

Comment on resistivity vs. viscosity 2/2010 (rv1)

When Boltzmann equation is integrated separately over electrons and ions for a plasma, one obtains the "two fluid" approximation to plasma physics. When collisions are included, an additional collisional integral must be included for ion-electron collisions:

e.g. $\frac{Df_e}{Dt} = (\text{electron-electron collision terms}) + (\text{ion-electron collision terms})$

for electrons $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \nabla + \frac{\vec{F} \cdot \nabla}{T}$ must include Electromagnetic force $e(\vec{E} + \vec{v}_e \times \vec{B})$

The ion-electron collision terms cause a drag

on electrons as they move with respect to the ions and thus represent the source of resistive damping of the current. Let $\vec{V} = \frac{m_i n_i \vec{v}_i + m_e n_e \vec{v}_e}{n_i + n_e}$ be the bulk flow velocity where $\vec{v}_i, \vec{v}_e$ are bulk ion & electron velocities.

Roughly, in a steady state, subtracting the electron velocity fluid equation from that for the ions gives after some algebra:

$$\underbrace{e(\vec{E} + \frac{\vec{V} \times \vec{B}}{c})}_{\text{electromagnetic force}} + \underbrace{m_e \nu_{ei} \vec{V}_e - m_i \nu_{ie} \vec{V}_i}_{\text{drag force}} = 0 \quad (rv1)$$

ν_{ei} is frequency with which electrons encounter ions and make a $\frac{\pi}{2}$ deflection
 ν_{ie} is frequency with which ions encounter electrons and make a $\frac{\pi}{2}$ deflection $\rightarrow$

$$V_{ie} = \frac{m_e}{m_p} V_{ei}$$

electron thermal speed

outer impact radius
Debye length
(charge shield scale) (rv2)

where $V_{ei} = n_e \sigma_{ei} U_{the}$

$$\text{and } \sigma_{ei} \approx \frac{1}{2\pi} \left(\frac{e^2}{U_{the}^2 m_e} \right)^2 \ln \left(\frac{\lambda_D}{b\pi^{1/2}} \right) \quad (rv2)$$

inner impact radius deflection by $\theta/2$

so then (rv1) becomes

$$e \left(E + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) = + m_e n_e \sigma_{ei} V_e (V_i - V_e) \quad (rv3)$$

since definition of current density is

$$\mathbf{J} = + n_e e (V_i - V_e)$$

(rv3) $\Rightarrow$

$$(E + \frac{\mathbf{v}}{c} \times \mathbf{B}) = \underbrace{\frac{m_e \sigma_{ei} U_{the}}{e^2}}_{\eta} \mathbf{J}$$

$$\Rightarrow \eta = \frac{m_e U_{the}}{e^2} \frac{1}{2\pi} \left(\frac{e^2}{U_{the}^2 m_e} \right)^2 \ln \left(\frac{\lambda_D}{b\pi^{1/2}} \right)$$

$$= \frac{e^2}{2\pi U_{the}^3 m_e} \ln \left(\frac{\lambda_D}{b\pi^{1/2}} \right)$$

(rv4)

$\propto T_e^{-3/2}$ and independent of density

$\uparrow$ electron temp.



(rv5)

so Ohm's law is:

$$\vec{E} + \frac{\vec{v}}{c} \times \vec{B} = \eta \vec{J}$$

(rv5)

Induction equation

$$\frac{\partial \vec{B}}{\partial t} = -c \vec{\nabla} \times \vec{E}$$

 $\Rightarrow$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) - c \vec{\nabla} \times \eta \vec{J}$$

(rv6)

but $\vec{J} = \frac{c \vec{\nabla} \times \vec{B}}{4\pi}$ so (rv6) $\Rightarrow$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) - \frac{c^2}{4\pi} \vec{\nabla} \times \eta (\vec{\nabla} \times \vec{B})$$

(rv7)

for $\eta = \text{constant}$, and using $\vec{\nabla} \cdot \vec{B} = 0$, (rv7) $\Rightarrow$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \underbrace{\frac{\eta c^2}{4\pi}}_{\nu_m} \nabla^2 \vec{B}$$

(rv8)

$\nu_m = \text{magnetic diffusivity}$

take curl

$$\Rightarrow \frac{\partial \vec{J}}{\partial t} = \vec{\nabla} \times (\vec{\nabla} \times (\vec{v} \times \vec{B})) + \nu_m \nabla^2 \vec{J}$$

(rv9)

ν_m has units of $\frac{(\text{length})^2}{\text{time}}$ just like viscosity
BUT $\rightarrow$

remember that from above, magnetic diffusivity (5)

$$\nu_m = \frac{\eta c^2}{4\pi} \sim \frac{c^2 e^2}{8\pi^2 u_{th,e}^3 m_e} \sim \frac{\sigma_{ei} u_{th,e} m_e}{e^2}$$

independent of density

whereas:

$$\frac{\mu}{\eta} = \nu \sim u_{th,i} \lambda_{mfp} \approx u_{th,i} \frac{1}{n_i \sigma_{ii}}$$

viscosity in fluid momentum equation

thermal velocity of ions (since they carry the inertia)

mean free path of ions

depends on density and inverse dependence on cross section

- viscosity randomizes bulk flow by the action of large random excursions by inertia carrying particles. Thus "less interactions" favor HIGHER VISCOSITY
- magnetic diffusivity dissipates currents by damping relative motions between ions and electrons. Thus "more interactions" (between ions & electrons) favor HIGHER MAGNETIC DIFFUSIVITY