Almost all we know about the astronomical universe comes from radiation emanating from faint sources. We need to know how to interpret this radiation.

**Electromagnetic spectrum, wavelength and frequency:**

\[ \nu \lambda = c, \]  
\( \nu \) is frequency, \( \lambda \) is wavelength. The speed of light depends on the index of refraction.

**Justification for Macroscopic Treatment of Radiation**

Scale of system \( \gg \lambda \) of the radiation, the radiation can been considered to travel in “straight lines” =rays. \( dA \gg \lambda^2 \)

Amount of energy passing through source \( dA \) is in time \( dt \) is \( F dA dt \) where \( F \) is the flux. Units are \( \text{erg/cm.s}^2 \) in CGS.

**Flux from an isotropic source**

Isotropic means energy emitted in all directions. Consider 2 different spherical surfaces \( S_1 \) and \( S_2 \). (fig 1.)

\[ F_1 dA_1 dt_1 = F_2 dA_2 dt_2 \]  
but \( dt_1 = dt_2 \) so that

\[ F_1/F_2 = dA_2/dA_1 = r_2^2/r_1^2. \]

This is the inverse square law.

**Intensity**

Flux measures energy from all rays in given area. A more detailed approach is to consider energy carried from individual rays, or sets of rays differing infinitesimally from the initial ray. (fig 2.)
\[ dE = (\hat{k} \cdot \hat{n}) I_\nu dAdtd\Omega d\nu, \]  
where \( I_\nu \) is the Specific Intensity and represents Energy / (time \times\) area \times sold angle \times frequency).

Now suppose we have an isotropic radiation field. The differential flux at a given frequency is
\[ dF_\nu = (\hat{k} \cdot \hat{n}) I_\nu d\Omega = I_\nu \cos \theta d\Omega. \]  
(5)

But if \( I_\nu \) is isotropic, then \( F_\nu = 0 \).

**Constancy of Intensity in Free Space:**

Specific intensity is constant along a ray in free space.

Here’s why: Consider all rays passing through both \( dA_1 \) and \( dA_2 \) and use conservation of energy: (fig 3)

\[ dE_1 = I_{\nu,1} dA_1 dt_1 d\Omega_1 d\nu_1 = dE_2 = I_{\nu,2} dA_2 dt_2 d\Omega_2 d\nu_2 \]  
(6)

Note that \( dt_1 = dt_2 \), \( d\nu_1 = d\nu_2 \). Note also that \( d\Omega_1 \) is the solid angle subtended by \( dA_2 \) at \( dA_1 \) and \( d\Omega_2 \) is the solid angle subtended by \( dA_1 \) at \( dA_2 \). Thus we have
\[ d\Omega_1 = dA_2/r^2 \]  
(7)

and
\[ d\Omega_2 = dA_1/r^2. \]  
(8)

Thus
\[ I_{\nu,1} = I_{\nu,2}. \]  
(9)

The intensity is constant along free space.
Application to a Telescope (fig 3.)

Assume large distance between object and lens $r \gg f$, where $f$ is the focal length. Determine image intensity $I_i$ given object intensity $I_0$. Infinitesimal amount of surface area of object $dA_0$ has intensity $I_0$. We have

$$I_0dA_0d\Omega_{T,0} = I_0A_TdA_0/r^2.$$  \hfill (10)

where $d\Omega_{T,0}$ is the solid angle of the telescope as measured from the $dA_0$ of the object. All photons from $dA_0$ must strike $dA_1$ on focal plane.

Thus

$$I_0dA_0d\Omega_{T,0} = I_0dA_0A_T/r^2 = I_idA_id\Omega_{T,i} = I_idA_iA_T/f^2,$$  \hfill (11)

but solid angle sub-tending image from telescope equals solid angle sub-tending object from telescope so

$$dA_0/r^2 = dA_i/f^2,$$  \hfill (12)

and as expected, $I_i = I_o$.

**Do telescopes measure flux or intensity?**

If resolution of telescope is crude, and we cannot resolve object, then we measure flux. If telescope can resolve object, then we measure intensity.

**Why?** Consider the case when the source is unresolved. Now imagine pushing the source to farther distance. As the distance increases, the number of photons falls as $r^2$. The flux is measured.

If instead the the source is resolved, then as the source is pushed farther away, more area of the source would be included with the solid angle, which compensates for the the increased distance and the collected number of photons remain the same.

**Flux from a uniformly bright sphere:** read in text.

**Relationship of Intensity to Stat Mech Quantities:**

Consider photon distribution function $f$ such that $f_\alpha(x, p, t)d^3xd^3p$ is the number of photons in $x, p$ space, with spin index $\alpha$. 
\[ p = h \hat{k} = (h\nu/c)\hat{k} \]  

so

\[ dE = \sum_{\alpha=1}^{2} h\nu f_\alpha(x, p, t) d^3x d^3p. \]  

But photons traveling in direction \( \hat{k} \) for time \( dt \), through an element of area \( dA \) whose normal = \( \hat{n} \), occupy

\[ d^3x = c dt (\hat{k} \cdot \hat{n}) dA \]  

and

\[ d^3p = p^2 d\Omega dp = (E^2/c^2)d\Omega dE/c = (\hbar^3 \nu^3/c^3)d\Omega d\nu, \]  

where we have used \( E = pc \) and \( E = \hbar \nu \). Thus,

\[ dE = (\hat{k} \cdot \hat{n}) \sum_{\alpha=1}^{2} (\hbar^4 \nu^3/c^3) f_\alpha(x, p, t) dAdtd\Omega d\nu \]  

so

\[ I_\nu = \sum_{\alpha=1}^{2} (\hbar^4 \nu^3/c^3) f_\alpha(x, p, t). \]  

In stat mech, the occupation number for each photon is

\[ N_\alpha = \hbar^3 f_\alpha, \]  

which is dimensionless so

\[ I_\nu = \sum_{\alpha=1}^{2} (\hbar^3/c^2) N_\alpha(x, p, t). \]  

Later we will see for example that \( I_\nu \) corresponds to \( B_\nu \), the Planck function for b-body radiation. The occupation number corresponds to \( N_\alpha = (e^{h\nu_\alpha/kT_\alpha} - 1)^{-1} \) for Bose-Einstein stats.

**Radiation Pressure**

photon carries momentum \( p_\nu = E_\nu/c \). (fig. 4)

Can compute pressure by considering incident photons reflecting off of a wall: The angle of incidence equals the angle of reflection. The change in the \( z \) component of momentum of
photon between frequency $\nu$ and $\nu + d\nu$ reflected in time $dt$ from area $dA$ is
\[
dpnu d\nu = [(p_{\nu}) - (p_{\nu})]d\nu = (1/c)(E_{\nu} \cos \theta - (E_{\nu} \cos \theta))d\nu = (2/c)E_{\nu} \cos \theta d\nu = \left(\frac{2}{c}\right) I_{\nu} dv dt dA \cos^2 \theta d\Omega,
\]
where we used (4). Now the change in momentum per unit area per unit time is a differential force/area= differential pressure. Integrating over solid angle gives the total pressure:
\[
P_{\text{rad,}\nu} = \frac{dp_{\nu}}{dt dA} = \frac{1}{c} \int I_{\nu} \cos^2 \theta d\Omega_{\text{hem}} = \frac{1}{c} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_{\nu} \cos^2 \theta \sin \theta d\theta d\phi.
\]
Now imagine removing the reflecting surface. Thus instead of the factor of 2 in momentum, we would have photons coming in from the other side. Thus we can remove the factor of 2, and integrate over the full sphere.
\[
P_{\text{rad,}\nu} = \frac{dp_{\nu}}{dt dA} = \frac{1}{c} \int I_{\nu} \cos^2 \theta d\Omega = \frac{1}{c} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_{\nu} \cos^2 \theta \sin \theta d\phi.
\]
**Energy Density in a Cylinder**

Consider a cylinder of cross section $dA$. (fig 5)

\[
dE = u_{\nu}(\Omega) dv d\Omega = u_{\nu}(\Omega) c dt dv dA = I_{\nu} dA d\Omega dtd\nu
\]
so the energy density is
\[
u_{\nu}(\Omega) = (I_{\nu}/c).
\]
Then
\[
u_{\nu} = \int u_{\nu}(\Omega) d\Omega = (1/c) \int I_{\nu} d\Omega = 4\pi T_{\nu}/c
\]
For isotropic radiation field
\[
T_{\nu} = I_{\nu}
\]
Note also that the pressure for an isotropic radiation field from (23) and (26) is
\[
P = (1/c) \int I_{\nu} \cos^2 \theta d\Omega = (T_{\nu}/c) \int \cos^2 \theta d\Omega = \frac{1}{3} \int u_{\nu} d\nu.
\]