Comment on resistivity vs. viscosity 2/2010 (rv1)

when Boltzmann equation is integrated separately over electrons and ions for a plasma, one obtains the "two fluid" approximation to plasma physics. When collisions are included, an additional collisional integral must be included for ion-electron collisions:

e.g. $\dfrac{Df_e}{Dt}$ = (electron-electron collision terms) + (ion-electron collision terms)

for electrons $\dfrac{D}{Dt} \equiv \dfrac{\partial}{\partial t} + \vec{v}\cdot\nabla + \vec{F}\cdot\vec{\nabla}$

↑ must include Electromagnetic force $e(E + v_e \times B)$

The ion-electron collision terms cause a drag on electrons as they move with respect to the ions and thus represent the source of resistive damping of the current. Let $\vec{V} = \dfrac{m_i n_i \vec{v}_i + m_e n_e \vec{v}_e}{n_i + n_e}$ be the bulk flow velocity where $\vec{v}_i$, $\vec{v}_e$ are bulk ion & electron velocities.

Roughly, in a steady state, subtracting the electron velocity fluid equation from that for the ions gives after some algebra

$$\underbrace{e\left(E + \frac{V \times B}{c}\right)}_{\text{electromagnetic force}} + \underbrace{m_e \nu_{ei} V_e - m_p \nu_{ie} V_p}_{\text{drag force}} = 0 \qquad \text{(rv1)}$$

ν_{ei} is frequency with which electrons encounter ions and make a $\frac{\pi}{2}$ deflection

ν_{ie} is frequency with which ions encounter electrons and make a $\frac{\pi}{2}$ deflection →

$$\nu_{ie} = \frac{m_e}{m_p} \nu_{ei}$$

outer impact radius (rv2)
Debye length (charge shield scale)

electron thermal speed

where $\nu_{ei} = n_e \sigma_{ei} u_{th,e}$

and $\sigma_{ei} \approx \frac{1}{2\pi} \left(\frac{e^2}{u_{th,e} m_e} \right)^2 \ln \left(\frac{\lambda_D}{b_{\pi/2}} \right)$ (rv2)

inner impact radius deflection by $\pi/2$

so then (rv1) becomes

$$e \left(E + \frac{v}{c} \times B \right) = + m_e n_e \sigma_{ei} v_e (v_i - v_e)$$ (rv3)

since definition of current density is

$J = + n_e e (v_i - v_e)$

(rv3) =>

$$\left(E + v/_c \times B \right) = \underbrace{\frac{m_e \sigma_{ei} u_{th,e}}{e^2}}_{\eta} J$$

$$\Rightarrow \eta = \frac{m_e u_{th,e}}{e^2} \frac{1}{2\pi} \left(\frac{e^2}{u_{th,e}^2 m_e} \right)^2 \ln \left(\frac{\lambda_D}{b_{\pi/2}} \right)$$

$$= \frac{e^2}{2\pi u_{th_e}^3 m_e} \ln \left(\frac{\lambda_D}{b_{\pi/2}} \right)$$ (rv4)

$T_e^{-3/2}$ and independent of density

electron temp.

→

so Ohm's law is:

$$\vec{E} + \frac{\vec{v}}{c} \times \vec{B} = \eta \vec{J} \qquad (rv5)$$

Induction equation

$$\frac{\partial \vec{B}}{\partial t} = -c \vec{\nabla} \times \vec{E}$$

$\Rightarrow$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) - c \vec{\nabla} \times \eta \vec{J} \qquad (rv6)$$

but $\vec{J} = \frac{c \vec{\nabla} \times \vec{B}}{4\pi}$ so (rv6) $\Rightarrow$

$$\frac{\partial B}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \frac{c^2}{4\pi} \vec{\nabla} \times \eta (\vec{\nabla} \times \vec{B}) \qquad (rv7)$$

for η = constant, and using $\vec{\nabla} \cdot \vec{B} = 0$, (rv7) $\Rightarrow$

$$\frac{\partial B}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \underbrace{\frac{\eta c^2}{4\pi}}_{\nu_m} \nabla^2 \vec{B} \qquad (rv8)$$

ν_m = magnetic diffusivity

take curl

$$\Rightarrow \frac{\partial \vec{J}}{\partial t} = \nabla \times (\nabla \times (v \times B)) + \nu_m \nabla^2 \vec{J} \qquad (rv9)$$

dissipation term for $\vec{J}$

ν_m has units of $\frac{(\text{length})^2}{\text{time}}$ just like viscosity

<u>BUT</u> $\rightarrow$

(rvt)

remember that from above, magnetic diffusivity

$$\nu_m = \frac{\eta c^2}{4\pi} \sim \frac{c^2 e^2}{8\pi^2 u_{th,e}^3 m_e} \sim \frac{\sigma_{ei}\, u_{th,e}\, m_e}{e^2}$$

independent of density

whereas:

$$\frac{\mu}{\rho} = \nu \simeq u_{th,i}\, l_{mfp} \simeq u_{th,i} \frac{1}{n_i \sigma_{ii}}$$

viscosity in fluid momentum equation

thermal velocity of ions (since they carry the inertia)

mean free path of ions

depends on density and inverse dependence on cross section

- viscosity randomizes bulk flow by the action of large random excursions by inertia carrying particles. Thus "less interactions" favor HIGHER VISCOSITY
- magnetic diffusivity dissipates currents by damping relative motions between ions and electrons. Thus "more interactions" (between ions & electrons) favor HIGHER MAGNETIC DIFFUSIVITY