Aspects of Nuclear Burning & Stellar Evolution

As mentioned in the previous lecture, a number of important high energy astrophysical sources emerge from the end states of stellar evolution. A brief review of how stars evolve is therefore relevant. In particular we will review the distinction between low & high mass stellar evolution and their end states.

nuclear energy basics

atoms: e^- confined to shells surrounding nucleus

nucleus: nucleons (protons, neutrons) incur transitions similar to e^- atomic transitions

nucleons, like e^- are fermions. Each has 2 spin states.

- nucleons are more tightly bound than e^-:

→ to ionize atoms requires $\simeq$ 10 eV to 10^4 eV

→ to unbind nuclei requires 10^6 eV ↔ 1.6×10^{-6} erg ↔ $\underline{10^{10}\text{K}}$

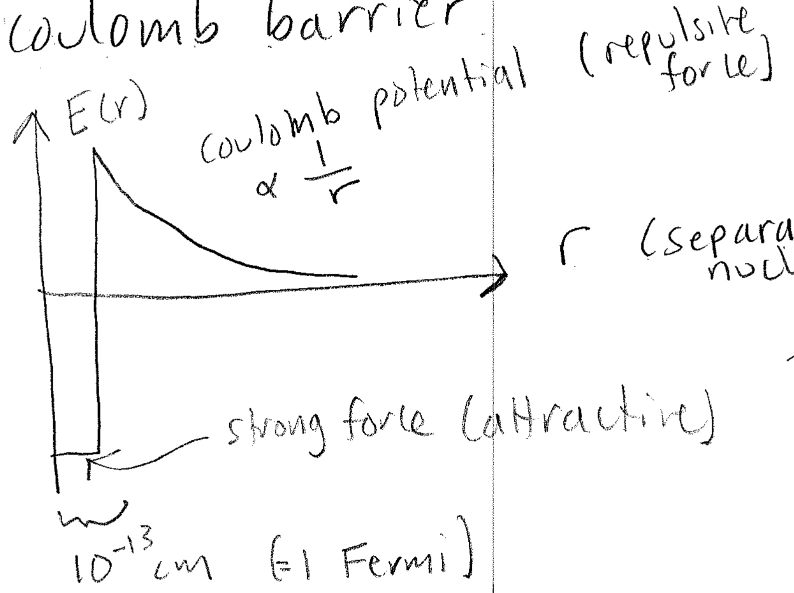
nuclei.

- even though free neutrons decay to protons with $t_{1/2} \simeq 10$ min, inside nuclei the available energy slots for the resulting protons from such decays correspond to loosely bound levels. It requires more energy to get proton into that state than energy gained by decay so bound nuclei don't easily decay.

- For stability, nucleus must have rest energy Mc^2 less than the sum of rest energies of its constituent masses if they were free
- (note: ^5Li is unstable by this criterion so was puzzle to explain heavy elements in stars if ^5Li was an intermediate stage; led to discovery of "triple α" process which solved the mystery (Hoyle et al.)

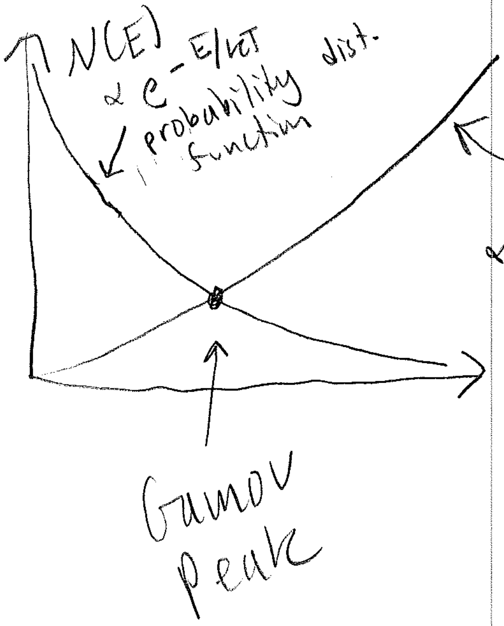
how does nuclear fusion release energy?

- at "low" temperatures nuclei are more stable with more binding energy
- central temperature of the sun is 10^7 K which is 10^{-3} times the nuclear binding energy for hydrogen
- In order to fuse, the interacting nuclei must have large enough energies to overcome their mutual coulomb barrier



need quantum tunneling to overcome coulomb barrier to reach attractive force regime.

Higher energy particles tunnel more easily but are rarer in the distribution:



$$\propto e^{-2\pi\left(\frac{m_r}{2}\right)^{1/2}\frac{q_1 q_2}{\hbar E^{1/2}}}$$

$m_r = \dfrac{m_1 m_2}{m_1 + m_2}$ = reduced mass

q_1, q_2 = charges of m_1, m_2

E = relative kinetic energy of m_1 & m_2

probability of particles being at relative energy E is N(E) which decreases with E, whereas penetration probability increases with E $\Rightarrow$ existence of <u>Gamov peak</u>.

Fusion probability from tunneling is then:

$$\propto \int_0^\infty e^{-E/kT - 2\pi\left(\frac{m_r}{2}\right)^{1/2}\frac{q_1 q_2}{\hbar E^{1/2}}}\,dE$$

maximum occurs where exponent is maximized:

$$Q(E) = \frac{-E}{kT} - 2\pi\left(\frac{m_r}{2}\right)^{1/2}\frac{q_1 q_2}{\hbar E^{1/2}}$$

$$\frac{dQ}{dE} = 0 \quad \text{where} \quad E = E_0 = \left[\left(\frac{m_r}{2}\right)^{1/2}\pi\frac{q_1 q_2}{\hbar}kT\right]^{2/3}. \quad (1)$$

for hydrogen $= \left[\left(\dfrac{m_p}{4}\right)^{1/2}\dfrac{\pi e^2 kT}{\hbar}\right]^{2/3}.$

$\longrightarrow$

Now for increasing T, the N(E) curve flattens and the Gamow peak moves up.

Thus we can estimate a characteristic T above which a significant fraction of nuclei participate in fusion by setting $E = kT$ in (1). This gives $T_{ch} \simeq 10^6$ K.

Minimum mass of a star

Given a minimum T above which star can support gravity via fusion, we can get a minimum mass of a star. We demand that separation between particles is larger than the electron De Broglie wavelength because otherwise electron degeneracy pressure limits further compression, whereas systems undergoing "steady fusion" are compressing & releasing heat.

This condition is:

$$\bar{r} = \left(\frac{m_p}{\bar{\rho}}\right)^{1/3} \geq \lambda_e = \frac{\hbar}{\Delta p} \simeq \frac{\hbar}{(2 m_e k T)^{1/2}} \qquad (2)$$

Where we assume a uniform density sphere and

$$\bar{\rho} = \frac{M}{4/3 \pi R^3} \qquad (2a)$$

→

From the virial theorem, which states that in equilibrium between gravity and internal energy

$$2K + U = 0 \qquad (3)$$

internal kinetic energy; grav potential energy

Grav pot of unit density sphere

$$2\left|\frac{3}{2}\frac{kTM}{\mu m_p}\right| = \left|\frac{3}{5}\frac{GM^2}{R}\right| \qquad (4)$$

$$\left(\mu m_p \equiv \frac{n m_p + m_e n_e}{n + n_e} = \text{mass per free particle}\right)$$

$n \equiv n_p + n_n$ = number density of nuclei

solving (4) =>

$$T = \frac{1}{5}\frac{GM\mu m_p}{kR} \qquad (5)$$

Using (5) and (2a) in (2) =>

$$\left(\frac{4/3\,\pi R^3 m_p}{M}\right)^{1/3} > \frac{\hbar}{(2m_e\cancel{k})^{1/2}}\left(\frac{5\cancel{k}R}{GM\mu m_p}\right)^{1/2}$$

$$\boxed{\Rightarrow \left(\frac{M}{M_\odot}\right)^{1/6} \gtrsim \left(\frac{R_\odot}{R}\right)^{1/2}\frac{0.1?}{\mu^{1/2}}} \qquad (6)$$

minimum mass star → can eleminate R for T: →

$$\left(\begin{aligned} M_\odot &\equiv 2\times10^{33}\,g \\ R_\odot &= 7\times10^{10}\,cm \end{aligned}\right)$$

Using (5) in (6) =>

$$\left(\frac{M}{M_\odot}\right)^{1/6} \gtrsim \left(\frac{5TkR_\odot}{GM_\odot \mu m_p}\right)^{1/2} \left(\frac{0.1}{\mu^{1/2}}\right)$$

$$\Rightarrow \boxed{M \gtrsim 0.06 \left(\frac{T}{10^6 K}\right)^{3/4} \left(\frac{0.5}{\mu}\right)^{3/2} M_\odot}$$

temperature derived for fusion

for Hydrogen plasma

Objects below this mass are planets or Brown dwarfs, no fusion.

Thus, end states we will study require initial masses larger than this. (and, actually $\gtrsim 0.5\, M_\odot$ to evolve during age of universe!)

Next we review basic evolution of low mass and high mass stars.

Note that the end states of low mass stars ($M \lesssim 8\, M_\odot$) turn into <u>planetary nebulae</u> with white dwarf cores whereas $M \gtrsim 8\, M_\odot$ evolve into <u>supernovae</u> with neutron star or black hole cores.

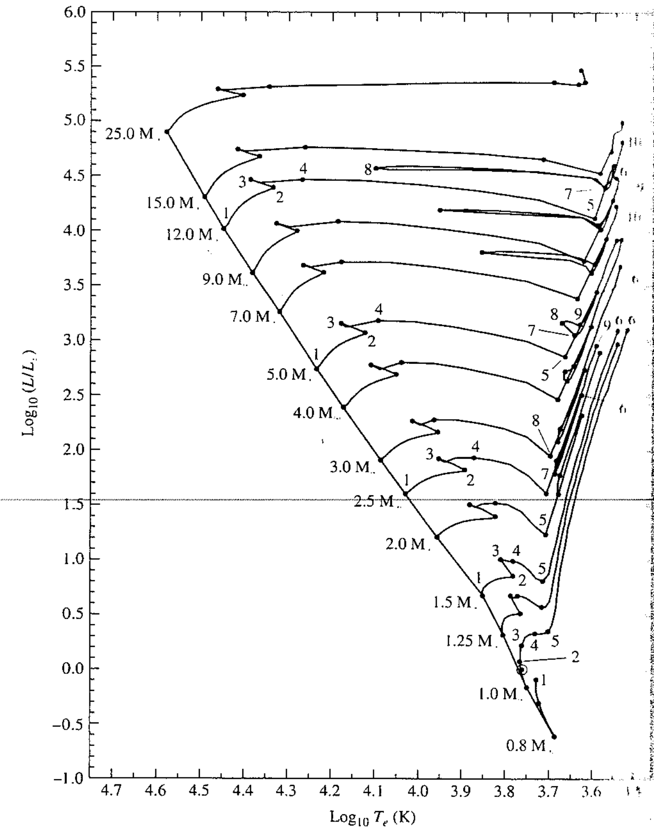


FIGURE 13.1 Main-sequence and post-main-sequence evolutionary tracks of stars with [illegible] composition of $X = 0.68$, $Y = 0.30$, and $Z = 0.02$. The location of the present-day Sun (see [illegible]) is depicted by the solar symbol ($\odot$) between points 1 and 2 on the 1 $M_\odot$ track. The elapsed [illegible] points indicated on the diagram are given in Table 13.1. To enhance readability, only the [illegible] the evolutionary tracks for 0.8, 1.0, 1.5, 2.5, 5.0, and 12.0 $M_\odot$ are labeled. The model [illegible] include mass loss and convective overshooting. The diagonal line connecting the locus of [illegible] is the zero-age main sequence. For complete, and annotated, evolutionary tracks of 1 M [illegible] stars, see Figs. 13.4 and 13.5, respectively. (Data from Schaller et al., *Astron. Astrophys.* [illegible] 269, 1992.)

TABLE 13.1 The elapsed times since reaching the zero-age main sequence to the indicated points in Fig. 13.1, measured in millions of years (Myr). (Data from Schaller et al., *Astron. Astrophys. Suppl.*, *96*, 269, 1992.)

Initial Mass ($M_\odot$)	1 6	2 7	3 8	4 9	5 10
25	0 6.51783	6.33044 7.04971	6.40774 7.0591	6.41337	6.43767
15	0 11.6135	11.4099 11.6991	11.5842 12.7554	11.5986	11.6118
12	0 16.1150	15.7149 16.4230	16.0176 16.7120	16.0337 17.5847	16.0555 17.6749
9	0 26.5019	25.9376 27.6446	26.3886 28.1330	26.4198 28.9618	26.4580 29.2294
7	0 43.4304	42.4607 45.3175	43.1880 46.1810	43.2291 47.9727	43.3388 48.3916
5	0 95.2108	92.9357 99.3835	94.4591 100.888	94.5735 107.208	94.9218 108.454
4	0 166.362	162.043 172.38	164.734 185.435	164.916 192.198	165.701 194.284
3	0 357.310	346.240 366.880	352.503 420.502	352.792 440.536	355.018
2.5	0 595.476	574.337 607.356	584.916 710.235	586.165 757.056	589.786
2	0 1148.10	1094.08 1160.96	1115.94 1379.94	1117.74 1411.25	1129.12
1.5	0 2910.76	2632.52	2690.39	2699.52	2756.73
1.25	0 5588.92	4703.20	4910.11	4933.83	5114.83
1	0 12269.8	7048.40	9844.57	11386.0	11635.8
0.8	0	18828.9	25027.9		

Low Mass Stars evolution off of the main sequence

- Consider e.g. Sun
- After H is burned in the core

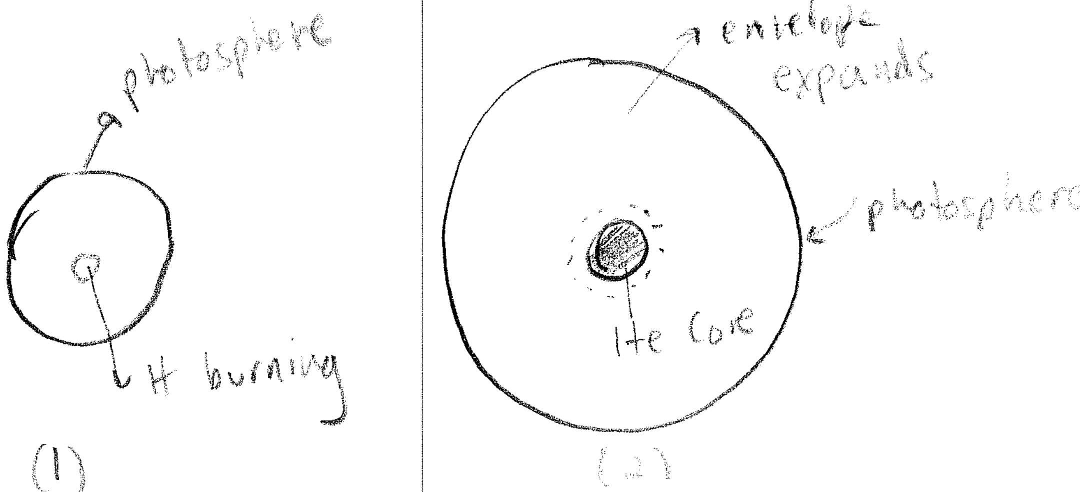
Star collapses gravitationally

- as core contracts, it heats up
- hydrogen now begins to burn from larger radii, resupplying fuel
- He core (the product of H burning) is produced
- gravitational contraction leads to faster H burning, (more compact core => higher temp & pressure required to balance gravity)
- But photons released are limited in radiative zone by photon diffusion
- Intermediate layers heat up, => expansion

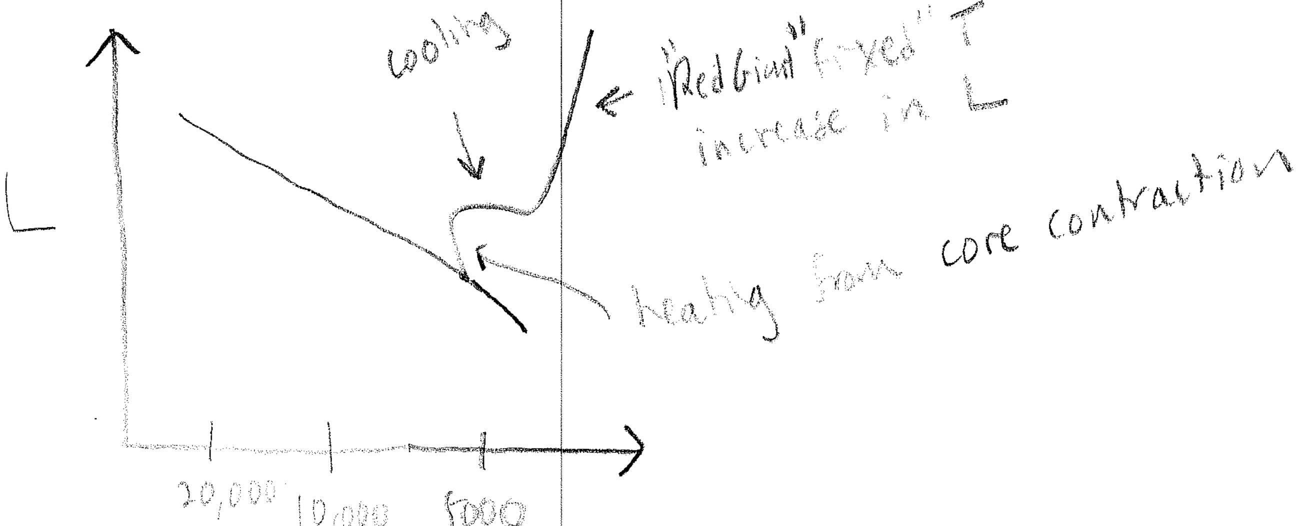
=> increase in R, at constant L

$L \simeq 4\pi R^2 \sigma T_e$ => Te decreases

→ red subgiant horizontal motion on H-R diag



- as star expands, T cannot fall arbitrarily low because opacity is sensitive to T
- leads to minimum temperature T_c
- leads to convective instability and upward path on H-R diagram (Red Giant Branch)



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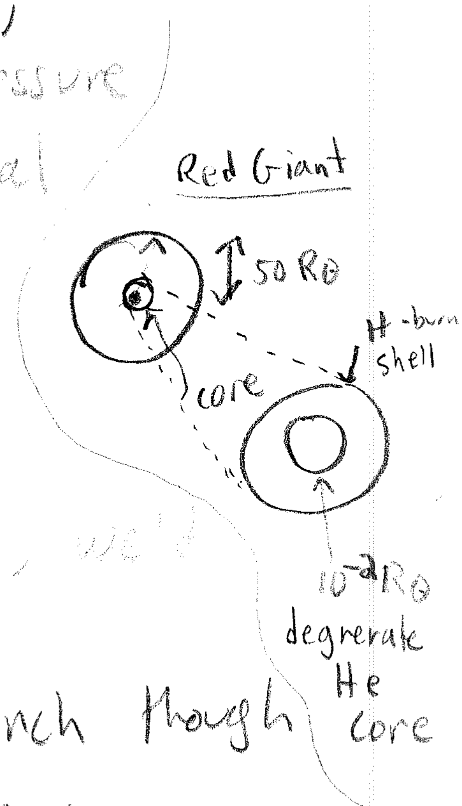
- during envelope exansion on Red Giant branch
  => red giant, convectively unstable
- the core continues to contract
- electrons become "degenerate";
  high density -> dominant pressure
  from e^- is quantum mechanical
  No more "fusion in core.
  More on WD later!
- thus if we could peel off
  outer layers of Red Giant, we'd
  see He white dwarf
- at top of red giant branch though
  $T \rightarrow 10^8$ K in core, He can
  burn



- But when degeneracy is dominant pressure source,
  burning is different than for non-degenerate
  conditions

-> non degenerate: increase T => increase P => decrease T
(by expansion)

-> degenerate: increase T => increase P = INCREASE T
(since no expansion)

=> runaway flash!

(this occurs because, under degenerate conditions
pressure is not due to thermal motions so expansion
does not accompany temperature increase!)

(Schwarzschild & Harm showed explicitly)

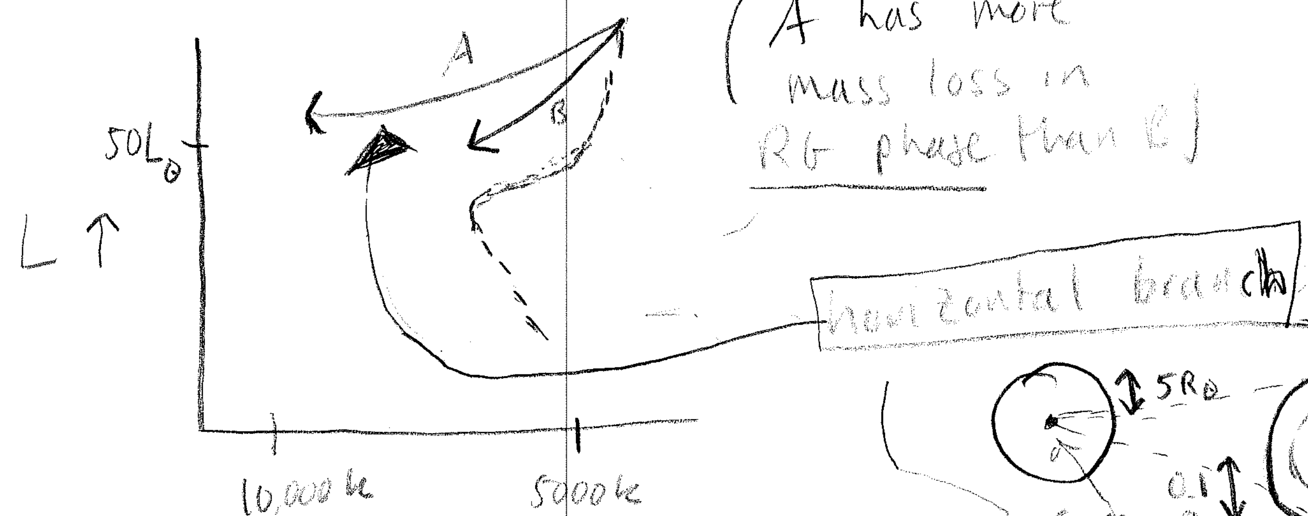
→ the "flash" removes the degeneracy

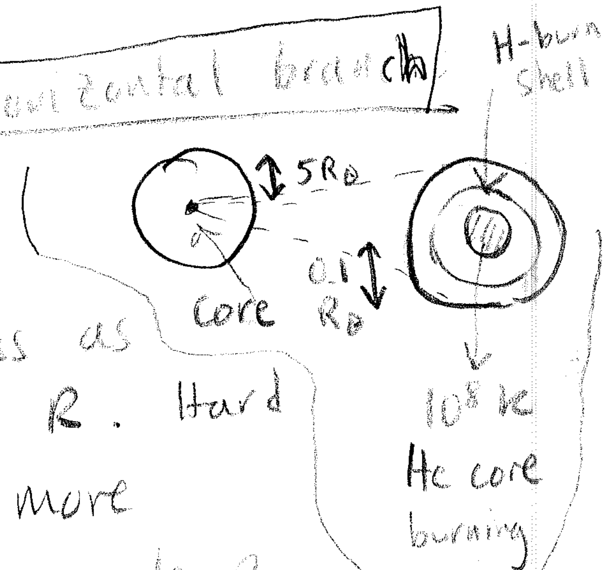
→ Normal thermal pressure then dominates & core expands

→ star has He burning core and H burning in shell, but H burning is weakened such that it produces less Luminosity because star is distended by He burning. Mass is also lost.

→ star shrinks & gets dimmer

(A has more mass loss in RG phase than B)





→ distended red giant loses mass as gravity is weak at large R. Hard to predict how much, but more mass loss ⇒ higher average temp at same L (see figure above and compare A & B cases)

- Asymptotic Giant Branch

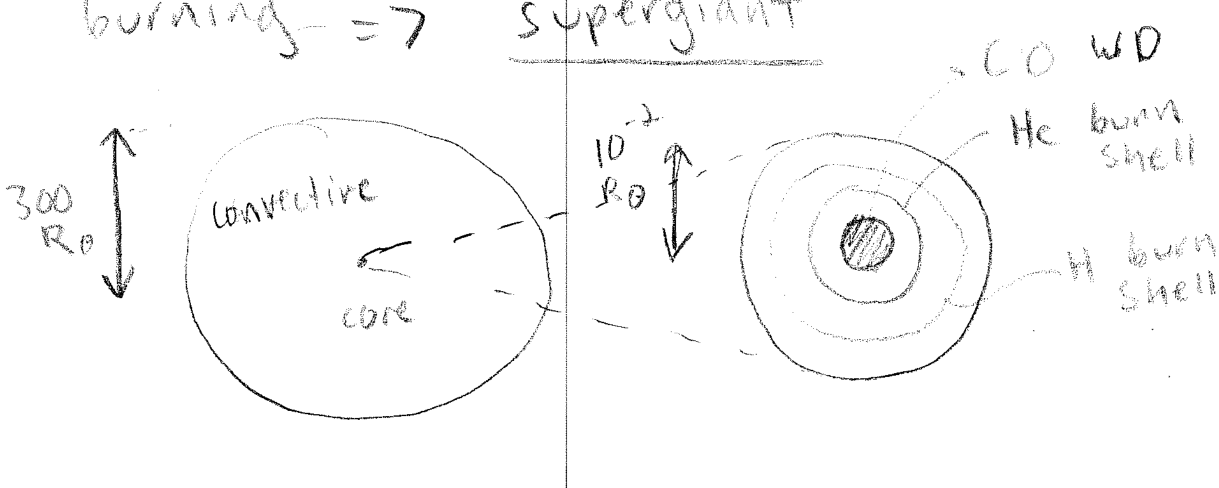
- after He core is burned out into C & O

core contracts → overlying layers
burn He, and layers above that burn H
"double shell burning"

- core contracts, burning increases
→ expansion as before
⇒ double shell burning Giant
= "asymptotic giant branch" phase (AGB)

- eventually core again becomes degenerate

- core ⇒ Carbon Oxygen White Dwarf
more mass ⇒ smaller radius WD
(more later!)
gravity is large, high amount of shell
burning ⇒ supergiant



- thermal relaxation runaway / oscillilations occur: shells burn and are spatially thin, though shell tries to expand it is too weak
- overlying material provides more fuel and shell cannot expand to cool, so Temp increases & runs away
- (Period between pulses: $\simeq 10^5 \left(\frac{M}{M_\odot}\right)^{-2}$ yr)

Afterwards, convection takes away excess heat.

=> relaxation oscillations as star tries to return back to double shell burning state between thermal pulses

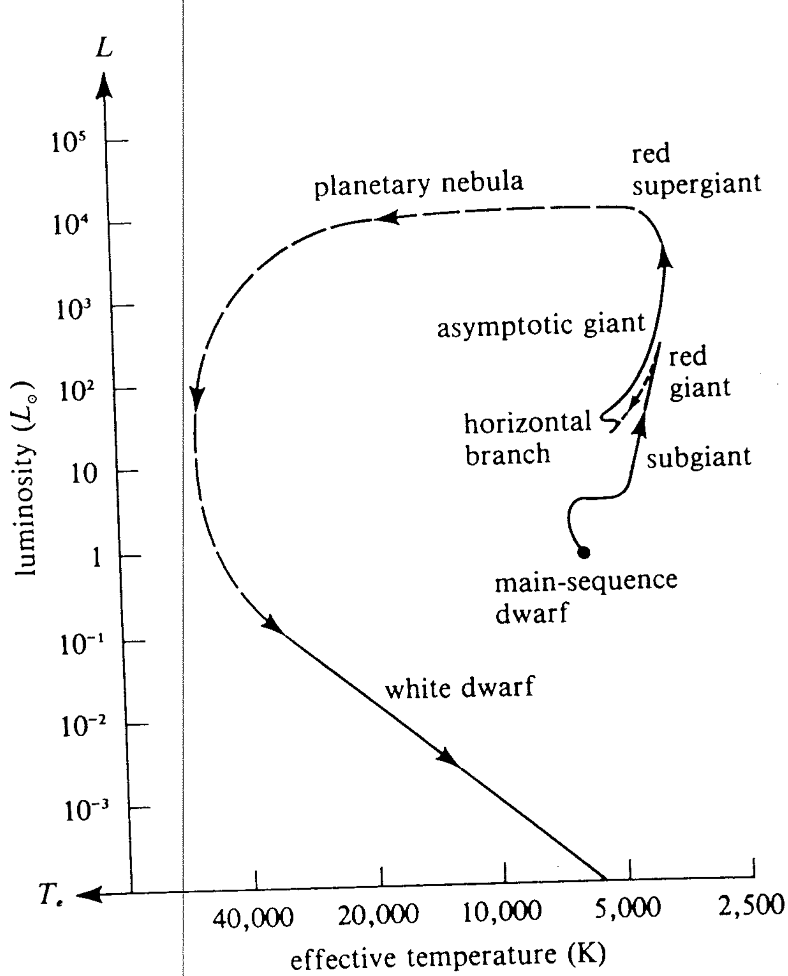
- Mass loss may also occur on AGB branch, likely radiation driven, aided by dust (possibly B-fields)
- for M initially < 6 $M_\odot$, 80 % of mass can be lost (observed), eventually

=> PNe and Carbon Oxygen WD

planetary nebulae

(31)

low mass
Stellar evolution
$M < 8 M_\odot$



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write it in the form

$$m - M = 5 \log r - 5 + A + K, \tag{1.21}$$

where K is called the K correction. The actual form of this correction depends on the shape of the spectrum of the object.

1.4 Overview of Stellar Evolution

We next overview the key features of stellar evolution that will be elaborated on in Chaps. 3–7. Figure 1.2 summarises the entire evolutionary sequence of a

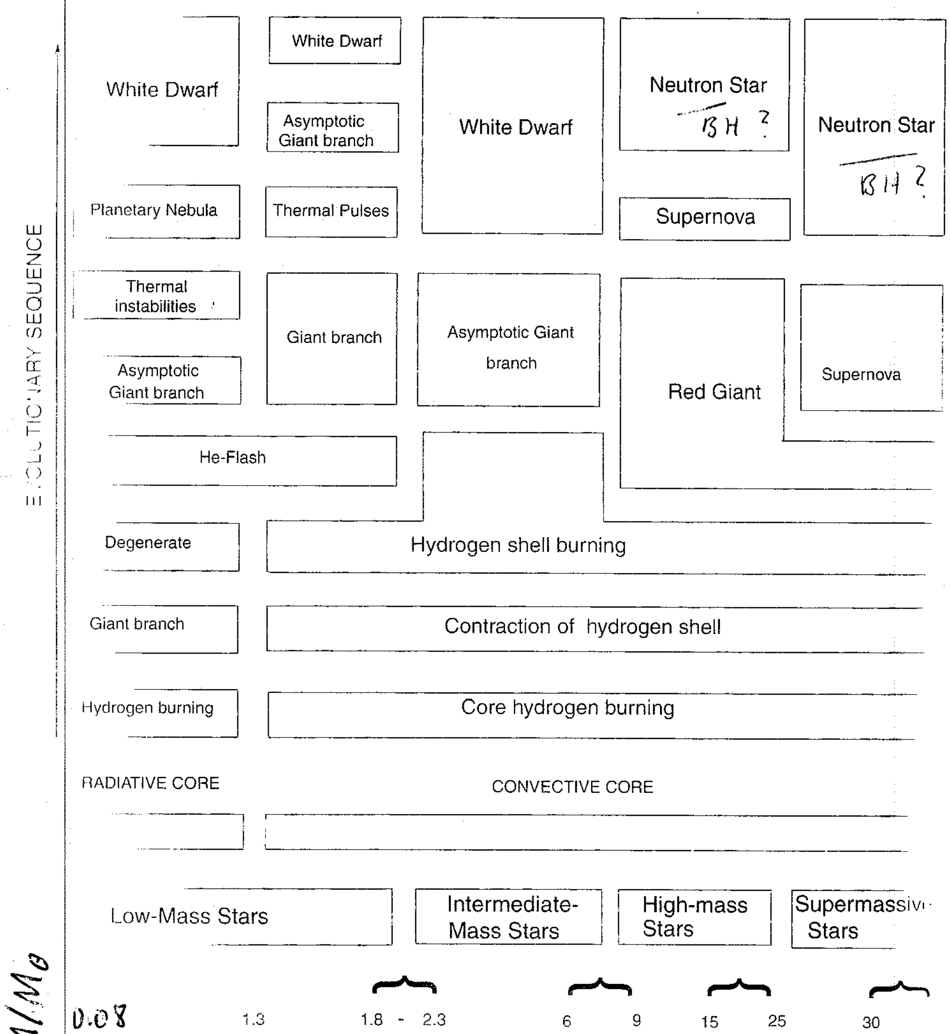

Fig. 1.2. Summary of stellar evolution; see text for discussion.