Shock-Fermi acceleration has

been widely appreciated as an effective mechanism for particle acceleration

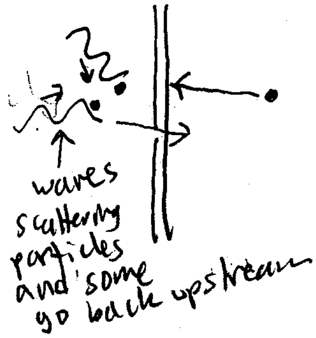
Basic Idea is this:

Consider a (e.g.-supernova) shock moving through ISM with some flux of high energy particles both ahead of and behind the shock. Assume this population of particles is seeded with enough energy initially so that their gyro-radius is larger than the characteristic scale of the thermal gyro-radius determining the shock thickness.

As particles cross the shock, they are "assumed" to scatter by "resonant wave particle interactions" on either side of the shock. The waves can be generated the particles themselves.

$\rightarrow$

down down up



the waves serve to isotropize the particle velocity in the frame where fluid is at rest locally. Now I argue that this leads to power law acceleration:

Let shock move with velocity U from left to right. (Unshocked material)

ρ_2, T_2, P_2, V_2 || ρ_1, T_1, P_1, V_1

(shocked material)

U ← lab frame

measured in shock frame

for maximally strong shock

For gas with $\gamma = \frac{5}{3}$, $\frac{\rho_2}{\rho_1} = 4$, $\frac{V_2}{V_1} = \frac{1}{4}$

In frame when shock is at rest, velocity into shock $V_1 = U$ from right to left. Then $V_2 = \frac{U}{4}$ by jump conditions

Now consider comparison of up and downstream frames:

In frame in which upstream is at rest, shocked material moves from left to right at speed $|V_2 - V_1| = \frac{3}{4}U$;

~~An upstream~~ particle will cross the shock front which moves toward the particle. For particle that crosses into downstream region, we can now get the energy of particle as it passes into downstream region:

downstream frame

upstream frame

$$E' = \Gamma_v (E + p_x V_d)$$

relative velocity between up and downstream $V = |V_1 - V_2| = \frac{3}{4}U$

momentum of particle toward shock

for $\Gamma_v \simeq 1$ (non-relativistic shock)

but relativistic particle ($E = pc$)

we have
$$E' = E + \frac{VE}{c}\cos\theta \qquad (116)$$

or $\Delta E = \frac{V}{c}\cos\theta$. Now, the probability that particles cross plane of shock and hit it at an angle θ is proportional to product of number of particles incident between the angles θ and $\theta + d\theta$ times the rate at which particles cross the shock.

this is $\propto$ $\underbrace{c\cos\theta}_{x\text{-component of velocity}}$ $\sin\theta\, d\theta\, d\phi$ ← not important due to symmetry

solid ∡ element

so the $P(\theta) \propto \cos\theta \sin\theta \, d\theta$ (114)

properly normalized distribution (i.e. such that $\int_0^{\frac{\pi}{2}} P(\theta)\,d\theta = 1$)

$P(\theta) = 2\cos\theta \sin\theta \, d\theta$, so that

$$\left\langle \frac{\Delta E}{E} \right\rangle = \int P(\theta) \frac{\Delta E}{E} d\theta = \frac{V}{C}\int_0^{\frac{\pi}{2}} 2\cos^2\theta \sin\theta \, d\theta = \frac{2}{3}\frac{V}{C} \quad (117)$$

Thus particles gain energy when moving from upstream to downstream.

Now consider downstream frame to be at rest. Then particles downstream see upstream flowing toward it with speed $|V_1 - V_2| = \frac{3}{4}U$

The same argument above holds, so the Net energy gain in going down to upstream is the same. ==> total fractional energy gain is

$$2 \times \frac{2}{3}\frac{V}{C} = \frac{4}{3}\frac{V}{C} = \frac{U}{C} \equiv \frac{\Delta E_{tot}}{E} \quad (118)$$

in moving back and forth across the shock. Note that this requires the isotropization of particle velocity vectors on each side of the shock, so that their "average" motion across the shock is determined by the flow speeds not their particle speed.

Now we can get the accelerated particle energy spectrum:

let acceleration increase energy by factor β each full trip (across and back across the shock)

- Let τ be mean time between two occurences of "full trips", so the rate is $\frac{1}{\tau}$ $\left(= \frac{\#\text{ full trips}}{\text{time}} \right)$
- Assume diffusive nature of the scattering process (that isotropizes the particle momentum) so probability is $P(t) = e^{-t/T}$ for particle to remain in accelerating region for duration t, where T is time scale for escape. The number of particles that remain in accelerating region during $t + dt$ is

$$n(t)\,dt = N_0 e^{-t/T} d(t/T) \qquad (118)$$

$\rightarrow$

Since mean number of occurences of process in time t is $\left(t/\tau\right)$,

$\uparrow$ time per single process

typical energy will evolve as

$$E(t) = E_0 \beta^{t/\tau}$$

$$= E_0 \exp\left[\frac{\ln\beta}{\tau} t\right]$$

Then $t = (\tau/\ln\beta)\ln\left(E/E_0\right)$ (119)

so (118) and (119) $\Rightarrow$

$$n(E)\,dE = n(t)\frac{dt}{dE}\,dE \simeq \frac{\tau}{\ln\beta}\left(\frac{E_0}{E}\right)\exp\underbrace{\left[\frac{-\tau}{T\ln\beta}\ln\frac{E}{E_0}\right]}_{-t/T} dE/E_0 \quad (120)$$

the probability for particle to remain in accelerating region after 1 occurence of process is $P_0 = e^{-\tau/T}$, so

$\frac{\tau}{T} = -\ln P_0$. Using this in (120)

gives $n(E) \propto \frac{\exp\frac{\ln P_0 \ln E}{\ln\beta}}{E}\,dE$

$$\simeq E^{-1+\frac{\ln P_0}{\ln\beta}}\,dE$$

$$= E^{-q}\,dE$$

$$q = 1 + \frac{\tau}{T\ln\beta}$$

$$= 1 - \frac{\ln P_0}{\ln\beta} \quad (121)$$

$\rightarrow$

to ~~to~~ get value of power law index, we need β and P_0:

Since $\frac{\Delta E}{E} = \frac{4V}{3c}$ we know that

$$\beta = 1 + \frac{\Delta E}{E} = 1 + \frac{4}{3}\frac{\overset{\swarrow \frac{3}{4}U}{V}}{c} = 1 + \frac{\overset{\swarrow \text{shock speed in lab frame}}{U}}{c} \qquad (122)$$

To get P_0, we use following argument:
number of particles per area crossing from upstream to downstream is $\underset{\uparrow \text{number density of high energy particles moving toward shock}}{n_{\gamma,u}} c$ which in downstream becomes $\frac{n_{\gamma,d}\, c}{4}$ due to density compression. Downstream, these particles are swept away at rate $n_{\gamma,d} V = \frac{n_{d,\gamma} U}{4}$.
Fraction lost = probability for escape:
This is $P_{esc} = 1 - P_{trap} = \frac{\frac{n_{d,\gamma} U}{4}}{\frac{n_{d,\gamma}\, c}{4}} = \frac{U}{c}$, so that

$$\left.\begin{aligned} P_{trap} &= 1 - \frac{U}{c} \\ &= \text{probability for trapping per crossing} \\ &= P_0 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} \ln P_0 &\simeq \ln\left(1 - \frac{U}{c}\right) \overset{\text{for } \frac{U}{c} \ll 1}{\simeq} -\frac{U}{c} \\ \ln \beta &\simeq \ln\left(1 + \frac{U}{c}\right) \simeq \frac{U}{c} \end{aligned}$$

then plugging into (121) for q

gives for $q = 1 - \frac{\ln P_0}{\ln \beta} \simeq 1 + \frac{V/c}{V/c}$

$= 2$

$\Rightarrow \quad n(E)\,dE \propto E^{-2}\,dE$

So we get a power law with index 2 for a strong shock!

When placed into the sychrotron radiation formula, the flux

$S_\nu \propto \nu^{-(p-1)/2} \propto \nu^{-1}$ consistent with observations!

<u>Exercise</u>: derive q for arbitrary shock compression ratio $1 \leq \frac{\rho_2}{\rho_1} \leq 4$