely to strike taller objects is threefold, although two of the reasons are actually related.

1. They are simply closer. It takes less energy to ionize a shorter column of air for the charge to pass through.

2. Tall objects tend to be "pointier" in that the have a smaller radius of curvature. This isn't always true (a huge sports dome, clearly, has a smaller radius than my head) but on average is.

    (a) A smaller radius of curvature means a larger charge density on the surface (if the object is a conductor).

    (b) A smaller radius means a higher local voltage at the surface, assuming the same total charge.

Why are (a) and (b) true? It turns out that (a) is a lot more difficult to demonstrate than I remember. You'll have to be satisfied with a bit of "hand waving" rather than a derivation, as it would be impenetrable anyway. (b) we'll still show explicitly.

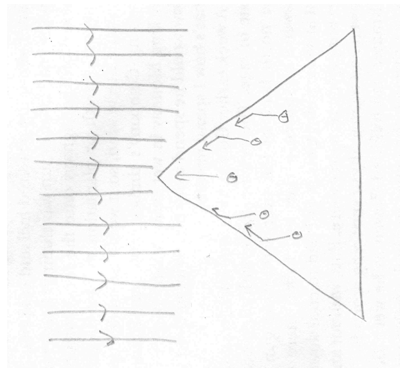
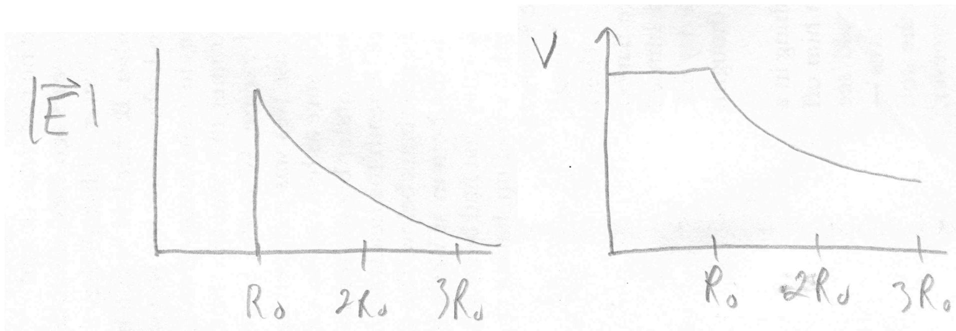


Figure 4.2: Charge Density of a Point

The reason that objects with points have larger induced surface charges is the same reason that induced charges happen in the first place. In the absence of an applied field, the free electrons in a conductor will distribute themselves evenly due to their repulsive forces. The external field overcomes this mutual repulsion (somewhat), pushing the electrons together on one side. The external field wants to create a certain charge imbalance in order to cancel the applied field. When you have a pointed end, however, the conductor basically acts like a funnel to the electrons. Each electron feels a sum of forces from the applied field and its neighbors. They equilibrate where these contributions all balance out. However, when there exist corners or regions of curvature, there is a difference from point to point in the effect of the conductors edge. Remember that electrons flow freely *within* a conductor, but the edge of that same conductor acts like a wall to them. If one part of the surface curves differently than another, electrons in different places will be pressed up against the wall at different angles. The forces that keep the electrons spread out are the components of their mutual repulsion which happen to be perpendicular to the walls (since they are all pressed up against the wall). When an electron is being pushed against the wall at an angle rather than straight-on, that means that the external force will be driving it to one side or the other (towards the point in Fig[4.2]). This then means that it will require either more charges or more closely spaced charges on the side of the tip relative to the flat side in order to balance the forces.



(a) Electric Field (b) Scalar Potential

Figure 4.3: Field and Potential of a Conducting Sphere

The same logic applies in reverse if we rotate the triangular conductor and a larger density of positive charge accumulates in the tip instead of negative. I realize this explanation isn't the clearest. We will revisit this question again after we discuss equipotentials to demonstrate their descriptive power.

(b) is by comparison quite simple. Consider a conducting sphere of radius R_0 with charge Q. We know the electric field outside of this sphere, which means we can calculate the potential difference as well. Let us take our point of comparison to be the point at infinity ($V_\infty = 0$) such that $V(\vec{r}) = V_r - V_\infty$ (see definition above for measurement from a given 0 point.

$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}$

There are two regions for $\vec{E}$: $r < R_0$ and $r > R_0$. Recall that

$\vec{E}\,(r < R_0) = 0$ and $\vec{E}(r > R_0) = k\frac{Q}{r^2}\hat{r}$.

So

$V\left(|\vec{r}| > R_0\right) = V_{r\infty} = -kQ\int_\infty^r \frac{dr'}{r'^2}$

There is no integral from 0 to r because the field is 0 there.

This is a simple enough integral $\left(\int r^{-2}dr = -r^{-1}\right)$ and gives us

$V\left(|\vec{r}| > R_0\right) = -kQ\left(-\frac{1}{r} + \frac{1}{\infty}\right) \Rightarrow k\frac{Q}{r}$

$V\left(|\vec{r}| < R_0\right) = k\frac{Q}{R_0}$.

We could easily have defined $V(0) = 0$ instead, such that then $V(\infty) = -k\frac{Q}{R_0}$. The difference is the only meaningful number, and it remains the same.

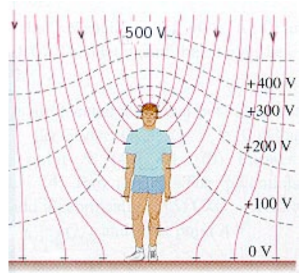


Figure 4.4: A Person on the ground during a storm

Fig[4.4] shows the situation for an individual standing outside on a field while a charge differential is building up between the sky and ground. Notice that the field lines concentrate on his head: this is the area with the smallest radius of curvature. This figure also labels equipotential lines: you can come back to it later.

Point charge inside a conducting shell (see worked examples)

From the worked examples:

$\vec{E}\,(r < R_1) = k\frac{q}{r^2}\hat{r}$ (Region I)

$\vec{E}\,(R_1 < r < R_2) = 0$ (Region II)

$\vec{E}\,(r > R_2) = k\frac{q+Q}{r^2}\hat{r}$ (Region III)

To find the potential difference, we need to integrate through each of

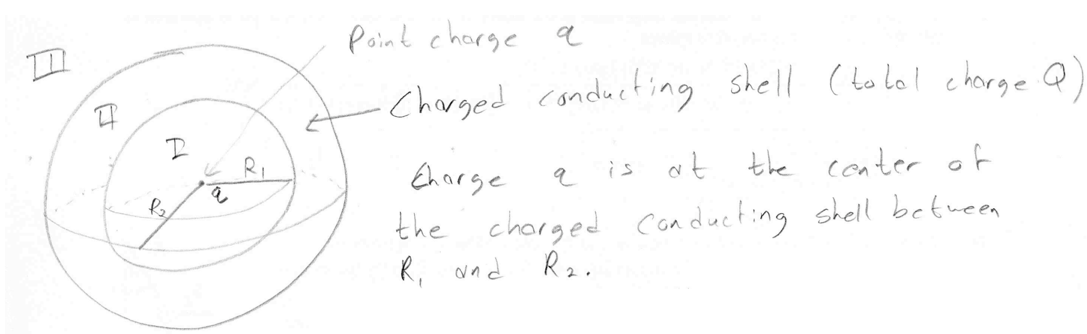


Figure 4.5: Point charge inside a charged conducting shell

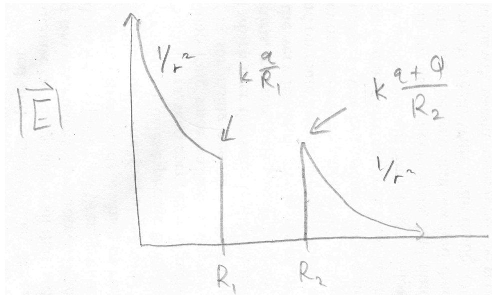


Figure 4.6: Electric field of composite spherical structure

these reasons and add it together. So first we find the contribution to V from the $r > R_2$ region.

$V_{III} - V_\infty = -\int_\infty^r k\frac{q+Q}{r'^2}\hat{r}\cdot dr'\hat{r} = -k\left(q+Q\right)\left(-\frac{1}{r}+\frac{1}{\infty}\right) = k\frac{q+Q}{r}$

Once we get inside of R_2, this portion of the field stops making additional contributions but has already raised the potential from 0 at ∞ to $k\frac{q+Q}{R_2}$. For the region from R_1 to R_2 we need to add the contribution from region III to the integral over the field in region II. In this particular case, that integral is 0 because the field is 0. So,

$V_{II} - V_\infty = k\frac{q+Q}{R_2}$.

To find the electric potential in region I, add the integral over the field to the value of the electric potential in region II:

$V_I - V_\infty = V_{II}\left(R_1\right) + -\int_{R_2}^r k\frac{q}{r'^2}\hat{r}\cdot dr'\hat{r} = k\frac{q+Q}{R_2} - kq\int_{R_2}^r \frac{dr'}{r'^2}$

$V_I - V_\infty = k\frac{q+Q}{R_2} - kq\left(-\frac{1}{r}+\frac{1}{R_1}\right)$

$V_I - V_\infty = k\left(\frac{q+Q}{R_2} - \frac{q}{R_1} + \frac{q}{r}\right)$

Charged Insulated Cylinder

4.4 Equipotential Lines/Surfaces

One of the powerful things about the electric field is that you can draw it and look at it and think about the behavior of the field and charges based on that graphical representation. For many people, having such a visual

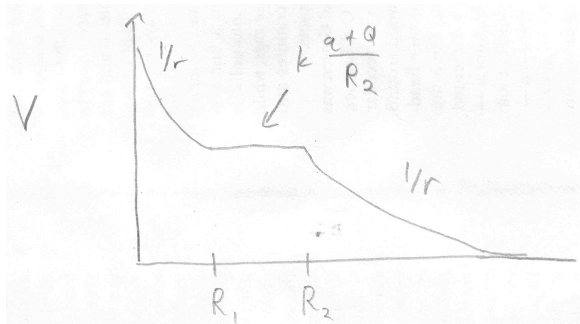
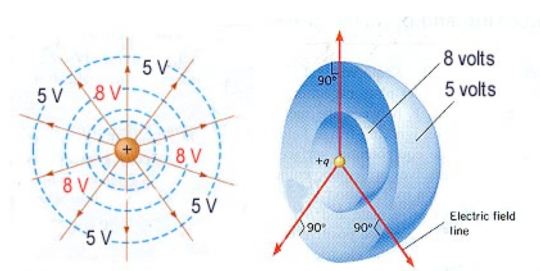


Figure 4.7: Electric Potential of Compound Spherical System

handle on the physics is extremely helpful to understanding. The electric potential has a similar visual representation. Since V is a scalar rather than a vector, things are a bit different. Not nearly as different as you might expect, however.

Instead of field lines, we draw equipotential lines or surfaces. Equipotential lines are exactly analogous to the elevation lines on a topological map (or the Terrain view on `maps.google.com` if you ignore the colors). The lines on a topological map connect points of equal elevation while lines on an equipotential of equal electric potential (hence the name). Equipotential surfaces come in when we deal with 3D systems but are built on the exact same concept.



(a) 2D (b) 3D

Figure 4.8: Equipotentials and Fields around a Point Charge

Fig[4.8] shows the equipotential lines for a point charge. Notice that the electric field lines and the equipotential lines are perpendicular. This is always true for a static charge distribution! This fact makes for a really easy sanity check for both. Sometimes its easier to see what the equipotentials should look like, others the field. If you think about both and find sets of orthogonal lines, there is a good chance you've gotten it right.

Notice in Fig[4.8] that the equipotential lines get further apart far from the point charge. This is because the electric potential is varying more slowly further out. If you saw a region on a elevation map with very spread out elevation lines versus one with more closely spaced lines, you would conclude that the first region had a much more gentle slope and that a mass sitting on the ground would feel a weaker force dragging it downhill than a mass placed in the region of narrowly spaced elevation lines (the side of a steep hill). The same thing happens here: a charge placed in a region with narrowly spaced equipotential lines will feel a stronger electrostatic force. If the charge is positive, this force will be "down hill", while for negative charges, they will be pushed "up hill".

The example of the elevation map should sound familiar. It is one of the examples I used for a scalar field before introducing the electric field. This brings up a point of nomenclature: it would actually make more sense to refer to $\vec{E}$ as the electric vector field, and to call V the electrical scalar field. This is even more clearly true when you study more advanced topics in electromagnetism. Sadly, the historical names are with us to stay and V is the electrical potential, tho I may still refer to it as the "scalar potential".

4.4.1 Examples

Electric Dipole

Dipoles are one of the places where the scalar potential is much easier to deal with than the vector field. In order to calculate the general field around a dipole using the techniques we've learned so far would require us to add the fields from two point charges with different origins in spherical coordinates. The general case of this results in a huge mess of varying unit vectors and expressions which are too complex to easily interpret, which is why so far we have restricted ourselves to points which are equidistant between the

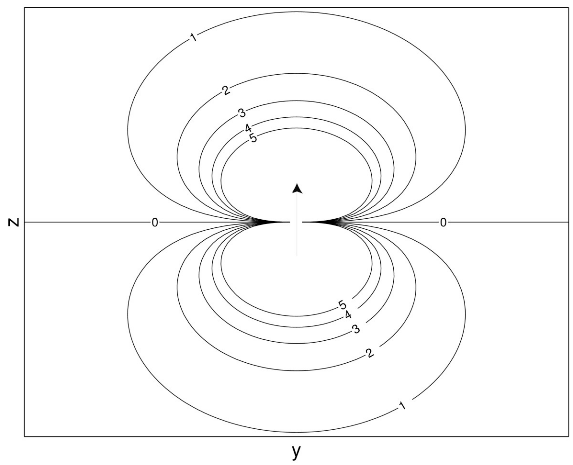


Figure 4.9: Equipotentials around a Dipole

two charges. With the scalar potential, however, all we need to know is the distance of our test point P from each charge. We needn't know the direction of a unit vector which has been converted, much less how to add them. We simply adjust the distances.

$V_P - V_\infty = V_{P\infty}(q_+) + V_{P\infty}(q_-)$

$V_{P\infty} = k\dfrac{q_+}{r_+} + k\dfrac{q_-}{r_-} = kq_+\left(\dfrac{1}{r_+} - \dfrac{1}{r_-}\right)$

(Note that the text uses a different but equivalent method to arrive at the limiting case. This way takes you through the exact result. While this result is a little ugly, it is usable and far easier to arrive at than the analogous $\vec{E}$.

In Fig[4.10] we see that the general result for r_+ is

$r_+ = r_-\sqrt{\left(\frac{\ell}{r_-}\right)^2 + 2\left(\frac{\ell}{r_-}\right)\cos\theta_- + 1}$

which would give as the general result for the scalar potential

$V_{P\infty} = \frac{kq_+}{r_-}\left[\left(\left(\frac{\ell}{r_-}\right)^2 + 2\left(\frac{\ell}{r_-}\right)\cos\theta_- + 1\right)^{-1/2} - 1\right]$

If we want to know the simplified behavior far from the dipole, we are considering cases where $\frac{\ell}{r_-}$ is small. We could just set it to 0, but then the whole potential would be 0. That's a sign that we have been too eager in our attempts to simplify, but is correct as far as it goes: far enough away,

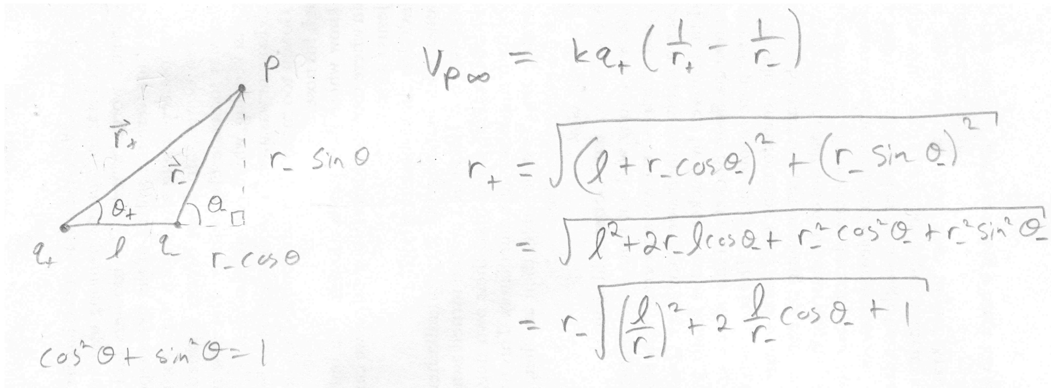


Figure 4.10: Scalar Potential of a Dipole

the scalar potential does in fact drop to 0. In order to keep a little more information, however, lets only set $\left(\frac{\ell}{r_-}\right)^2$ to 0.

$$\overset{\lim}{\ell \to 0}\left(\frac{r_+}{r_-}\right) = \sqrt{2\left(\frac{\ell}{r_-}\right)\cos\theta_- + 1}$$

we can expand this as a power series (if you aren't familiar with power series, don't worry, I'm not going to ask you to use them). To refresh, the power series for a function is:

$$f(x) = f(0) + f'(x)\big|_{x=0}\, x + \left.\frac{f''(x)}{2!}\right|_{x=0} x^2 + \mathcal{O}\left(x^3\right)$$

The idea being that each successive term will be less and less significant if x is a small number. We are going to expand in a series where ℓ is the variable, because that is the thing which is small.

$$\begin{aligned}
\frac{r_+}{r_-} &= 1 + \left(-\frac{1}{2}\right)\left(2\frac{\ell}{r_-}\cos\theta + 1\right)^{-3/2}\frac{2}{r_-}\cos\theta\bigg|_{\ell=0}\ell + \mathcal{O}\left(\left(\frac{\ell}{r_-}\right)^2\right) \\
&= 1 - \frac{1}{2}\frac{2}{r_-}\cos\theta\ell + \mathcal{O}\left(\left(\frac{\ell}{r_-}\right)^2\right) \\
&= 1 - \frac{\ell}{r_-}\cos\theta
\end{aligned}$$

so then for the scalar potential we get

$$
\begin{aligned}
V_{P\infty} &= \frac{kq_+}{r_-}\left[1 - \frac{\ell}{r_-}\cos\theta - 1\right] \\
&= -\frac{kq_+\ell}{r_-^2}\cos\theta \\
&= -k\frac{p_q}{r^2}\cos\theta
\end{aligned}
$$

The sign difference between this solution and that in the text is due to the fact that the positions of the charges is swapped.

4.5 Getting $\vec{E}$ from V

As I've mentioned a few times, equipotential lines are perpendicular to field lines. The fact that there is such a consistent and simple relationship between them means that there must be some way to calculate the vector field from the scalar potential. It should not be too surprising to discover that since we integrate the scalar potential to find $\vec{E}$, we perform a derivative to find V from $\vec{E}$. Differentiation and integration are inverse operations, after all.

Start with the expression for V from $\vec{E}$:

$V = \int dV = -\int \vec{E} \cdot d\vec{\ell}$

This suggests

$dV = -\vec{E} \cdot d\vec{\ell} = -E_\ell d\ell$

where E_ℓ is the component of $\vec{E}$ in the $\vec{\ell}$ direction. Algebra then gives

$\frac{dV}{d\ell} = -E_\ell$

In other words, the component of $\vec{E}$ in a given direction is equal to the derivative of the scalar potential with respect to that direction. In order to perform these operations, we introduce (if you haven't already seen it) the concept of a partial derivative. Despite the scary name, a partial derivative just means you only worry about the "explicit" dependence on a variable, rather than any and all dependence. For instance, imagine that I am working in a spherical coordinate system. The meaning of $\hat{\phi}$ depends on θ, but this dependence isn't explicit: $\hat{\phi}$ has a meaning all by itself. If this is confusing, a good rule of thumb is that you don't need to bother with the chain rule on coordinates if you are taking partial derivatives, even tho you might if it were a total derivative (what we've always just called a derivative until now).

The notation for a partial derivative is $\frac{\partial f(x)}{\partial x}$, as compared to the total derivative $\frac{df(x)}{dx}$. We read this "partial f partial x" rather than "dee$f$ dee x". Be careful not to write the partial symbol ∂ as a 2! This is a common and annoying mistake. The curl doesn't extend past the vertical swoop, and it certainly doesn't loop back up as a 2 might.

In this notation, we find:

$E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$.

Unfortunately, these expressions aren't so simple for cylindrical and spherical coordinates. They exist and are used, but we'll avoid them for this course.

4.6 Electric Potential Energy

Recall the original definition of electrical potential,

$V_{ba} = \frac{U_{ba}}{q}$.

This means that if we know the scalar potential difference between two points, we can easily calculate the change in potential energy, or work, needed to move a charge from points a to points b, $U_{ba} = qV_{ba}$.

There are two different ways in which this comes into play. First, if we have an external electrical potential set and are moving a charge around with in it, we apply this formula directly. Simply multiply the test charge's charge by the electrical potential difference, and you've found the difference in potential energy. Secondly, and more interestingly, we can use this to find the amount of energy it takes to assemble a system of charges.

First, we start with a single charge by itself, assuming that all other charges are infinitely far away. We then bring in charges one at a time to assemble our charge distribution. The first charge establishes a scalar potential through which the second must move. Once we have 2 charges, they establish a scalar potential through which the third must be moved. And so on and so forth, each new charge having to be brought in through the potential of the already accumulated charges.

$V_2 = k \frac{q_1}{r_{12}}$

$V_3 = k \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right)$

$V_4 = k \left(\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}} \right)$

$V_n = k \sum_{i=1}^{n-1} \frac{q_i}{r_{in}}$

$U = qV$

so

$U_{12} = k \frac{q_1 q_2}{r_{12}}$

$U_{123} = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$

$U_{1 \cdots n} = \sum_{i=2}^{n} q_i V_i$