Resolving Twin Paradox from different perspectives

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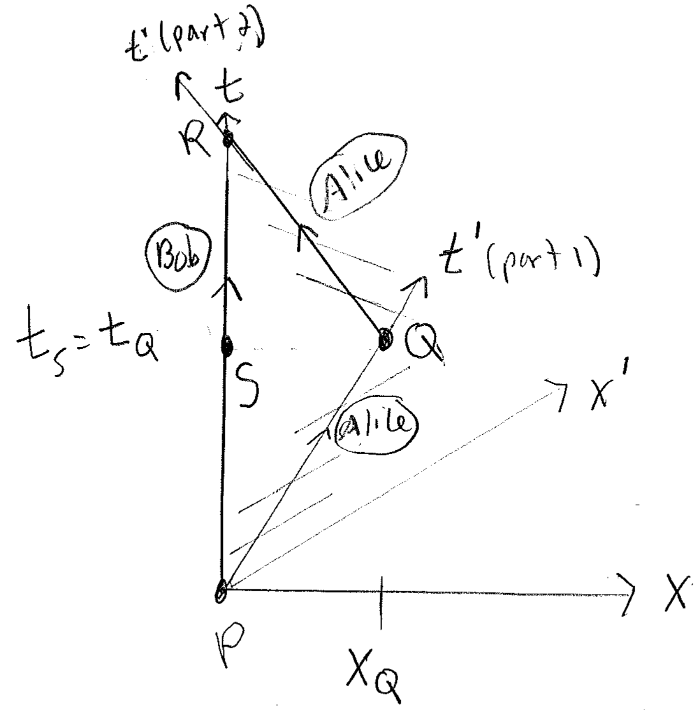
The usual space time diagram for resolving the twin paradox is this:



• In diagram, the diagnal hatches are lines of simultinaety for each part of Alice's trip in her coordinates

(Alice moves with velocity +V out and −V back)

If Alice travels for 10 years on her clock $(t'_{PQ} = 10\,yr = t'_{QR})$ away from Bob, and returns with same speed, the diagram above applies when Bobs axes are taken as the rest axes. Note that the turnaround point Q for Alice is simultaneous with the halfway point as measured by Bob. On Alices return trip her axes rotate. Alice's world line is along her time axis t' as she is at rest in her frame. If Alice travels 10 yr out then she measures her age to be 20 yr. During each half of Alice's trip Bob ages

$$t_{PS} = t_{PQ} = \gamma\left(t'_{PQ} + \frac{V}{c^2}\cancel{x'_{PQ}}\right) = \gamma t'_{PQ} = 10\gamma$$

$\cancel{x'_{PQ}}$ = 0

So Bob ages a total of $\boxed{20\gamma \text{ years}}$

$\gamma \equiv (1 - \frac{V^2}{c^2})^{-1/2}$

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the spacetime distance between spacetime points is invariant so:
this result can also be derived by considering the spacetime separation between P and Q:

$$s_{PQ}^2 = \underbrace{-c^2 t'^2_{PQ}}_{\text{Alices coords}} = \underbrace{-c^2 t_{PQ}^2 + x_{PQ}^2}_{\text{Bobs coords}} \qquad (*)$$

$$x_{PQ} = \gamma(\underbrace{x'_{PQ}}_{0} + \underbrace{v t'_{PQ})}_{\text{Alices coords}}$$

$$x_{PQ} = \gamma v t'_{PQ}$$

plug into $*$: $t'^2_{PQ} = t_{PQ}^2 - \gamma^2 \frac{v^2}{c^2} t'^2_{PQ}$

$$\Rightarrow t'^2_{PQ} = \frac{t_{PQ}^2}{1+\gamma^2 v^2/c^2} = \frac{t_{PQ}^2}{\left(1+\frac{v^2/c^2}{1-v^2/c^2}\right)} = \frac{t_{PQ}^2}{\left(\frac{1}{1-v^2/c^2}\right)} = \frac{t_{PQ}^2}{\gamma^2}$$

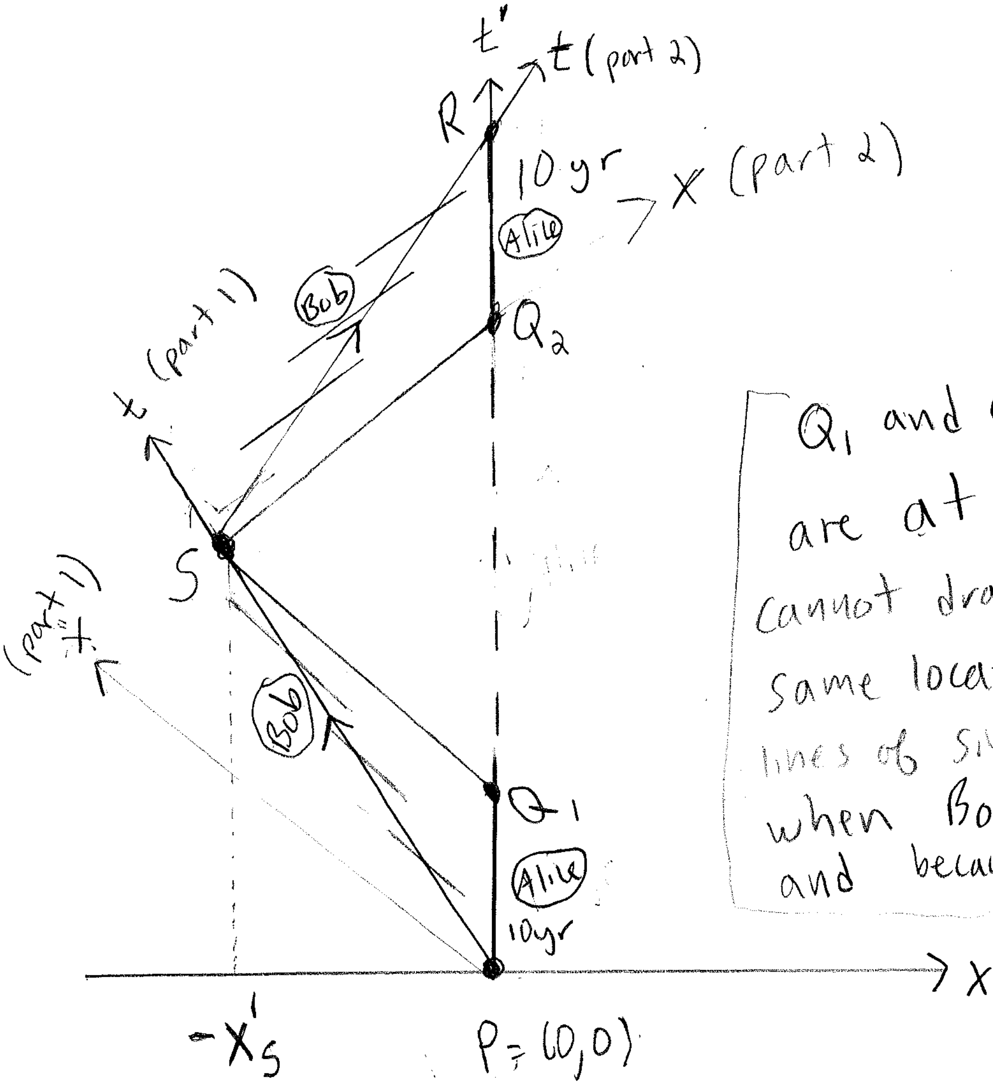
so again

$$t'_{PQ} = \frac{t_{PQ}}{\gamma} = \text{Alices age}$$

$\Rightarrow$ Bob is older by factor of γ.

Now: we can see the same result from point of view of rest frame of Alice: $\Longrightarrow$

Calculation with Alice's coordinates perpendicular on page: Now Bob is moving.



Q_1 and Q_2 are at same t' but cannot draw them at same location because: lines of simultinaety rotate when Bob turns around and because lines SQ_1 or SQ_2 are not lines of simultinaeily in Alices coords.

$x'_{PQ_1} = 0$

$t'_{PQ_1} = 10$

$t_{PQ_1} = t_{PS} = \gamma\left(t'_{PQ} + \cancelto{0}{\frac{v x'_{PQ_1}}{c^2}}\right) = \gamma t'_{PQ}$

$\Rightarrow t_{PS} = \gamma t'_{PQ} = \gamma\, 10\,yr$

$\Rightarrow$ total age of Bob $= 2\gamma\, 10\,yr$

$= 20\gamma$ years ! Same as approach when Bob's world line is drawn vertically

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so note that we get the same result (4)
independent of whether we draw a
spacetime diagram with Bob's rest frame
as orthogonal axes, or Alice's axes as
the orthogonal axes. The key point is that
we state Alice travels for 10 years as
measured on her clock and then turns
around. So Alice's proper time of 10 years
determines the turnaround point in Bob's frame.
Alice just measures two trips of 10 years
each. During each trip Bob measures
the time to be $\gamma 10$. The awkwardness
of the diagram in Alice's frame highlights
the simplicity of presenting the diagram
in a frame where Bob's axes are orthogonal because
the turnaround point is simultaneous with the 1/2 way point
on his world line in such a frame.