Some aspects of shock propagation through supernova envelope and ambient interstellar medium

- Deep in star where energy from outward propagating material comes from radioactivity, thermalization occurs with temp in opt-uv range.
- When outflow becomes optically thin, effective "temperature" goes up (that is, x-ray photons are not down scattered efficiently so we see high energy non-thermal emission).
- Source of energy eventually changes from radioactive decay, to conversion of bulk flow energy at shock (remembers shocks are sites of bulk flow dissipation).
- Forward shock and reverse shocks are present:

  free ejecta

  compressing ejecta rapidly cools

  reverse shock

  compressed ambient material

  contact discontinuity

  forward shock

Shocks propagate away from the highest density regions. Because of rapid cooling by Bremsstrahlung in the compressed regions, the high density region also supersonically migrates "backward" into the free ejecta in rest frame.
• note that the ejecta, contact discontinuity and reverse shock are all moving outward in the lab frame, but in the frame of the contact discontinuity there are shocks propagating both outward forward and inward toward the explosion point = reverse shock.

• forward & reverse shocks are important concepts throughout supersonic astrophysics (jets, GRB, etc.)

• The supernova Remnant SNR (scales > 1000AU) emits by conversion of bulk flow energy at shock; ejecta has kinetic energy

\[ \leq 10^{51} \text{ erg} = \frac{1}{2} \text{M}_{ej} \text{ V}_{ej}^2 \]

where \( M_{ej} \approx 2M_{\odot} \), \( \rightarrow \) \( \text{V}_{ej} \geq 10^{18} \text{ cm}^2 \) \( \Rightarrow \) \( \text{V}_{ej} \approx 10^{9} \text{ cm} \)

\[ \Rightarrow \text{ initial } "\text{temperatures}" \text{ as high as } 10^8-10^9 \text{ K} \]

(Using \( V = \left( \frac{k \text{E}}{m_p} \right)^{1/2} \)).

• but there is an important subtlety as the shock reaches these scales \( \geq 1000 \text{AU} \)

Let's look a bit at the shock physics
Recall from our brief discussion that shocks form as waves steepen non-linearly. Waves are calculated as linear perturbations of the hydro equations. They move at speed $c_s$ for unmagnetized plasma. Because pressure disturbance from ejecta moves at $V_{eject} \approx (\frac{T_{ISM}}{\rho_{ISM}})^{1/2}$ waves pile up:

$\frac{\partial V}{\partial t} = -V \cdot \nabla V - \nabla P + \nabla \cdot \nabla \nabla V$

The role of "nonlinearly" arises in the Navier-Stokes equation (fluid momentum)

Viscosity is always approximately $\propto$ speed $\times$ length. Typically, for ambient ISM into which shock propagates $c_s \ll c_{s, ISM}$. Because "non-linear" effects induce dissipation $\nabla \cdot \nabla V = 0$ where $|V - \nabla V| = \nabla^2 V$
\[
\begin{align*}
\text{(94)} \quad \Delta V &= v_{\text{eff}} = \sqrt{\frac{2}{m_p}} \sqrt{\frac{E}{m_p}} \\
\text{or} \quad \ell &= \frac{v_{\text{eff}}}{c_s \sqrt{V_{\text{tot}}}} \\
\text{In vicinity of shock, the velocity} \\
\text{transits from} \quad V_1 > c_s \quad \text{to} \quad V_2 \leq c_s. \quad \text{Thus} \quad m_p \text{ (93) } \Rightarrow \ell \approx \ell_{\text{eff}} \text{ should be the scale over} \\
\text{over which the flow changes from upstream to downstream.} \\
\text{Typically, therefore, we expect the shock thickness} \\
to \text{be} \approx \ell_{\text{eff}}. \quad \text{In reality, instabilities} \\
\text{broaden the shock somewhat, but put that aside for} \\
\text{the moment).} \quad \text{Now let us estimate this for} \\
\text{Supernova Remnants:} \quad \text{At ejection velocity} \\
V_{\text{ej}} = 10^9 \text{cm/s} \quad \text{Kinetically energy per proton is} \quad 3 \times 10^{50} \text{erg} \\
\text{is about} \quad 2 \text{MeV.} \quad \text{As these protons hit an} \\
H \text{ atom of the ISM, the latter will ionize.} \\
\text{Cross section of interaction is} \quad \sigma_{\text{i}} = 10^{-17} \text{cm}^2 \left( \frac{\text{MeV}}{m_p} \right)^2 \\
\text{Energy lost per ionization is} \quad \approx 50 \text{eV,} \\
\text{(which represents the inelastic part of the collision).} \\
\text{The stopping distance of the impinging protons is therefore} \\
\ell_{\text{eff}} \frac{dE}{d\ell} = \frac{E}{d\ell} \text{d}m_p = \frac{2 \text{MeV}}{50 \text{eV} \sigma_{\text{i}} n_{\text{ion}}} \\
\text{for} \quad n = 1 \text{cm}^{-3} \\
\Rightarrow \quad \ell_{\text{eff}} \approx 4 \times 10^4 \text{ to } 10^{11} \text{ cm} \approx 10^3 \text{ pc}. 
\end{align*}
\]
But shock thicknesses observed are much smaller than 10^3 pc. In fact the entire remnants become invisible (merged with ambient medium) on scales of 50 pc. Thus, how can thin shocks form if the scale left were actually 10^3 pc?

Here the answer is magnetic fields.

Calculate the Larmor radius for microgauss field: \( r_L = \frac{mc\nu_{th}}{eB} = \frac{(10^{-24}) (3 \times 10^{10} \text{ cm/s}) (10^9 \text{ cm})}{(4 \times 10^{-10}) (3 \times 10^{-6} \text{ G})} \)

\( \approx 2.5 \times 10^{10} \text{ cm} \approx 10^{-8} \text{ pc} \)

\( r_L \) is very small, strength field of ISM.

\( B \)-fields are fundamental for "collisionless shocks" in astrophysics. They make the effective mean-free path equal to the Larmor radius which is much smaller than the collisional mfp even for extremely weak magnetic fields.
Assume that the shock represents a 'thin discontinuity'. (This was justified in part last lecture):

Conservation of mass, energy & momentum can all be written \( \partial_t \rho + \nabla \cdot \mathbf{F}_\rho = 0 \)

If we integrate such a conservation law across the thin discontinuity using the "pill box" as shown:

\[
\begin{align*}
\text{in steady state:} \\
\partial_t \rho + \nabla \cdot \mathbf{F}_\rho &= 0 \\
0 &= \nabla \cdot \mathbf{F}_\rho \\
\text{but volume is arbitrary} \\
\text{so that} \\
\int \nabla \cdot \mathbf{F}_\rho \, d^3x &= 0 = \int \mathbf{F}_\rho \cdot d\mathbf{S}
\end{align*}
\]

by Gauss' theorem

For mass continuity:

\[
\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0 \quad \Rightarrow \quad \oint \rho \mathbf{v} \cdot d\mathbf{s} = 0
\]

\[
\int_{s_1} \rho_1 \mathbf{v}_1 \cdot ds_1 = \int_{s_2} \rho_2 \mathbf{v}_2 \cdot ds_2 \quad \text{for pill box}
\]

\[
\Rightarrow \int_{s_1} \rho_1 \mathbf{v}_1 = \int_{s_2} \rho_2 \mathbf{v}_2
\]
Similarly, for flows in which B-field is energetically negligible:

\[ w_1 + \frac{1}{2} V_1^2 = w_2 + \frac{1}{2} V_2^2 \quad \text{energy conservation} \]  
\[ p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 \quad \text{momentum flux conservation} \]  

\[ w = \text{enthalpy density} = \frac{\Gamma}{\Gamma - 1} \frac{p}{\rho} = \frac{c_{s1}^2}{\rho} \]

(96–98) are the Rankine-Hugoniot jump conditions for a shock. Define \( M_{1}^2 = V_{1}^2/c_{s1}^2 \)

Solving (96–98) (I leave as exercise):

\[ \frac{p_2}{p_1} = \frac{(\Gamma + 1) M_1^2}{(\Gamma + 1) + (\Gamma - 1)(M_1^2 - 1)} = \frac{V_1}{V_2} \]  
(99)

\[ \frac{p_2}{p_1} = \frac{(\Gamma + 1) + 2\Gamma (M_1^2 - 1)}{\Gamma + 1} \]  
(100)

\[ \frac{c_{s2}^2}{c_{s1}^2} = \frac{T_2}{T_1} = \frac{[\Gamma(\Gamma + 1) + 2\Gamma (M_1^2 - 1)][(\Gamma + 1) + (\Gamma - 1)(M_1^2 - 1)]}{[(\Gamma + 1)^2 M_1^2]} \]  
(101)
Assume flow is supersonic on side 1

so \( M_1 = \frac{V_1}{c_{1s}} > 1 \).

Then

\[
\frac{P_2}{P_1} > 1, \quad \frac{T_2}{T_1} > 1, \quad \frac{V_2}{V_1} < 1, \quad \frac{T_2}{T_1} > 1.
\]

Strongest shock \( \Rightarrow M_1^2 \gg 1 \)

\[
\Rightarrow \quad \frac{P_2}{P_1} = \frac{\Gamma+1}{\Gamma-1}; \quad \frac{T_2}{T_1} \gg 1; \quad \frac{T_2}{T_1} \gg 1
\]

Limiting relation as \( M_1^2 \rightarrow \infty \) \( \Rightarrow \) for \( \Gamma = 5/3 \Rightarrow \frac{P_2}{P_1} = 4 \)

Note: momentum conservation and mass conservation are usually satisfied as in 96 & 98, but energy conservation can have important radiative terms, chemical reaction terms, thermal conduction ... we ignore these for the moment.

The above treatment assumes that the viscous terms operate only in the thin layer of the shock itself; this gets back to our notion from the previous lecture that the shock thickness can be estimated by comparing dissipative & bulk velocity terms.
In momentum equation, compare $V \nabla V$ term to $V \nabla^2 V$ term: (see page 93)

$$\Rightarrow \frac{\nabla V}{\nabla^2 V} = \frac{\text{V eff}}{\text{L eff}},$$

where $V \text{ eff}$ is the effective "viscosity" at the shock.

Now across the shock, the bulk energy of the flow in $V_1$ gets converted to random thermal energy such that $C_s \approx V_1$. As discussed on p. 95 of this lecture, the previous lecture, $C_s$ is determined by multiple of Larmor radius rather than collisional mean free path.

The shock is actually a "current sheet": when B-field included in jump conditions. This is because Maxwell's equations require that tangential component of $E$ is conserved across the shock:

Consider "pill surface" crossing shock across the shock:

from Maxwell's equations:

$$\frac{\partial B}{\partial t} + \nabla \times E = 0 \Rightarrow \int (\nabla \times E) \cdot dS = 0$$

0 in steady state

$$\Rightarrow \int \vec{E} \cdot d\vec{l} = 0$$

for arbitrarily thin pill surface only.

Sides contribute:

$$\Rightarrow \int \vec{E} \cdot d\vec{l} = 0 = E_{1+}d - E_{1-}d = 0$$

$$\Rightarrow [E_{1+}, E_{2+}]$$
Since Ohms law implies
\[ E = -\nabla \times B + nJ \]
then
\[ E_{1,T} = E_{2,T} \]
\[ \Rightarrow -\left( \nabla \times B + nJ \right)_{1,T} = \left( -\nabla \times B + nJ \right)_{2,T} \]

but \( J = \frac{e}{4\pi} \nabla \times B \) and away from shock, \( ne \nabla \times B \) can be considered small.

\( n \) is the resistivity and most astro-plasmas have low resistivity. However, near the shock \( \nabla \times B = \frac{B}{\Gamma_{eff}} = \frac{B}{\Gamma} \) \( \text{Larmor radius} \)

The gradient scale is small and near the shock \( nJ \) is important. This is why a shock is a "current sheet." Magnetic Reconnection provides another example of a current sheet based on same principle.

\[ \text{Interface: Exercise: show that interface is a current sheet if interface is thin!} \]
Now back to the evolution of the expanding SN shock: Transition to Sedov phase

During the early stages of the propagation of the optically thin phase of the shock's progress through the envelope and into ISM, the ejecta material has much more inertia than the ISM with which it interacts. The ejecta speed $V_e$ is thus constant.

**But:** there exists a critical radius $R_c$ at which the ejecta mass $M_{ejecta} = \frac{4\pi}{3} \rho_{ISM} R_c^3$.

At this point the blast enters the Sedov phase. Now the mass is piling up behind the shock and this mass starts to dominate the total mass of the ejecta.

The mass piles up behind the shock, but instead of the contact discontinuity:

\[ \text{pre-shocked ISM} \rightarrow 0 \]
Once the Sedov phase is underway, the speed of the blast wave is no longer constant. In the Sedov phase mass is dominated by that accumulated from ISM $E_{\text{ISM}}$ so the energy is

$$E = \frac{1}{2} \frac{4\pi}{3} \rho_{\text{ISM}} r^3 V_1^2 = \text{constant} \quad (103)$$

Just the initial explosion energy

$$E \propto r^3 V_1 \rightarrow r^3 \left( \frac{\text{d}r}{\text{d}t} \right)^2 = \text{constant}$$

$$r^3 \text{d}r = \text{d}t$$

$$r = (\text{constant}) t^{2/3} \quad (104)$$

Another way to arrive at this is to note that $s_{\text{ISM}}$ and $E$ are constant and

$$E = \frac{1}{2} M \left( \frac{r}{t} \right)^2 = \text{const} \quad (105)$$

$$s_{\text{ISM}} = \frac{M}{4\pi r^3} = \text{const} \quad (106)$$

$$\Rightarrow \frac{E}{s_{\text{ISM}}} = \text{const} = \frac{2\pi}{3} \frac{r^5}{E^2} \Rightarrow r = \left( \frac{E t^2}{s_{\text{ISM}}} \right)^{1/5} \quad (107)$$
\[ r = \left( \frac{E}{\rho_{\text{ISM}}} \right)^{1/5} t^{2/5} = 3 \text{ pc} \left( \frac{E}{10^{51} \text{ erg}} \right)^{1/5} \left( \frac{1}{300 \text{ yr}} \right)^{2/5} \]  
\[ (103) \]

(applying only for)

\[ R > \frac{c}{\sqrt{2}} c_{\text{crit}} = \left( \frac{3 M_{\odot}}{3 \times 10^4 \text{ M}_{\odot}} \right)^{1/3} \]

\[ \Rightarrow V = \frac{c}{t} = 3 \times 10^3 \text{ km} \left( \frac{E}{10^{51} \text{ erg}} \right)^{1/5} \left( \frac{1}{300 \text{ yr}} \right)^{2/5} \]

\[ (109) \]

Using \( V = c_{\text{S2}} \Rightarrow \)

\[ T = \frac{M_p}{V} \frac{c^2}{\rho} = \frac{M_p}{V} \frac{V^2}{V} = 9 \times 10^8 \text{ kpc} \left( \frac{E}{10^{51} \text{ erg}} \right)^{3/5} \left( \frac{1}{300 \text{ yr}} \right)^{-6/5} \]

\[ (110) \]

\[ \Rightarrow \text{at } t = 3.5 \times 10^4 \text{ yr, } T = 3 \times 10^6 \text{ kpc} \]

Thus if SNR is observed with \( T = 3 \times 10^6 \text{ kpc} \) (as is Cygnus Loop), the time in Sedov phase to reach that stage is, from (110)

\[ t_{\text{Sedov}} = 3.5 \times 10^4 \text{ yr} \left( \frac{T}{3 \times 10^6} \right)^{-5/6} \left( \frac{E}{10^{51} \text{ erg}} \right)^{1/3} \left( \frac{1}{3 \times 10^4 \text{ kpc}} \right)^{-1/3} \]

\[ (111) \]

\[ \Rightarrow \text{for given } V \text{ or } T \text{ and } r \text{ observed (to determine age can be determined if } r \text{ or } \rho_{\text{ISM}}) \]
Now, as deceleration becomes significant, outer shells of expanding sphere decelerate first ⇒ material in the inner region catches up with material in the outer regions.

Region A is supersonic with respect to B ⇒ reverse shock moves "backward" in frame of contact discontinuity. In lab frame everything is moving outward.

At the reverse shock, kinetic energy of ejecta is re-heated by reverse shock dissipation as it passes through ⇒ implies some of the bulk energy of the ejecta goes back into heat of ejected material. The forward shock converts some of the bulk energy into heating ambient ISM material. (X-ray emission is visible from both shocked regions.)
We mentioned, and will discuss later, the Rayleigh Taylor instability, which takes place during the Sedov phase. The Rayleigh-Taylor fingers:

\[ \text{low density ISM} \rightarrow \text{expansion direction} \]

Both forward and reverse directions have equivalent of "heavy" fluid falling on "light" fluid.

Radiative phase of SNR blast wave

Once radiative cooling time becomes short compared to Sedov age, we have radiative phase. Sedov age is given by (111).

For cooling time, note that for \( T < 10^6 \) keV, \( N_{ij0} \) gain \( e^- \) and become atomic.

Cooling by atomic cascade of \( e^- \) falling to lower levels.

\[ n_{H_2} \Lambda(T) = 10^{-32} \text{erg/cm}^3 \text{s} \quad N_{H_2} \left( \frac{T}{10^6 \text{K}} \right)^{1/2} \]  (112)
$$t_{\text{cool}} = \frac{n_1 z T}{n^2 \Lambda(T)} \approx 2 \times 10^5 \left( \frac{E}{3 \times 10^6} \right)^{3/2} \frac{\eta_H}{n_{\text{ISM}}^{1/3}} \frac{\eta}{\text{yr}}$$ \hfill (108)

$$t_{\text{cool}} \leq t_{\text{Selov}} \text{ when from (111) and (113)}$$

$$T^{7/3} < \frac{2 \times 10^5}{3.5 \times 10^4} \left( 3 \times 10^6 \right)^{7/3} \frac{\eta_H}{n_{\text{ISM}}^{1/3}} \left( \frac{E}{10^5 \text{ erg}} \right)^{1/3}$$

or

$$T < \left( \frac{3.5 \times 10^4}{2 \times 10^5} \right)^{3/4} \left( 3 \times 10^6 \right)^{(1/4)} \left( \eta_H \right)^{(1/4)} \left( \frac{E}{10^5 \text{ erg}} \right)^{(1/4)}$$

Compression ratio across shock just before cooling becomes important (see eqn. 102)

$$T \lesssim 5.7 \times 10^6 \text{ K} \left( \frac{\eta_H}{n_{\text{ISM}}} \right)^{(3/4)} \left( \frac{E}{10^5 \text{ erg}} \right)^{(1/4)} \hfill (114)$$

or

$$V = \left( \frac{kT}{m} \right)^{1/2} \leq 240 \text{ km s}^{-1} \left( \frac{E_{51} \eta_H^2}{5} \right)^{1/4}$$

notice the weak dependence on $E$ and $\eta_H$!
In radiative phase, shock becomes isothermal as it evolves.

Hot interior region but a cooled, isothermal interior shell: In frame of contact discontinuity:

\[
\begin{align*}
\text{cold} & \quad \text{"hot"} & \text{ISM} & \quad \text{cold, fast} \\
\text{slower} & \quad \text{slow} & \text{\(T_2 > T_1\)} & \text{\(V_1\) - \(V_1\)} \\
\text{\(T_1 = T_3\)} & & & \\
\text{T \ll 10^6 \text{K}} & \text{T > 10^6 \text{K}} & \text{T \ll 10^6 \text{K}}
\end{align*}
\]

\(T \ll 10^6 \text{K}\)

\(T > 10^6 \text{K}\)

\(T \ll 10^6 \text{K}\)

Cooling takes away most of the shock energy but momentum is conserved because radiation is essentially isotropic. Thus

\[
\frac{d}{dt} \left( \frac{4\pi G M}{3} r^3 \dot{r} \right) = 0 \quad \text{in radiative phase}
\]

\(\Rightarrow r^3 \dot{r} = \text{constant for } \frac{dG \mu m}{dt} = 0\).

\[
\Rightarrow r^3 dr = dt
\]

\[
\Rightarrow r \propto t^{1/4}
\]

\[
\dot{r} \propto t^{-3/4}
\]

and

\[
\sqrt{r} = \frac{240 \text{km/s}}{(E_{\text{SN}} n H^2)^{1/4} \frac{t}{(5 \times 10^4 \text{yr})^{-3/4}}}
\]

using (114) & (112)
A supernova explosion from the surrounding ISM, a gaseous nebular region that the star in the presupernova heat and ionise such a region. An expanding luminous ring from OIII was detected around the centre of the expansion. Supernovas also lead to lig phenomena discussed in Vo. Two light echoes were detected approximately 1 yr after the explosion.

A supernova emits X-rays material behind the shock. The temperature in the plasma at a temperature arises during phase 3, as well. The material is heated by the remnant in the material with a temperature of the radiating atoms. In add remnant are also strong so it spiraling in the magnetic field. Vol. I, Chap. 6, Section 6.11, electrons per unit volume is 1 then the total flux of an optic be expressed as

$$S_v = \frac{G}{d^2} V K B^{(1+p)/2} v^{-\frac{p}{2}}$$

where $V$ is the volume of the star. $K$ is a numerical factor. In the core is strongly ionised during the supernova. Frozen into the plasma fluid. It is strongly ionised during the supernova.

If the energy of individual relativistic electron increases the pressure of relativistic electron. To give $e \propto r^{-4}$. The total energy...