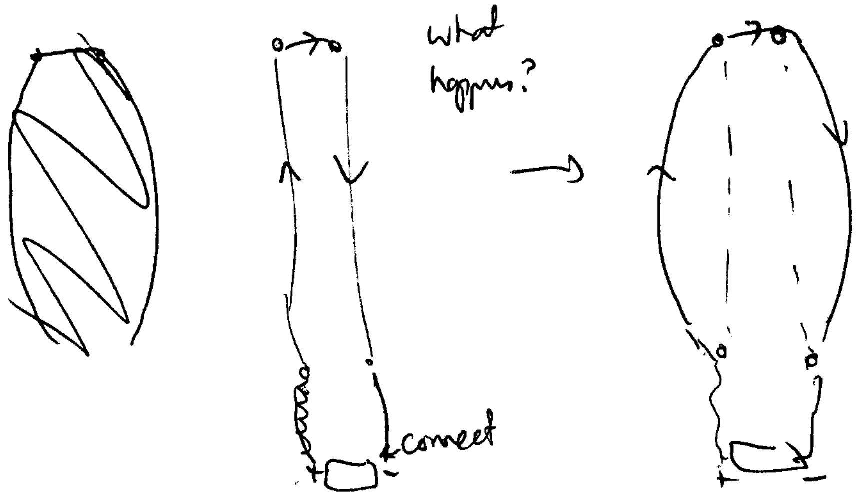
* note I'm mixing ch 27 + 28

Magnetism

imagine:



they repel!

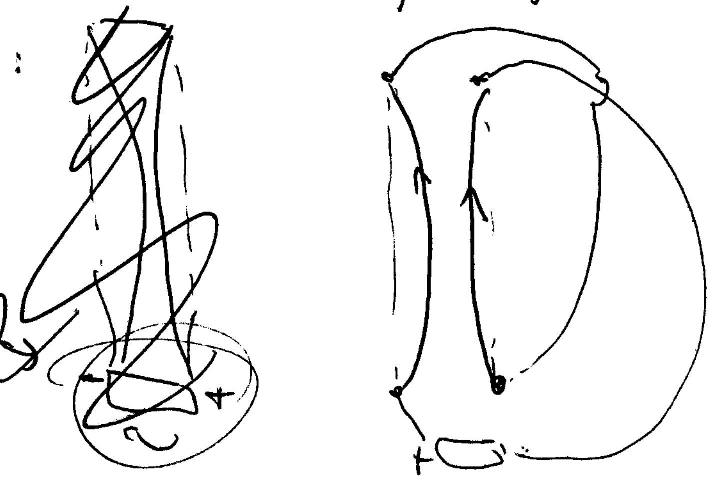
Discuss:

Electrostatic explanation?

charges repel — wrong — there are just as many stationary "+ charges" as many e^- in any given segment — wire is neutral!

now consider:

switch direction



they attract!

What is happening? Magnetic fields + forces.

Source of E field: pt charges

source of magnetic field: current (ie moving charges)

a compass measures magnetic field. If you measured field near wire, what would you find?

(use RHR)

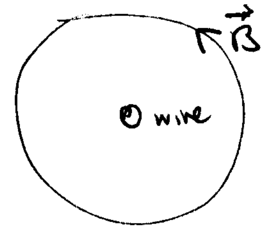
magnetic field lines are curly!

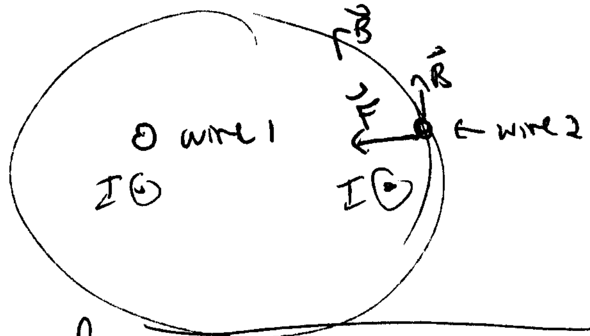
recall: E field lines ~~pt~~ pointed in direction charge would move.

What if I put a charge on a mag. field line?

nothing happens. What about opening example? force was felt. why?

there was moving charge





$\vec F$ is $\perp$ to $\vec B$

in general, $\boxed{\vec F_{mag} = q(\vec v \times \vec B)}$ Lorentz force law

→ Cross product, discuss RHR

recall:

So if a charged particle moves in with E and B fields,

$$\vec F = q\vec E + q(\vec v \times \vec B)$$

Q - can a Mag field be used to stop a charged particle?

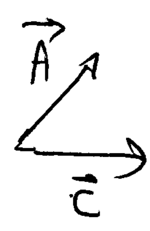
no, In fact Mag. fields do no work on charge particles

RHR: 1) point 4 fingers in direction of $\vec v$

2) curl to dir. of $\vec B$

3) $\vec F$ points in dir. of thumb.

eg.



$\vec D = \vec A \times \vec C$

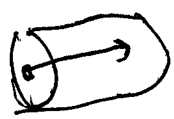
direction of D?

Mag 3.

what is the force on a wire?

A current can be thought of as a line charge moving along with a velocity $\vec{v}$

$$I = \frac{\Delta Q}{\Delta t}$$



length $v\Delta t = \Delta l$

$$\Delta Q = \lambda \Delta l$$
$$= \lambda v \Delta t$$

$$= \frac{\lambda v \Delta t}{\Delta t}$$

$$\vec{I} = \lambda \vec{v}$$

$$\vec{v} = \frac{\vec{I}}{\lambda}$$

Alternatively

$$\vec{I} = \frac{dQ}{dt} \qquad Q = \lambda l$$

$$= \frac{d}{dt}(\lambda l) = \cancel{\frac{d\lambda}{dt}}\, l + \lambda \frac{dl}{dt}$$

$$= \lambda \vec{v}$$

$$\vec{F} = q\, \vec{v} \times \vec{B}$$
$$= q\, \frac{\vec{I}}{\lambda} \times \vec{B}$$

charge/length l

$$\boxed{\vec{F} = l\, \vec{I} \times \vec{B}}$$

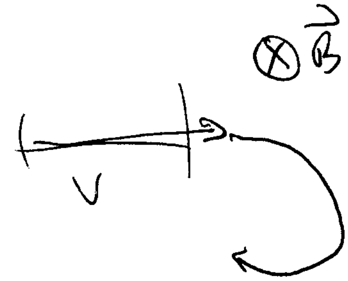
$$\boxed{\vec{F} = I\, \vec{l} \times \vec{B}}$$ (typically, and in book)

explain that path is circular!

p27-19 example A doubly charged helium atom whose mass is 6.6×10^{-27} kg is accel. by a voltage of 2700V. (a) what will be its radius of curvature if it moves in plane ⊥ to a uniform 0.340 T field? b) what is its period of revolution

potential ↔ kinetic energies

$$qV = \frac{1}{2} m v^2 \qquad V \neq v$$



$$v = \sqrt{\frac{2qV}{m}}$$

$\vec{F}_{mag} = q\vec{v} \times \vec{B}$ $\sin\theta = \sin 90° = 1$

$|\vec{F}| = qvB$

circular motion $F = \frac{mv^2}{r}$

$q\cancel{v}B = \frac{mv^{\cancel{2}}}{r}$

$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}} =$

$= \frac{1}{0.34T}\sqrt{\frac{2(6.6\times10^{-27}kg)(2700V)}{2(1.6\times10^{-19}C)}}$

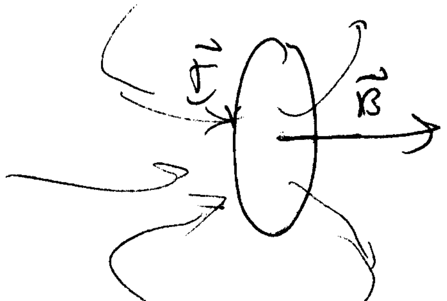
$= 3.1\times10^{-2}\,m$

b) $v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T}$

$T = \frac{v}{2\pi r} = \frac{v}{2\pi} B\sqrt{\frac{q}{2mV}} = \frac{2\pi m}{qB} = \ldots = 3.8\times10^{-7}s$

$v = \sqrt{\frac{2qV}{m}}$

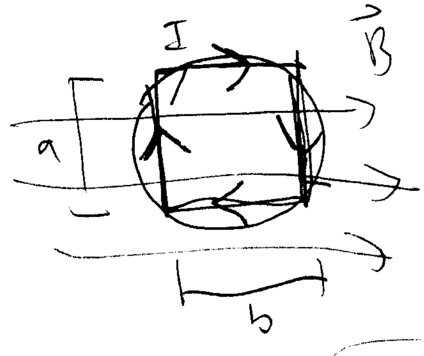
What is the direction of magnetic field for a loop of wire?



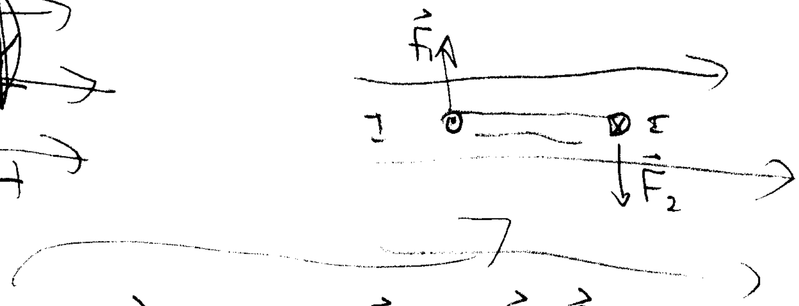
"magnetic dipole"

Torque $\longrightarrow$ next page

What is torque on a loop of wire? (in an external field)



side View:



$$\vec{\tau}_1 = \vec{r} \times \vec{F} = \frac{b}{2} I a B \otimes$$

for one

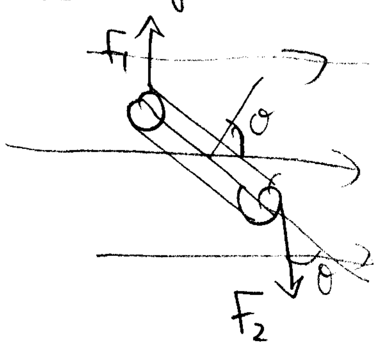
$$\vec{F} = I \vec{l} \times \vec{B}$$

$$|F| = I l B$$

$ba = A$

$$\vec{\tau}_{total} = 2\tau_1 = b I a B = \boxed{AIB}$$

as it pushes:



$$\vec{F} = I \vec{l} \times \vec{B} = IV$$

$$|F| = I l B \quad l = a$$

$$= I a B$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= |r||F| \sin\theta$$

$$= \frac{b}{2} I a B \sin\theta + \frac{b}{2} I a B \sin\theta$$

$$= b I a B \sin\theta \qquad ab = A = \text{area}$$

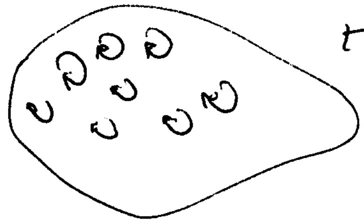
$$\tau = I A B \sin\theta$$

$\Rightarrow$ Magnetic field will tend to align loops of current

May 5

What about magnets?

Imagine a material made of loops of current:

$\vec{B} = \otimes$  ← if all the same way, then mag. field

Since electrons orbit, can think of them as loops of current.

Then why are not all mat'ls magnetic?

Reality - much more complicated

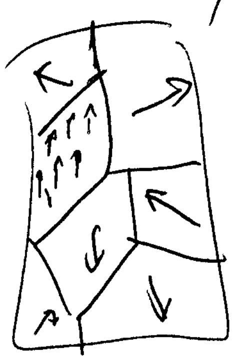
Materials exhibit 3 characteristics:

(1) Paramagnetism - dipoles associated w/ the spins of unpaired electrons experience a torque tending to line them up ∥ to field.

(2) Diamagnetism - the orbital speed of the e^- is altered in such a way as to change the orbital dipole moment in a direction opposite the field

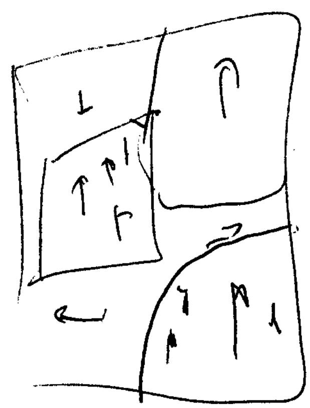
Named after ~~Atoms~~ Fe

(3) Ferromagnetism - favorable for spins to align. Domains are nearly completely aligned:



$\xrightarrow{\text{put in field}}$

= torque on loops



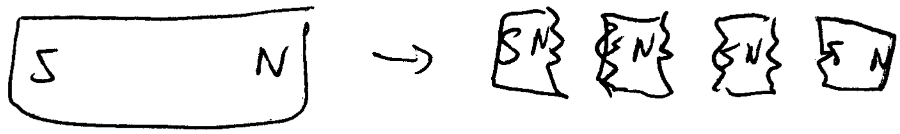
← domains grow so that one direction is dominant

take away field, and still have magnet

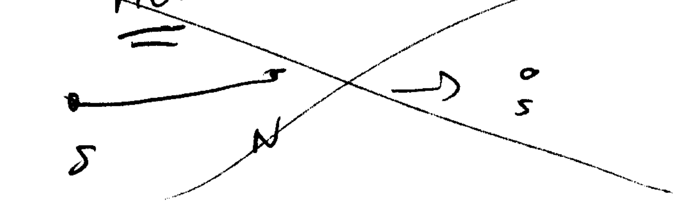
May 6

Magnetic ~~Monopoles~~?

if you break up a magnet, you just get more:



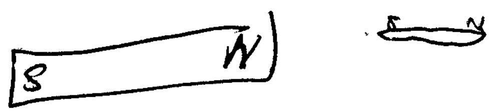
~~not~~



major diffrnce btwn Electricity + magnetism

Why do magnets pick up things?

- small opposite field induced in mat'l by strong field from magnet.



Do Demo

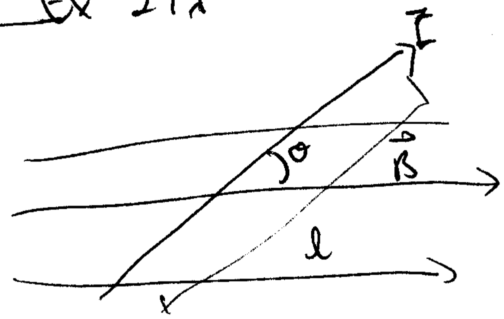
magnetic field lines form closed loop

E fields do not

May 7

Ex 27d

$\vec{B}$ is uniform btwn magnets



$I = 30\,A$

$l = 12\,cm$

$\theta = 60°$

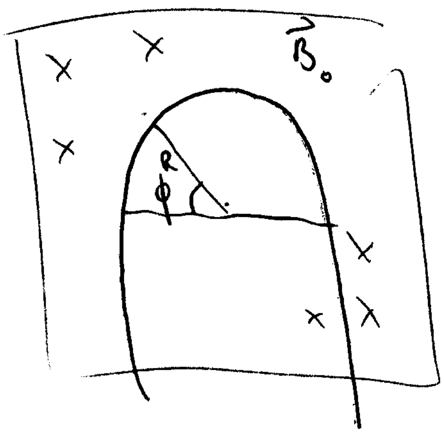
$B = 0.9\,T$ find $|\vec{F}|$

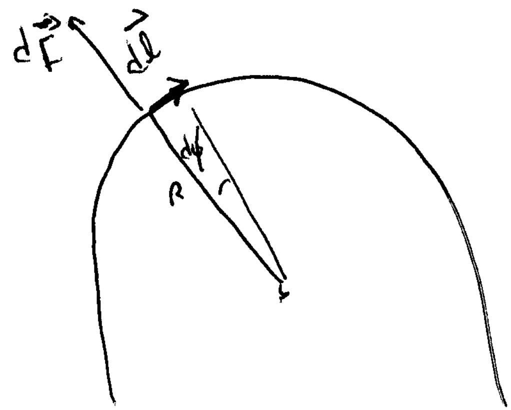
$\vec{F} = I\,\vec{l} \times \vec{B}$

$|F| = I l B \sin\theta$

$= (30A)(0.12\,m)(0.9T)\sin(60) = 2.8\,N$

Ex 27-3





$\vec{F} = \int d\vec{F} = \int dF_x \hat{i} + \int dF_y \hat{j}$

$d\vec{F}_x = dF\cos\phi$ $\quad dF_y = dF\sin\phi$ $\quad d\vec{F} = I\,d\vec{l} \times \vec{B}$ $\quad \theta = 90°$

$dF \quad I\,dl \cdot B \cdot (1)$

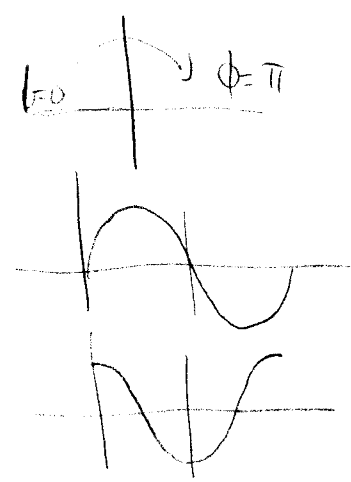
$\vec{F} = \int I\,dl\,B\cos\phi\,\hat{i} + \int I\,dl\,B\sin\phi\,\hat{j}$

$dl = R\,d\phi$

$= IB\left[\cancel{\int_0^\pi R\,d\phi\cos\phi}^{\,0}\,\hat{i} + \int_0^\pi R\,d\phi\sin\phi\,\hat{j}\right.$

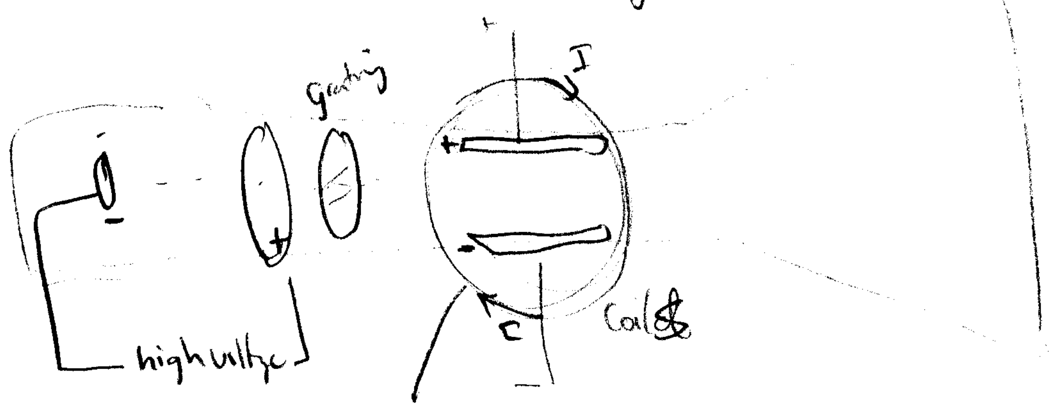
$= IBR\,[-\cos\phi]_0^\pi\,\hat{j}$

$\boxed{\vec{F} = IBR(2)\,\hat{j}}$



(27-7) Mag 8

CRT- Cathode Ray Tube:



- what would e^- do if plates and coils off? → $\vec{F} = q\vec{E} = eE$ up
- what if I turn on a potential difference? (e⁻ go up)
- what if I turn off " " → $F = q\vec{v}\times\vec{B} = evB$
  and turn on coils? (e^- go down)

$$F = \frac{mv^2}{r} = evB$$

Use CRT to measure charge-to-mass ratio of e^-

from above, $\frac{e}{m} = \frac{v}{Br}$ $v = ?$

turn on both fields, so that beam is undeflected

$\Sigma F = 0$

$F_{mag} - F_{el} = 0$

$F_{mag} = F_{el}$

$\cancel{e}vB = \cancel{e}E$

$v = \frac{E}{B}$

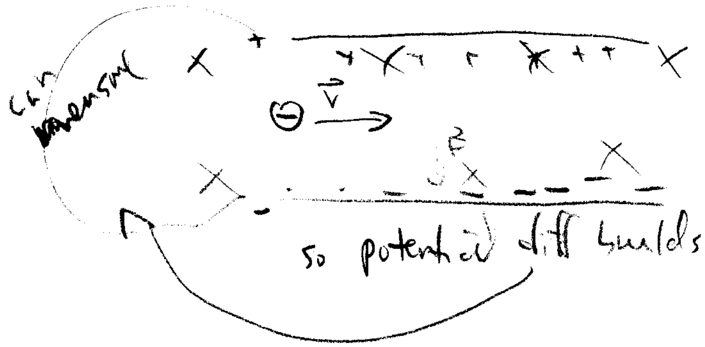
so $\boxed{\frac{e}{m} = \frac{1}{Br}\left(\frac{E}{B}\right) = \frac{E}{B^2 r}}$

$e/m = 1.76 \times 10^{11}$ C/kg

May 9

The Hall Effect (27-8)

- put current-carrying conductor in a B field
- B force on e^-
- creates a potential diff.



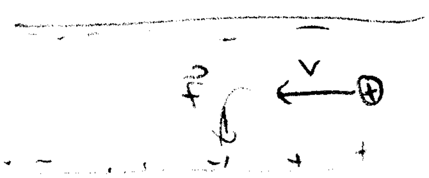
$\vec{F} = q\vec{v} \times \vec{B}$

$\therefore$ force is down

so potential diff builds up, until $\vec{F}_E = \vec{F}_B$

$eE_H = ev_d B$

what if we consider current to be the flow of +ive charges?



force is down.

gives opposite pot'l diff!

→ this first revealed that it is neg charges moving inside conductors

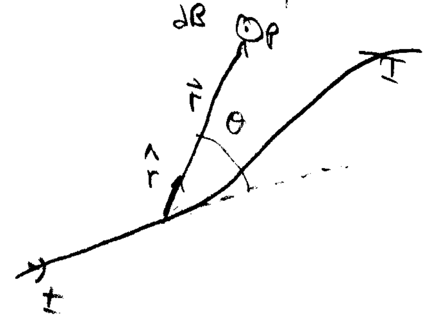
Q is it possible to measure B using the Hall effect?

Calculating Magnetic field

experimentally, $B \propto \frac{I}{r}$ ~~and~~ $\Rightarrow B = C\frac{I}{r}$ | for a wire

$$C = \frac{\mu_0}{2\pi} \Rightarrow \boxed{B = \frac{\mu_0}{2\pi}\frac{I}{r}}$$

similar to Coulomb's law for a point charge - gives only one situation - a straight wire.



(28-6) more generally, $\vec{B} = \int d\vec{B}$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l} \times \hat{r}}{r^2}}$$

also note: $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$

so

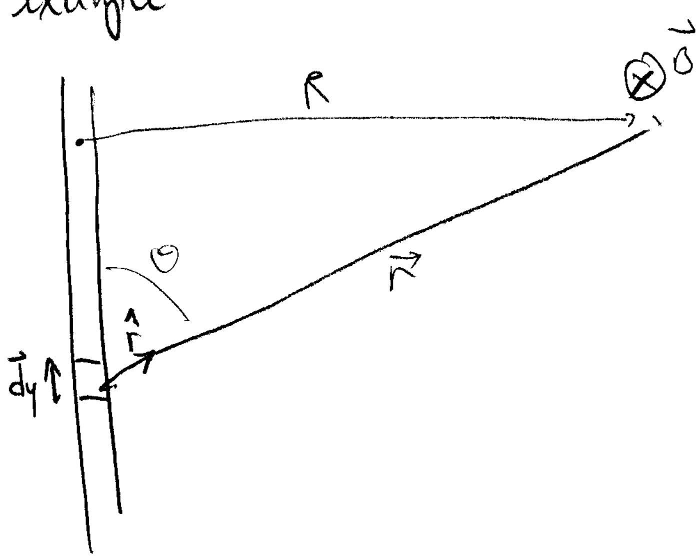
like Coulomb's law

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l} \times \vec{r}}{r^3}}$$

notice: vector sum

summing up mag. field from every current element

example



① draw good picture, w/ vectors
② consider direction (B constant dir)

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dy} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{dy\,(1)\sin\theta}{r^2}$$

r changing, y changing, θ changing

$$r^2 = R^2 + y^2$$

Mug 11

2 ways:

① $\sin\theta = \frac{R}{r}$ $r^2 = R^2 + y^2$ $r = (R^2+y^2)^{1/2}$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{dy(1)(R/r)}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{dy\, R}{(R^2+y^2)^{3/2}}$$

look up $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$

$a = R$ $x = y$

$$= \frac{\mu_0 I R}{4\pi}\left[\frac{y}{R^2(R^2+y^2)^{1/2}}\right]_{-\infty}^{+\infty}$$

$$= \frac{\mu_0 I}{4\pi R}\left[\frac{y}{(R^2+y^2)^{1/2}}\right]_{-\infty}^{\infty}$$

$$\lim_{y\to\pm\infty} \frac{y}{(R^2+y^2)^{1/2}} = \pm 1$$

$$= \frac{\mu_0 I}{4\pi R}[1-(-1)]$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}}$$

$\tan\theta = \frac{R}{-y}$

② $y = -\frac{R}{\tan\theta}$

$$dy = R\csc^2\theta\, d\theta = \frac{R\,d\theta}{\sin^2\theta}$$

$\sin\theta = R/r$

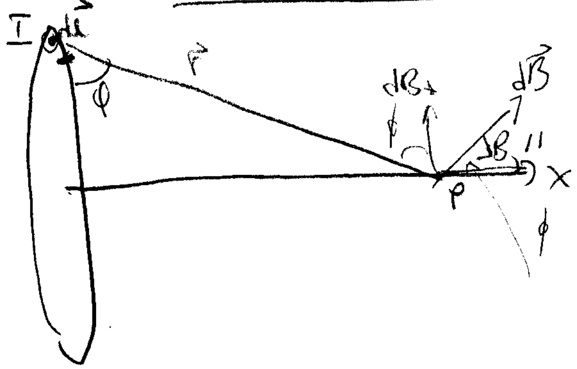
$$dy = \frac{R\,d\theta}{(R/r)^2} = \frac{r^2\,d\theta}{R}$$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{\cancel{r^2}\,d\theta}{R\,\cancel{r^2}}\sin\theta$$

$$= \frac{\mu_0 I}{4\pi R}\int_0^{\pi} d\theta\,\sin\theta$$

$$= -\frac{\mu_0 I}{4\pi R}\cos\theta\Big|_0^{\pi} = \frac{\mu_0 I}{2\pi R}$$

Another example:



$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\hat{r}}{r^2}$$

$$= \cancel{\int d\vec{B}_\perp} + \int dB_{\parallel}$$

$$= 0 + \int dB\cos\phi$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{dl}{r^2}\left(\frac{R}{r}\right)$$

May 10/12

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{dl\, R}{(R^2+x^2)^{3/2}}\,\hat{i}$$

$$= \frac{\mu_0 I R}{4\pi (R^2+x^2)^{3/2}}\int dl\,\hat{i}$$

$$= \frac{\mu_0 I R}{4\pi (R^2+x^2)^{3/2}}\,2\pi R\,\hat{i}$$

$$\boxed{\vec{B} = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}\,\hat{i}}$$

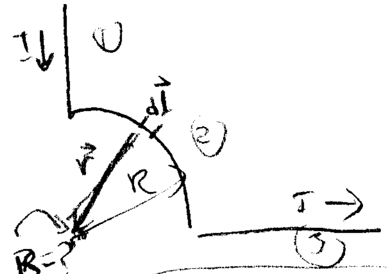
where is B a max? at x=0, the center of the loop!

compare to E field — E=0 @ center!

at $x=0$ $\quad B = \frac{\mu_0 I R^2}{2(R^2)^{3/2}} = \frac{\mu_0 I}{2R}$

Example 28-13

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\hat{r}}{|\vec{r}|^2} = \int d\vec{B}$$



$$= \cancel{\int_{\text{along }(1)} d\vec{B}} + \int_{\text{along }(2)} d\vec{B} + \cancel{\int_{\text{along }(3)} d\vec{B}}$$

= 0 b/cs $d\vec{l}\times\hat{r} = 0$ $(\sin(0)=0)$

for (2)? $|\vec{r}|^2 = R^2$

check $d\vec{l}\times\hat{R} = dl(1)$ into the pg

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{dl(1)}{R^2}$$

$$= \frac{\mu_0 I}{4\pi R^2}\left(\frac{2\pi R}{4}\right)$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{8R} \text{ into the board}}$$

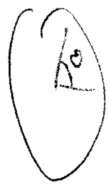
$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\vec{r}}{r^3}$$

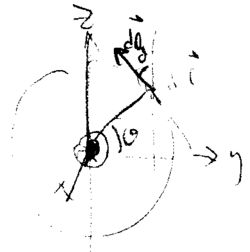
$\vec{r} = x\hat{i} - y\hat{j}$ $\quad +z\,\hat{i}!$

(x constant)

$|\vec{r}| = \sqrt{x^2+y^2}$

$d\vec{l} = R\,d\theta = ?$





$d\vec{l} = R\,d\theta\cos\theta\,\hat{k} - R\,d\theta\sin\theta\,\hat{j}$

$= -R\,d\theta\sin\theta\,\hat{j} + R\,d\theta\cos\theta\,\hat{k}$

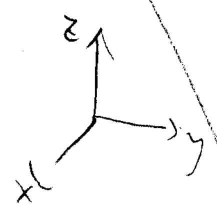
$d\vec{l}\times\vec{r} = (x\hat{i} - y\hat{j})\times(-R\,d\theta\sin\theta\,\hat{j} + R\,d\theta\cos\theta\,\hat{k})$

$\hat{i}\times\hat{i} = 0$

$\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{k} = \hat{i}$ etc

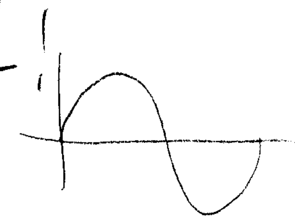
$\hat{k}\times\hat{i} = \hat{j}$



$$d\vec{l}\times\vec{r} = R\,d\theta\left[-x\sin\theta\,\hat{i}\times\hat{j} + x\cos\theta\,\hat{i}\times\hat{k} + \cancel{y\sin\theta\,\hat{j}\times\hat{j}} - y\cos\theta\,\hat{j}\times\hat{k}\right]$$

$$= R\,d\theta\left[-x\sin\theta\,\hat{k} + x\cos\theta\,(-\hat{j}) - y\cos\theta\,\hat{i}\right]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi}\frac{R\,d\theta\left[-x\sin\theta\,\hat{k} + x\cos\theta\,\hat{j} + y\cos\theta\,\hat{i}\right]}{(x^2+y^2)^{3/2}}$$



this is too complicated

Mag 13

Steps to remember:

- draw a careful diagram, labeling vectors
- consider separate segments, and symmetry
- figure out cross product
- figure out $|\vec{r}|$

put it together + integrate, being careful of limits

Is there an easier way? Remember Gauss' law...

Ampère's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

infinitesimal length vector

dot product takes only ∥ component of B — so any closed path will work

Strategy: pick loop (closed path) for which $\vec{B} \cdot d\vec{l} = B\,dl\,(\pm 1)$ or $B\,dl\,(0)$

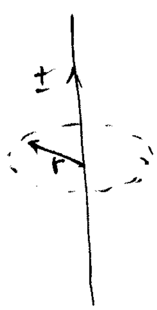
$\cos 0, \cos \pi$

$\cos(\frac{\pi}{2}), (-\frac{\pi}{2})$

for example: long straight wire

symmetry tells us B points ↺

pick loop in circle, + draw I and r



$\vec{B} \cdot d\vec{l} = B\,dl$

$\oint B\,dl = \mu_0 I_{encl}$ ← $I_{encl} = I$

$B \oint dl = \mu_0 I_{encl}$

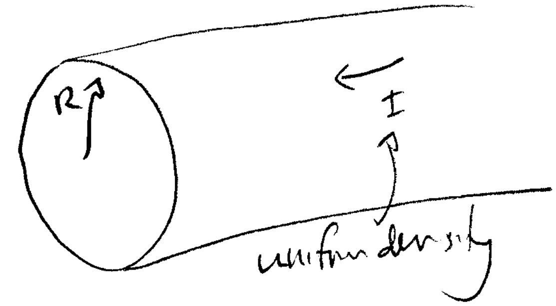
$B\, 2\pi r = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi r}$ ← like Gauss' law, only gives the magnitude

$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$ or around the wire

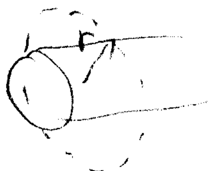
May 14

example (28-6)



symmetry: ...

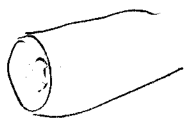
outside



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

$$B\, 2\pi r = \mu_0 I$$

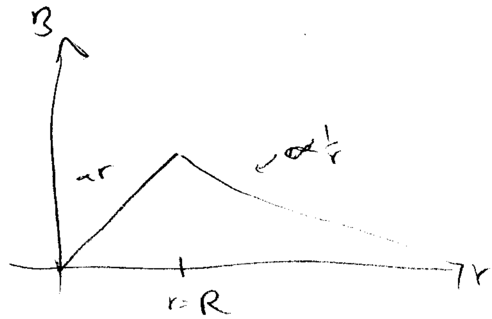
$$\boxed{B_{out} = \frac{\mu_0 I}{2\pi r}}$$



inside $I_{encl} = ?$

some fraction enclosed. uniform density, so take area ratio

$$I_{enc} = I\,\frac{\cancel{\pi} r^2}{\cancel{\pi} R^2}$$

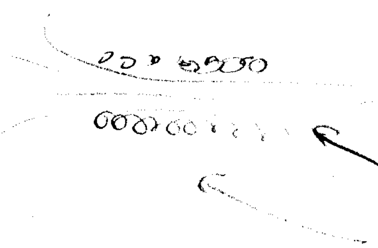


$$B\, 2\pi r = \mu_0 \frac{I\, r^2}{R^2}$$

$$\boxed{B_{in} = \frac{\mu_0 I r}{2\pi R^2}}$$

Solenoid (demo?)

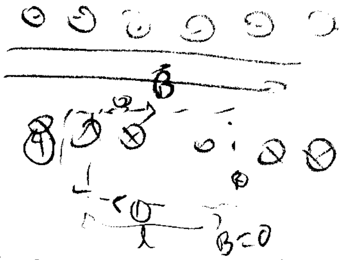
cut away



field really straight

field really zero

in simplified case, treat as



choose loop.

$$\oint \vec{B}\cdot d\ell = \int_1 + \int_2 + \int_3 + \int_4$$

$= \ell B = \mu_0 I_{encl}$

$I_{encl} = NI$

$\ell B = \mu NI$

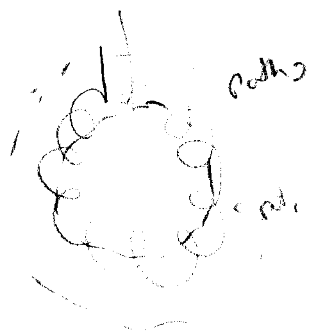
$B = \mu_0 \frac{N}{\ell} I$ $= n$ #/length

$$\boxed{B = \mu_0 n I}$$

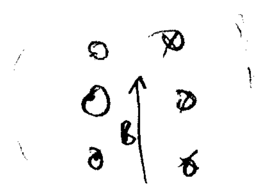
Toroid:

find B in and outside toroid.

have class try



outer loop: $I_{encl} = 0$



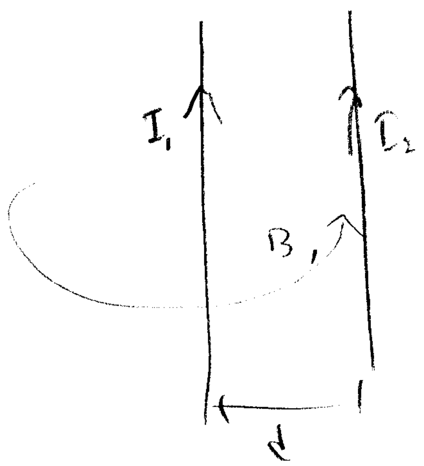
inside

$B 2\pi r = \mu_0 NI$

$B = \frac{\mu_0 NI}{2\pi r}$

concluded ch 28

Force btw 2 wires



$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

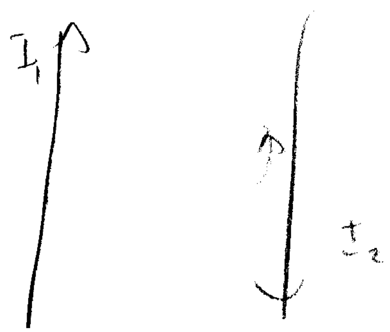
$$F_2 = I_2 B_1 l_2$$

↑ force on 2 by field B_1

$$F_2 = I_2 \left(\frac{\mu_0 I_1}{2\pi d} \right) l_2 \quad \text{to the left}$$

$$= \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} l_2$$

attractive



repulsive