



The Strömgren Sphere

lonization balance holds at each location within the region: volumetric ionization rate = $p-e^{-}$ recombination rate

Integrating over the sphere: total number of ionization events per unit time $(\mathcal{N}_*) =$ total recombination rate

 \mathcal{N}_* is the rate at which the star emits ionizing photons, i.e. those with with E > 13.6 eV. The corresponding minimum frequency is the Lyman limit of v_1 =3.29×10¹⁵ s⁻¹ or λ_1 =912 Å.

 \mathcal{N}_{FUV} is the star's far-UV output (6 < E < 13.6 eV).

Spectral	Mass	log \mathcal{N}_{*}	$\log \mathcal{N}_{\rm FUV}$
Туре	(M_{\odot})	(S ⁻¹)	(S⁻¹)
04	70	49.9	49.5
06	40	48.8	48.8
09	20	47.8	48.2
B0	18	47.1	48.1
B2	10	44.8	47.1

The volumetric rate at which free electrons and protons combine to produce atomic hydrogen can be written as:

$$\mathcal{R} = n_e n_p \alpha_{rec}(T)$$

$$= n_e^2 \alpha_{rec}(T)$$

$$= n_e^2 \alpha_{rec}(T)$$
using charge neutrality
in the ionized plasma

The total rate of recombinations within the sphere follows by integrating \mathcal{R} over the volume. If n_e and T are spatial constants, after balancing total ionizations and recombinations:

$$\mathcal{N}_{*} = \frac{4\pi}{3} n_{e}^{2} \alpha'_{rec}(T) R_{s}^{3}$$
Strömgren Radíus

 $\alpha'_{rec}(T)$ neglects recombinations to the ground (n=1) state, given that those produce another ionizing photon that is promptly absorbed nearby.

We will soon demonstrate that the ionization spreads so quickly to the Strömgren radius that the original cloud density is not able to change appreciably. Thus:

$$n_e = n_H^0$$
 the number density of H atoms external to R_s

[This simple approximation ignores electrons liberated from He]

Solving previous expression for R_s:

$$R_{s} = \left[\frac{3\mathcal{N}_{*}}{4\pi\alpha'_{rec} (n_{H}^{0})^{2}}\right]^{1/3}$$
$$= 0.4 \text{ pc} \left(\frac{\mathcal{N}_{*}}{10^{49} \text{ s}^{-1}}\right)^{1/3} \left(\frac{n_{H_{2}}^{0}}{10^{3} \text{ cm}^{-3}}\right)^{-2/3}$$

typical value of an O6 star; this also assumes value of $\,\alpha\,{}^{\prime}{}_{\rm rec}$ for gas at T~10⁴ K

Within an HII region, the probability per unit time of ionizing an atom is far greater than the probability for a proton to recombine. Thus, for the *volumetric* rates to match:

$$n_H \ll n_e$$

The interior of the HII region is nearly completely ionized

The neutral density becomes significant only when the photon flux becomes attenuated by the ionization it is creating. The transition to neutrality at the boundary thus occurs over some multiple of the photon mean free path within the fully neutral medium:



the HII region is very sharp-edged!



First and Second Expansion					
The zone of ionization expands with time. At the earliest stages, no HII region exists. The region first quickly spreads to the Strömgren radius, and then more slowly beyond.					
The moving edge of the HII region is called the ionization front. Its boundary is narrow enough to be considered a discontinuity.					
When $R < R_s$, the total rate of recombinations is insufficient to match the star's output of ionizing radiation \rightarrow surplus photons reach the front, ionize additional gas beyond this, and the front expands.					
$F_{\star}(t) = flux$ time After a time $n^{0}_{H2} \times \Delta R$ $2 \times n^{0}_{H2} \times \Delta F$	of ionizing photons reaching R at t. e Δt: molecules per unit area are dissociated, creating R atoms which are then ionized.	E > 14.7 eV $F_* \longrightarrow$ E > 13.6 eV	$\begin{array}{c c} H_2 & I \\ H_2 & I \\ / & I \\ H & H \\ I \\ H^+ & H^+ \\ I \end{array}$		
F₊ ×∆t	is the number of photons crossing the front.		$\begin{bmatrix} 1 \\ R \\$		



Combining the equations for dR/dt and \mathcal{N}_{\star} , we obtain the equation of motion for the ionization front:

$$\frac{dR}{dt} = \frac{\mathcal{N}_*}{6\pi n_H^0 R^2} - \frac{2}{9} n_H^0 \alpha'_{rec} R \qquad \boxed{\mathsf{A}}$$

Let's define a nondimensional radius $\lambda = R/R_s$ and time $\tau = t/t_{rec}$, with

$$t_{rec} = \frac{1}{n_H^0 \alpha'_{rec}} = 61 \text{ yr} \left(\frac{n_{H_2}^0}{10^3 \text{ cm}^{-3}}\right)^{-1}$$

Equation A now becomes:

$$\frac{d\lambda}{dt} = \frac{2}{9\lambda^2} - \frac{2\lambda}{9}$$

The solution of the above equation, obeying $\lambda(0)=0$ is then:

$$\lambda = [1 - \exp(-2\tau/3)]^{1/3}$$

First Expansion

The front is expanding at high speed initially and than slowly creeps towards the Strömgren radius: R reaches a fraction $(1-e^{-1})^{1/3} = 0.86$ of R_s in a time $3t_{rec}/2$, or 91 yr (using previous parameters). The corresponding average speed is 6×10^3 km s⁻¹.

So far, we have assumed that the mass density within the HII region matches the exterior value. This is OK at early times, when the ionization front velocity exceeds a_1 , the isothermal sound speed within the region (~10 km s⁻¹, for an ionized medium at 10⁴ K).

The pressure within the HII region is ~1000 times larger than outside. Once the radius approaces R_s , the pressure difference is able to drive a *second phase of expansion*. The ionization front is preceded by a shock wave that first expands into the ambient cloud.



The second phase begins when the **velocity of the ionization** front declines to the internal **sound speed**. At this point, pressure disturbances can cross the front and create an expanding shock.

External matter traversing this shock is compressed into a relatively thin, neutral shell. The shock and the ionization fronts move out at a common speed which soon falls below a_1 .

The mass of the ionized gas continues to rise (UV photons impinge on the inner side of the shell), while the interior density falls (expansion) \rightarrow the system gradually approaches pressure equilibrium.

Gas within the shell has the low cloud temperature at the leading edge and approaches 10⁴ K toward the HII region.





then modify the expression we found for the Strömgren radius to read:

$$\mathbf{\Psi}$$

$$R = \left[\frac{3\mathcal{N}_*}{4\pi\alpha'_{rec}(n_H^1)^2}\right]^{1/3}$$
Elimination of n¹_H between the above expression and
$$n_H^1 a_1^2 = n_H^0 \left(\frac{dR}{dt}\right)^2$$

$$\mathbf{\Psi}$$

$$\left(\frac{dR}{dt}\right)^2 = \frac{a_1^2}{n_H^0} \left[\frac{3\mathcal{N}_*}{4\pi\alpha'_{rec}R^3}\right]^{1/2}$$
Equation of motion during the second expansion
The velocity of the shell is thus $\mathbf{a}_1(R/R_s)^{-3/4}$. To find the radius at all times, let $\lambda = R/R_s$ and $\mathbf{r}' = \mathbf{a}_1 t/R_s$, so that:

$$\frac{d\lambda}{d\tau'} = \lambda^{-3/4}$$
adopting the initial condition $\lambda(0)=1$

$$\lambda = \left(1 + \frac{7\tau'}{4}\right)^{4/7}$$

Champagne Flows

In the final equilibrium state, n_{H}^{1} must fall to $(a_{0}/a_{1})^{2}n_{H}^{0}$, far below the external density. The final radius, becomes:

$$R_f = (a_1/a_0)^{4/3} R_s = (2T_1/T_0)^{2/3} R_s \sim 100 R_s !$$

In practice, such a large inflation never goes to completion, both because it takes too long (compared to the stellar lifetime) and, more realistically, because the ionization front reaches the boundary of the molecular cloud:

the HII region is density bounded rather than ionization bounded. The pressurized HII gas burst out of the cloud into the surrounding medium, creating a *champagne flow*.

If a portion of the ionizing volume crosses the edge into the lower-density gas, the front velocity increases. This part of the HII region runs out rapidly into a plume-like structure.

There is a large pressure discontinuity within the ionized gas and a shock rushes outward, accelerating material behind it to velocities exceeding the sound speed.



HII regions also emit continuum radiation. Some of this is in the optical and represents light from the central star scattered by dust grains. Dusty cloud gas may partially block nebular light. Long, optically thick columns (elephant trunks) may protrude into the HII region from the outside.

Example:

the Eagle Nebula, or M16. The columns in this optical image represent dense gas that has not yet been dissipated by UV radiation. Note the glowing ionization fronts at the top of each structure!



Ultracompact HII regions

Optically visible, or "classical" HII regions, such as the Orion Nebula, have $L \ge 10^{18}$ cm and $n_e \le 10^4$ cm⁻³. Precursors of these HII regions, are called **UCHII regions**, have $L \le 10^{17}$ cm and $n_e \ge 10^5$ cm⁻³. In the Far-IR they are the most luminous objects in the Galaxy (heated dust grains within the ionized gas).





Surveying the Galaxy, we find that about 2000 O stars are in this deeply embedded phase, or \sim 10% the **total** number of O stars.

$\mathbf{\Psi}$

A typical O star spends ~10% of its main sequence lifetime $(4 \times 10^6 \text{ yr})$ as an UCHII. Thus, **an UCHII region last for ~4x10⁵ yr.**

So, why the size is small ??

Assuming that the region is in its second expansion phase, then the ionization front advances at roughly the internal sound speed, or ~10 km s⁻¹. To cover the observed size of 0.1 pc would require only ~10³ yr!

There are far too many UCHII regions for their youthful dynamical state !!

Possibilities for confining ionization

1. High pressures in surrounding medium, including nonthermal forms of pressure (turbulence and magnetic fields)

2. Ram pressure because of an accretion flow to the star.

3. Ram pressure because of motion of the star: should lead to confinement only on one side (a "cometary" UCHII region).

4. Continual replenishment of neutral gas into the UCHII regions, either from remnant material left over after formation of the star or advection of fresh neutral gas, by accretion or turbulence.

