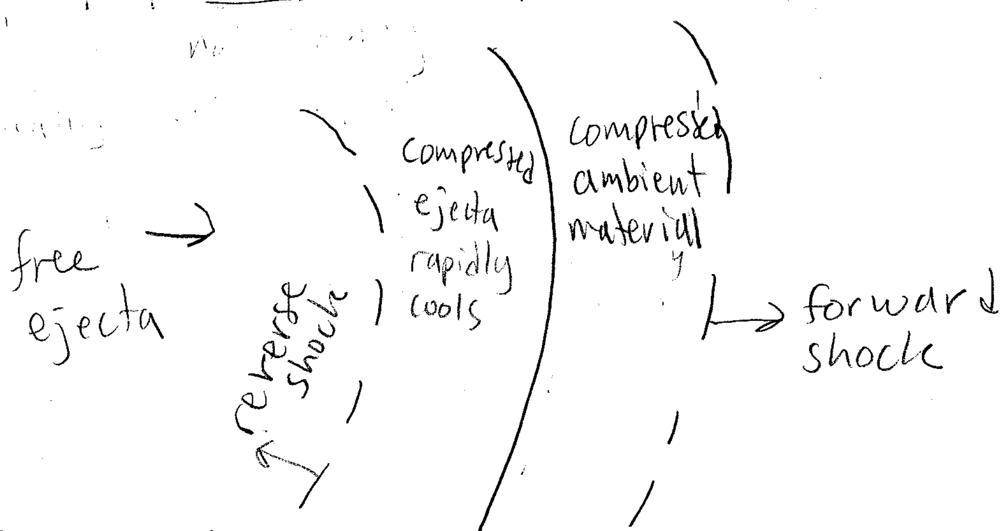
Some aspects of shock propagation through Supernova envelope and ambient Interstellar Medium (91)

- deep in star where energy from outward propagating material comes from radioactivity, thermalization occurs with temp in opt-UV range.
- when outflow becomes optically thin, effective "temperature" goes up (that is, γ and X-ray photons are not down scattered efficiently so we see high energy non-thermal emission
- source of energy eventually changes from radioactive decay, to conversion of bulk flow energy at shock (remember shocks are sites of bulk flow dissipation)
- Forward shock <u>and</u> Reverse shocks are present:



Shocks propagate away from the highest density regions. Because of rapid cooling by Bremsstrahlung in the compressed regions, the high density region also supersonically migrates "backward" into the free ejecta in rest frame of the contact discontinuity →

- note that the ejecta, contact discontinuity and reverse shock are all moving outward in the lab frame, but in the frame of the contact discontinuity there are shocks propagating both outward = forward and inward toward the explosion point = reverse shock.
- forward & reverse shocks are important concepts throughout supersonic astrophysics (jets, GRB, etc..)
- The supernova Remnant SNR (scales $\gtrsim 1000$ AU) emits by conversion of bulk flow energy at shock: ejecta has kinetic energy

$$\simeq 10^{51} \text{erg} = \frac{1}{2} M_{ej} V_{ej}^2$$

$$M_{ej} \simeq 4 M_\odot \Rightarrow V_{ej}^2 \gtrsim 10^{18} \frac{\text{cm}^2}{\text{s}^2} \Rightarrow V_{ej} \simeq 10^9 \frac{\text{cm}}{\text{s}}$$

$\Rightarrow$ Initial "temperatures" as high as 10^8–10^9 K (using $V = \left(\frac{kT}{m_p}\right)^{1/2}$).

- but there is an important subtlety as the shock reaches these scales $\gtrsim 1000$ AU

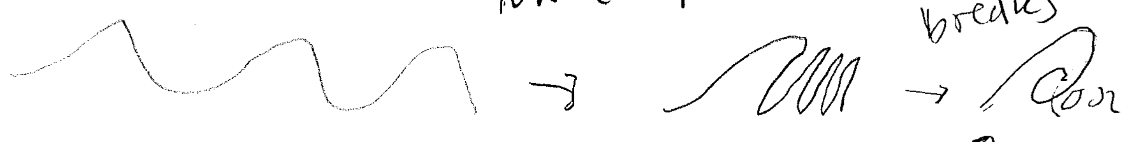
Lets look a bit at the shock physics

$\rightarrow$

- Recall from our brief discussion of shocks form as waves steepen non-linearly

waves are calculated as linear perturbations of the hydro equations. They move at speed $\simeq c_s$ for un-magneized plasma. Because pressure disturbance from ejecta moves at $V_{eject} \gg \left(\frac{T_{ISM}k}{m_p}\right)^{1/2}$ waves pile up:

10^3 K



randomization of ordered motion → heating. entropy increase

The role of "nonlinearily" arises in the Navier-Stokes equation (fluid momentum)

$$\frac{\partial V}{\partial t} = -\underbrace{V\cdot\nabla V}_{\text{non-linear term}} - \frac{\nabla P}{\rho} + \nu \nabla^2 V \qquad (91)$$

dissipation term, increases entropy

important when there are large gradients even for small viscosity ν.

Viscosity is always approximately $\simeq$ speed $\times$ length:
Typically, for ambient ISM into which shock propagates $\nu \simeq c_s l_{mfp}$.

c_s ← sound speed; l_{mfp} ← mean free path

Because "non-linear" effects induce dissipation we know shocks are important where $|V\cdot\nabla V| \simeq \nu\nabla^2 V$ (92)

→

eqn 92 $\Rightarrow$ $|v\cdot\nabla v| = \nu_{eff}\nabla^2 v$

$$\frac{v^{\cancel{2}}}{\cancel{\ell}} = \frac{\nu_{eff}\cancel{v}}{\ell^{\cancel{2}}}$$

some "effective" mean free path

or $$\ell = \frac{\nu_{eff}}{V_1} \simeq \frac{c_{s_2}\ell_{eff}}{V_1} \qquad (93)$$

In vicinity of shock, the velocity transits from $V_1 \gg c_{s_1}$ to $V_2 \lesssim c_{s_2}$. Thus eqn (93) $\Rightarrow$ $\ell \simeq \ell_{eff}$ should be the scale over which the flow changes from "upstream" to "downstream". Typically, therefore we expect the shock thickness to be $\sim \ell_{eff}$. (In reality, instabilities broaden the shock somewhat, but put that aside for the moment). Now let us estimate this for

Supernova Remnants: At ejection velocity $V_{ej} \simeq 10^9\,\mathrm{cm/s}$ kinetic energy per proton in the ejecta is about 2 MeV. As these protons hit an largely neutral H atom of the ISM, the latter will ionize.

Cross section of interaction is $\underline{\sigma_{ion}} \simeq 10^{-17}\,\mathrm{cm}^2 \left(\sim \frac{\hbar^2}{m_p^2 v_{th}^2}\right)$

Energy lost per ionization is $\simeq 50\,\mathrm{eV}$ (which represents the inelastic part of the collision). The stopping distance of the impinging protons is therefore

$$\ell_{eff} = \frac{E}{\frac{dE}{d\ell}} \simeq \frac{E}{\Delta E}\ell_{mfp} = \frac{2\,\mathrm{MeV}}{50\,\mathrm{eV}}\frac{1}{n\sigma_{ion}} \qquad (94)$$

for $n \simeq 1/\mathrm{cm}^3$

$$\Rightarrow \ell_{eff} \simeq 4\times10^4\, 10^{+17} \simeq 4\times10^{21}\,\mathrm{cm} \simeq 10^3\,\mathrm{pc}\ !$$

But shock thicknesses observed are MUCH smaller than 10^3 pc. In fact the entire remnants become invisible (merged with ambient medium) on scales of 50 pc. Thus, how can thin shock form if the scale ℓ_{eff} were actually 10^3 pc??

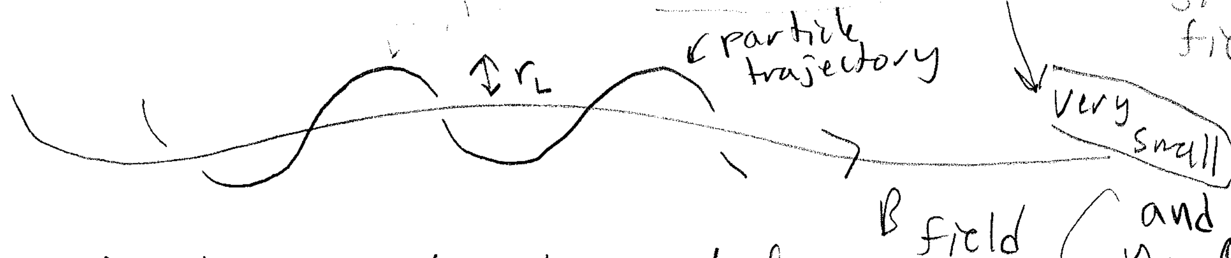
Here the answer is magnetic fields!

Calculate the larmor radius for microgauss field:

$$r_L \equiv \frac{mcV_{th}}{eB} \simeq \frac{(10^{-24}\,g)(3\times10^{10}\,\frac{cm}{s})(10^{9}\,\frac{cm}{s})}{(4\times10^{-10})(3\times10^{-6}\,G)}$$

$$\simeq 2.5\times10^{10}\,cm \simeq 10^{-8}\,pc\,!!$$

note very small strength field of ISM



(and replaces the ℓ_{eff} in (95) with much smaller value so ℓ_{eff} is then much smaller)

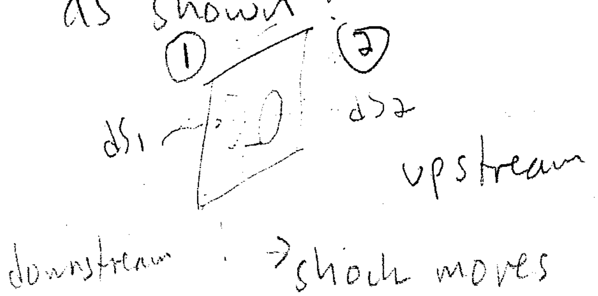
B-fields are fundamental for "collisionless shocks" in astrophysics. They make the "effective mean-free path" equal to the Larmor radius which is much smaller than the collisional mfp even for extremely weak magnetic fields.

~~Shock~~ More on Shock Jump conditions And Application to Supernova Blast Wave

Assume that the shock represents a thin discontinuity. (this was justified in part last lecture):

Conservation of mass, energy & momentum can all be written $\partial_t Q + \vec{\nabla}\cdot\vec{F}_Q = 0$

If we integrate such a conservation law across the thin discontinuity using the "pill box" as shown:



In steady state:

$$\underbrace{\cancel{\partial_t Q}}_{0} + \vec{\nabla}\cdot\vec{F}_Q = 0 \Rightarrow \vec{\nabla}\cdot\vec{F}_Q = 0$$

but volume is arbitrary

so that $$\int \vec{\nabla}\cdot\vec{F}_Q \, d^3x = 0 \underset{\uparrow}{=} \int_S \vec{F}_Q\cdot d\vec{S} \qquad (95)$$

by Gauss' theorem

For mass continuity:

$$\underbrace{\cancel{\partial_t \rho}}_{0} + \nabla\cdot(\rho u) = 0 \Rightarrow \oint \rho\vec{u}\cdot d\vec{S} = 0$$

$\rho_1 u_1 \cancel{dS_1} = \rho_2 u_2 \cancel{dS_2}$ $\quad$ $dS_1 = dS_2$ for pill box

$$\Rightarrow \boxed{\rho_1 u_1 = \rho_2 u_2} \qquad (96)$$

similarly: for flows in which B-field is energetically negligible:

$$w_1 + \tfrac{1}{2}V_1^2 = w_2 + \tfrac{1}{2}V_2^2 \qquad \text{energy conservation} \qquad (97)$$

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 \qquad \text{momentum flux conservation} \qquad (98)$$

$\left(w = \text{enthalpy density} = \frac{\Gamma}{\Gamma-1}\frac{P}{\rho} = \frac{c_s^2}{\Gamma-1} \right)$

<u>96-98</u> are the <u>Rankine-Hugoniot jump conditions</u> for a shock. Define $M_1^2 \equiv V_1^2/c_{s1}^2$

Solving (96-98) (I leave as <u>exercise</u>)

$$\frac{\rho_2}{\rho_1} = \frac{(\Gamma+1)M_1^2}{(\Gamma+1)+(\Gamma-1)(M_1^2-1)} = \frac{V_1}{V_2} \qquad (99)$$

$$\frac{P_2}{P_1} = \frac{(\Gamma+1)+2\Gamma(M_1^2-1)}{\Gamma+1} \qquad (100)$$

$$\frac{c_{s2}^2}{c_{s1}^2} = \frac{T_2}{T_1} = \frac{\left[(\Gamma+1)+2\Gamma(M_1^2-1)\right]\left[(\Gamma+1)+(\Gamma-1)(M_1^2-1)\right]}{\left[(\Gamma+1)^2 M_1^2\right]} \qquad (101)$$

$\rightarrow$

Assume flow is supersonic on side 1

so $M_1 = \frac{v_1}{c_{1s}} > 1$.

Then $\frac{P_2}{P_1} > 1$, $\frac{\rho_2}{\rho_1} > 1$, $\frac{v_2}{v_1} < 1$, $\frac{T_2}{T_1} > 1$.

strongest shock $\Rightarrow M_1^2 >> 1$

$$\Rightarrow \underbrace{\frac{\rho_2}{\rho_1} = \frac{\Gamma+1}{\Gamma-1}}_{\text{(limiting relation as } M_1^2 \to \infty!)} \; ; \quad \frac{P_2}{P_1} >> 1, \quad \frac{T_2}{T_1} >> 1 \qquad (102)$$

$\Rightarrow$ for $\Gamma = 5/3 \Rightarrow \frac{\rho_2}{\rho_1} = 4$

(Note: momentum conservation and mass conservation are usually satisfied as in 96 & 98, but energy conservation can have important radiative terms, chemical reaction terms, thermal conduction..., we ignore these for the moment.)

The above treatment assumes that the viscous terms operate only in the thin layer of the shock itself; this gets back to our notion from the previous lecture that the shock thickness can be estimated by comparing dissipative & bulk velocity terms:

In momentum equation, compare $V \cdot \nabla V$ term to $\nu \nabla^2 V$ term: (see page 93)

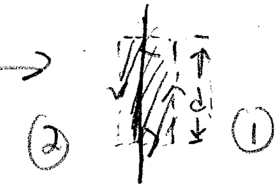
$$\Rightarrow \frac{V^{\cancel{2}}}{\cancel{\ell}} \sim \frac{\nu_{eff}}{\ell^{\cancel{2}}} \cancel{V} \Rightarrow V = \frac{\nu_{eff}}{\ell_{eff}}$$

, where ν_{eff} is the effective "viscosity" at the shock.

Now across the shock, the bulk energy of the flow in V_1 gets converted to random thermal energy such that $C_{S_2} \sim V_1$. As discussed on p 95 of this lecture the previous lecture, ℓ_{eff} is determined by multiples of larmor radius rather than collisional mean free path.

The shock is actually a "current sheet" when B-field included in jump conditions. This is because maxwells equation require that <u>tangential component</u> of E is conserved across the shock:

Consider "pill surface" crossing shock $\rightarrow$



from Maxwells equations:

$$\underbrace{\frac{1}{c}\frac{\partial B}{\partial t}}_{0 \text{ in steady state}} + \nabla \times E = 0 \Rightarrow \int (\nabla \times E) \cdot d\vec{S} = 0$$

stokes thm

surface is arbitrary

$$\Rightarrow \oint E \cdot d\ell = 0$$

for arbitrarily thin pill surface only the sides contribute:

$$\Rightarrow \oint E \cdot d\ell = 0 = E_{T,1} d - E_{T,2} d = 0$$

$$\Rightarrow \boxed{E_{1,T} = E_{2,T}} \longrightarrow$$

Since Ohms law implies

$$E = -\frac{V}{c}\times B + \eta J \quad \text{then}$$

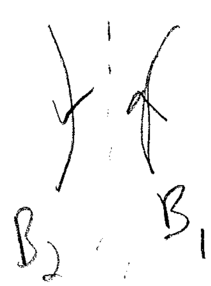
$$E_{1,T} = E_{2,T}$$

$$\Rightarrow \left(-\frac{V}{c}\times B + \eta J\right)_{1,T} = \left(-\frac{V}{c}\times B + \eta J\right)_{2,T}$$

but $J = \frac{c}{4\pi}\nabla\times B$ and away from shock, $\frac{\eta c}{4\pi}\nabla\times B$ can be considered small;

η is the resistivity and most astro-plasma have low resistivity. However, near the shock $\nabla\times B \simeq \frac{B}{\ell_{eff}} \simeq \frac{B}{r_L}$ ← (see eqn 95!)

The gradient scale is small and near the shock ηJ is important. This is why a shock is a "current sheet." Magnetic Reconnection provides another example of a current sheet based on same principle



(reconnection event)

← magnetic field annihilation at dotted interface: $\boxed{\text{Exercise}}$: show that interface is a current sheet if interface is thin!

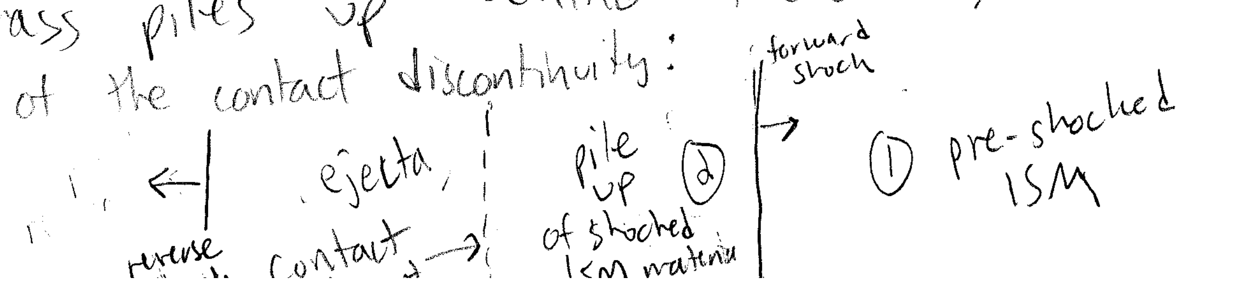
Now back to the evolution of the expanding SN shock: Transition to Sedov phase

During the early stages of the propagation of the optically thin phase of the shock's progress through the envelope and into ISM, the ejecta material has much more inertia than the ISM with which it interacts. The ejecta speed V_1, is thus constant.

BUT: There exists a critical radius r_c at which the ejecta mass $M_{ejecta} = \frac{4}{3}\pi \rho_{ism} r_c^3$.

At this point the blast enters the Sedov phase. Now the mass is piling up behind the shock and this mass starts to dominate the total mass of the ejecta.

The mass piles up behind the shock, but ahead of the contact discontinuity:



Once the Sedov phase is underway the speed of the blast wave is no longer constant: In the Sedov phase mass is dominated by that accumulated from ISM & so the Energy is

$$E \simeq \frac{1}{2}\frac{4\pi}{3}(\rho_{ISM} r^3) V_1^2 = \text{constant} \qquad (103)$$

radius of expanding shell

just the initial explosion energy

constant ρ_{ism} =>

$$E \propto r^3 V_1^2 \Rightarrow r^3\left(\frac{dr}{dt}\right)^2 = \text{constant}$$

$$\Rightarrow r^{3/2} dr = dt$$

$$\Rightarrow r = (\text{constant})\, t^{2/5}. \qquad (104)$$

Another way to arrive at this is to note that ρ_{ism} and E are constant and

$$E \simeq \frac{1}{2} M \left(\frac{r}{t}\right)^2 = \text{const} \qquad (105)$$

$$\rho_{ism} = \frac{M}{\frac{4\pi}{3} r^3} = \text{const.} \qquad (106)$$

$$\Rightarrow \frac{E}{\rho_{ism}} = \text{const} = \frac{2\pi}{3}\frac{r^5}{t^2} \Rightarrow r = \left(\frac{Et^2}{\rho_{ism}}\right)^{1/5} \qquad (107)$$

→

$$\Rightarrow \quad r = \left(\frac{E}{\rho_{ISM}}\right)^{1/5} t^{2/5} = 3\,\text{pc} \left(\frac{E}{10^{51}\,\text{erg}}\right)^{1/5} n_{ISM}^{-1/5} \left(\frac{t}{300\,\text{yr}}\right)^{2/5} \quad (108)$$

(applies only for $r > r_{crit} \simeq \left(\frac{3 M_{ejecta}}{4\pi \rho_{ISM}}\right)^{1/3}$)

$$\Rightarrow \quad V_1 \simeq \frac{r}{t} \simeq \underline{3\times10^3\,\frac{\text{km}}{\text{s}}} \left(\frac{E}{10^{51}\,\text{erg}}\right)^{1/5} n_{ISM}^{-1/5} \left(\frac{t}{300\,\text{yr}}\right)^{-3/5} \quad (109)$$

using $V_1 \simeq C_{s_2} \Rightarrow$

$$T \simeq \frac{m_p}{k} C_s^2 \simeq \frac{m_p}{k} V_1^2 \simeq 9\times10^8\,\text{K} \left(\frac{E}{10^{51}\,\text{erg}}\right)^{2/5} n_{ISM}^{-2/5} \left(\frac{t}{300\,\text{yr}}\right)^{-6/5} \quad (110)$$

$\Rightarrow$ $\underline{\text{at } t = 3.5\times10^4\,\text{yr},\ T \simeq 3\times10^6\,\text{K}}$

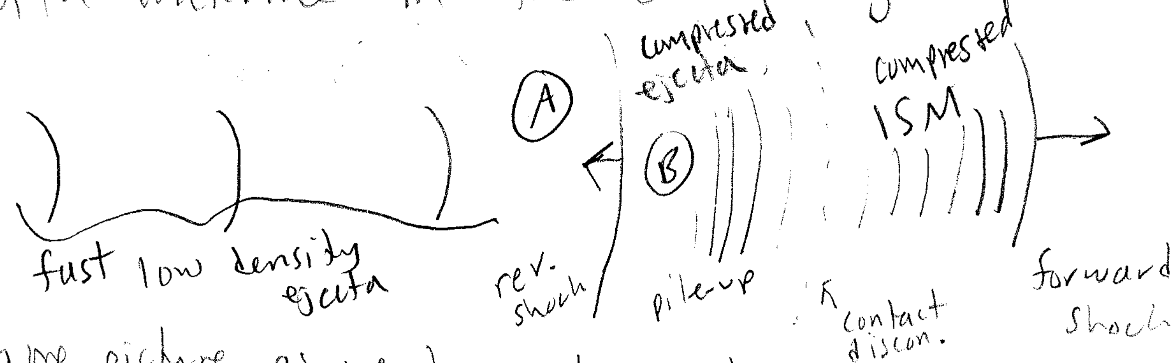
Thus if SNR is observed with $T \simeq 3\times10^6$ K (as is Cygnus Loop), the time in Sedov phase to reach that stage is, from (110)

$$t_{sedov} = 3.5\times10^4\,\text{yr} \left(\frac{T}{3\times10^6}\right)^{-5/6} \left(\frac{E}{10^{51}\,\text{erg}}\right)^{1/3} n_{ISM}^{-1/3} \quad (111)$$

$\Rightarrow$ for given V or T and r observed (to determine if $r > r_{crit}$) <u>age</u> can be determined

$\rightarrow$

Now, as deceleration becomes significant outer shells of expanding sphere decelerate first => material in the inner region catches up with material in the outer regions

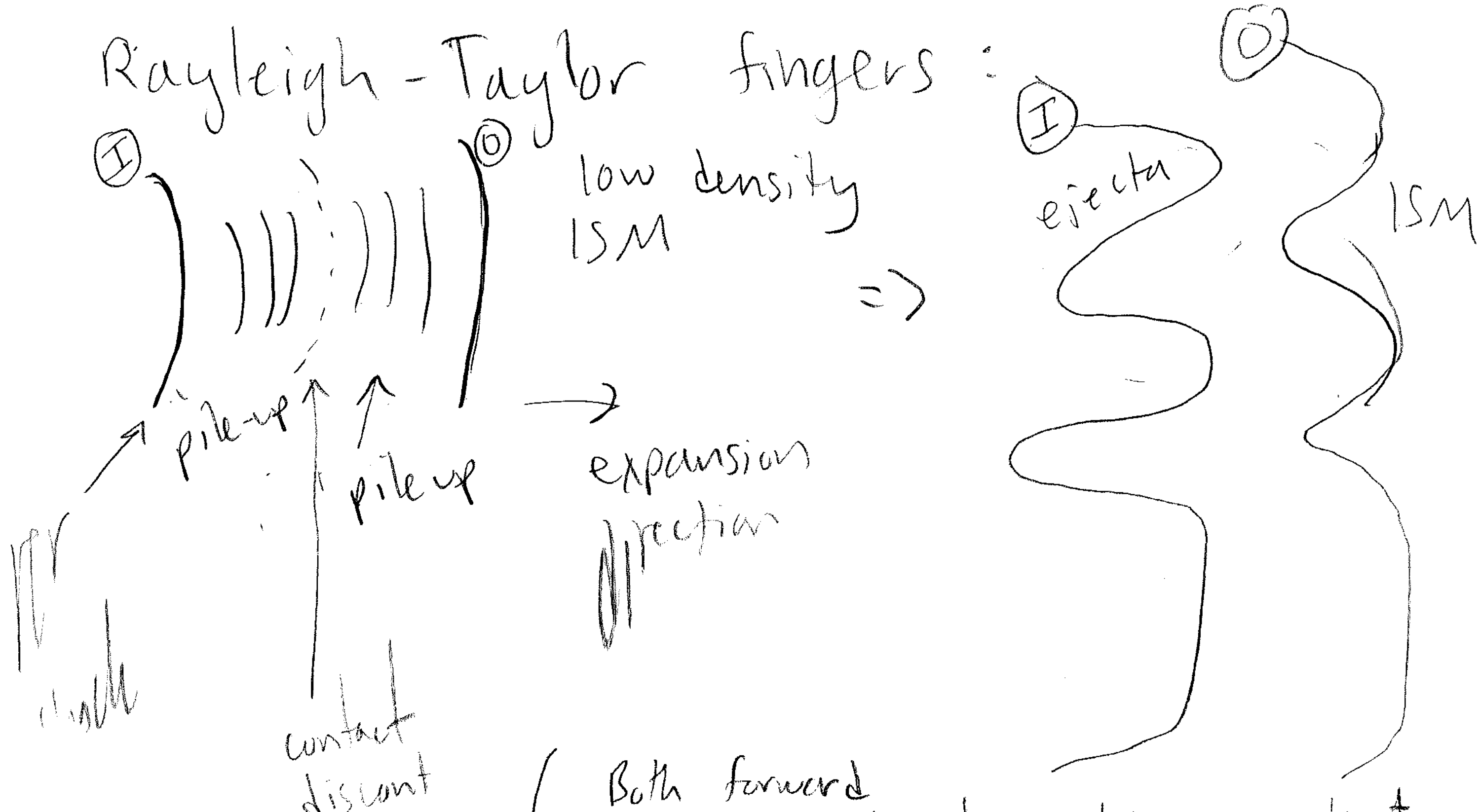


(same picture as we have disussed earlier)

Region A is supersonic with respect to B => reverse shock moves "backward" in frame of contact discontinuity. In lab frame everything is moving outward.

At the reverse shock, kinetic energy of ejecta is re-heated by reverse shock dissipation as it passes through → implies some of the bulk energy of the ejecta goes back into heat of ejected material. The forward shock converts some of the bulk energy into heating ambient ISM material. (X-ray emission is visible from both shocked regions) we mentioned, and will discuss later, the Rayleigh Taylor instability, which takes place during the Sedov phase. The

Rayleigh-Taylor fingers:



(Both forward and reverse directions have equivalent of "heavy" fluid falling on "light" fluid)

Radiative phase of SNR blast wave

once radiative cooling time becomes short compared to Sedov age we have radiative phase. Sedov age is given by (111).

For cooling time, note that for $T < 10^6 K$

C, N, O gain e^- and become atomic, cooling by atomic cascade of e^- falling to lower levels dominates:

$-1/2$

$$t_{cool} \simeq \frac{nkT}{n^2 \Lambda(T)} \simeq 2\times10^5 \left(\frac{T}{3\times10^6}\right)^{3/2} n_H^{-1} \ \text{yr} \qquad (113)$$

number density for compressed region

$t_{cool} < t_{sedov}$ when

from (111) and (113)

$$T^{7/3} < \frac{2\times10^5}{3.5\times10^4}\left(3\times10^6\right)^{7/3} \frac{n_H}{n_{ism}^{1/3}} \left(\frac{E}{10^{51}\,\text{erg}}\right)^{1/3}$$

or

$$T < \left(\frac{3.5\times10^4}{2\times10^5}\right)^{3/7} (3\times10^6)(4)\left(n_H^{2/3}\right)^{3/7} \left(\frac{E}{10^{51}}\right)^{1/7}$$

compression ratio across shock just before cooling becomes important (see eqn 102)

$$\boxed{T < 5.7\times10^6\,\text{K}\,\left(n_H^{2/7}\right)\left(\frac{E}{10^{51}\,\text{erg}}\right)^{1/7}} \qquad (114)$$

or

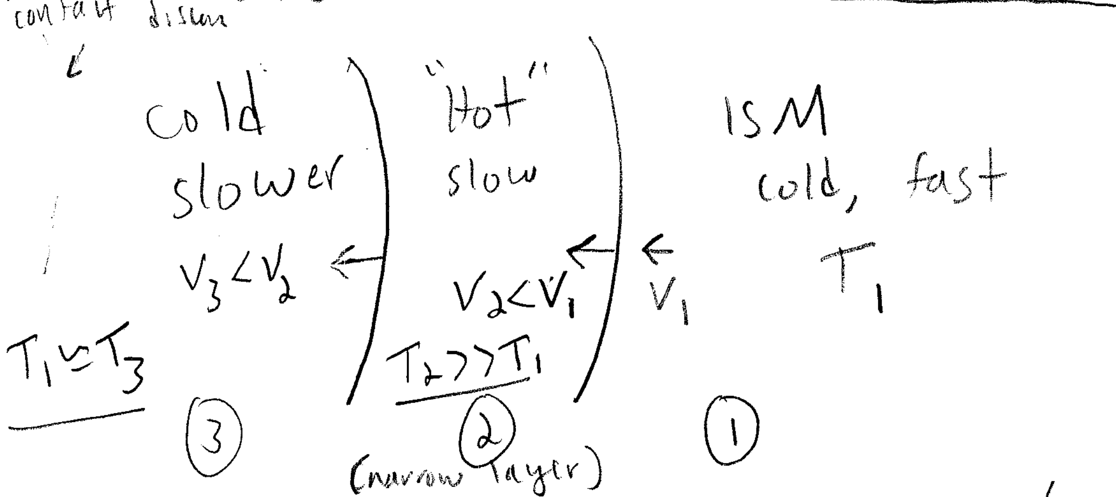
$$\boxed{v \simeq \left(\frac{kT}{m}\right)^{1/2} \lesssim 240\,\frac{\text{km}}{\text{s}}\left(E_{51}\, n_H^2\right)^{1/14}}$$

notice the weak dependence on E and n_H!

In radiative phase

shock becomes isothermal as it evolves.

Hot interior region but a cooled, isothermal interior shell: In <u>frame of contact discontinuity</u>:



$T_3 \ll 10^6 K$ $\quad$ $T_2 > 10^6 K$ $\quad$ $T_1 \ll 10^6 K$

cooling takes away most of the shock energy but <u>momentum is conserved</u> because radiation is essentially isotropic. Thus

$$\frac{d}{dt}\Big(\underbrace{\overbrace{\frac{4\pi \rho_{ISM}}{3} r^3}^{M_1} \dot{r}}_{\text{momentum}}\Big) \simeq 0 \quad \text{in radiative phase} \quad \ldots (115)$$

$\Rightarrow r^3 \dot{r} = \text{constant}$ for $\frac{d\rho_{ISM}}{dt} \simeq 0$.

$\Rightarrow r^3 dr \simeq dt$

$\Rightarrow r \propto t^{1/4}$

and $\dot{r} \propto t^{-3/4}$, $\boxed{\dot{r} \simeq 240 \frac{km}{s} \left(E_{51} n_H^2\right)^{1/14} \left(\frac{t}{5.2 \times 10^4 yr}\right)^{-3/4}}$ (116)

using (114) & (113)

from Padmanabhan, "Theoretical Astrophysics" Vol 2

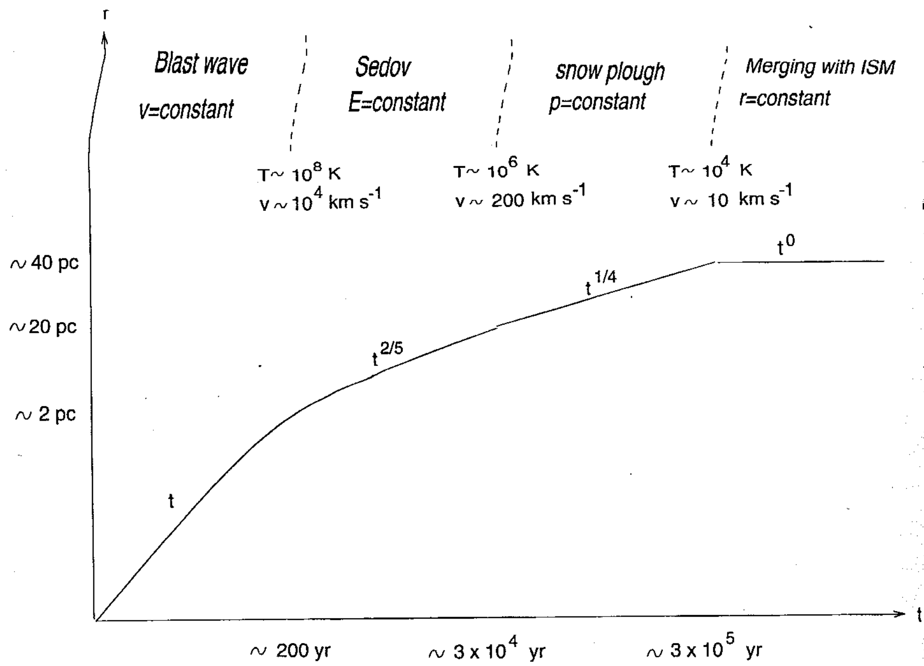


Fig. 4.6. The radius of the supernova shell as a function of time during the different phases.

This integrates to give

$$R = R_0\left[1 + 4\frac{v_0}{R_0}(t - t_0)\right]^{1/4}, \quad \dot{R} = v_0\left[1 + 4\frac{v_0}{R_0}(t - t_0)\right]^{-3/4}. \tag{4.107}$$

For large t, $R \propto t^{1/4}$ and

$$\dot{R} \propto t^{-3/4} \simeq 200 \text{ km s}^{-1} \,(t/3 \times 10^4 \text{ yr})^{-3/4}. \tag{4.108}$$

The time constant in relation (4.108) is fixed by equating the Sedov phase velocity of Eq. (4.101) to 200 km s^{-1}.

In the final phase, the speed of the shell drops below the sound velocity of the ISM, which is approximately (10–100) km s^{-1} in a time scale of $t \approx (1\text{–}5) \times 10^5$ yr. Around this time scale, the remnant loses its identity, and it is dispersed by random motions in the ISM. The evolution is shown schematically in Fig. 4.6.

It should be noted that supernova explosions and their eventual dispersion of ejected material have the effect of enriching the ISM with the material processed in stellar interiors. In particular, the heavy elements synthesised inside a star reach the ISM through this process. Because massive stars evolve at shorter time scales and also are more likely to end up as supernovas, the evolution of the first generation of massive stars changes the character of the ISM. Second and later generations of stars condense out of this enriched ISM and will have a higher proportion of heavier elements.