Duality Constrained Parameterization of Vector and Axial Nucleon Form Factors

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We present new parameterizations of vector and axial nucleon form factors. We maintain an excellent descriptions of the form factors at low momentum transfers, where the spatial structure of the nucleon is important, and use the Nachtman scaling variable ξ to relate elastic and inelastic form factors and impose quark-hadron duality constraints at high momentum transfers where the quark structure dominates. We use the new vector form factors to re-extract updated values of the axial form factor from neutrino experiments on deuterium. We obtain an updated world average value from ν_μd and pion electroproduction experiments of $M_A = 1.0144 \pm 0.0136\ GeV/c^2$. Our parameterizations are useful in modeling neutrino interactions at low energies (e.g. for neutrino oscillations experiments). The predictions for high momentum transfers can be tested in the next generation electron and neutrino scattering experiments.

PACS numbers: PACS numbers:13.40.Gp, 13.15.+g, 13.85.Dz, 14.20.Dh, 24.30.Bf, 25.30.Pt

The nucleon vector and axial elastic form factors have been measured for more than 50 years in $e^- N$ and νN scattering. At low Q^2, a reasonable description of the proton and neutron elastic form factors is given by the dipole approximation. The dipole approximation is a lowest-order attempt to incorporate the non-zero size of the proton into the form factors. The approximation assumes that the proton has a simple exponential spatial charge distribution, $\rho(r) = \rho_0 e^{-r/r_0}$, where r_0 is the scale of the proton radius. Since the form factors are related in the non-relativistic limit to the Fourier transform of the charge and magnetic moment distribution, the above $\rho(r)$ yields the dipole form defined by:

$$G_D^{V,A}(Q^2) = \frac{C^{V,A}}{\left(1 + \frac{Q^2}{M_{V,A}^2}\right)^2}.$$

Here $C^{V,A}$= (1,g_A), g_A = -1.267, M_V^2 = 0.71 $(GeV/c)^2$, and M_A = 1.015 GeV/c^2 (as discussed below).

Since M_A is not equal to M_V, the distribution of electric and axial charge are different. However, the magnetic moment distributions were assumed to have the same spatial dependence as the charge distribution (*i.e.*, form factor scaling). Recent measurements from Jefferson Lab show that the ratio of $\frac{\mu_p G_{Ep}}{G_{Mp}}$ falls at high Q^2 challenging the validity of form factor scaling [1] and resulting in new updated parameterizations of the form factors[2],[3]). In this paper we present parameterizations that simultaneously satisfy constraints at low Q^2 where the spatial structure of the nucleon is important, and at high Q^2 where the quark structure is important. A violation of form-factor scaling is expected from quark-hadron duality. We use our new vector form factors to re-extract updated values of the axial form factor from a re-analysis of previous neutrino scattering data on deuterium and present a new parameterization for the axial form factor within the framework of quark-hadron duality.

The new parameterizations presented in this paper are referred to as the duality based "BBBA07" parameterization. Our updated parameterizations feature the following: (1) Improved functional form that adds an additional Q^2 dependence using the Nachtman scaling variable ξ to relate elastic and inelastic form factors. For elastic scattering ($x = 1$) $\xi^{p,n,N} = \frac{2}{(1+\sqrt{1+1/\tau_{p,n,N}})}$, where $\tau_{p,n,N} = Q^2/4M_{p,n,N}^2$. Here $M_{p,n,N}$ are the proton (0.9383 GeV/c^2), neutron (0.9396 GeV/c^2), and average nucleon mass (for proton, neutron, and axial form factors, respectively). (2) Yield the same values as Arrington and Sick [4] for $Q^2 < 0.64 (GeV/c)^2$, while satisfying quark-hadron duality constraints at high-Q^2.

For vector form factors our fit functions are $A_N(\xi)$ (i.e. $A_{Ep}(\xi^p)$, $A_{Mp}(\xi^p)$, $A_{En}(\xi^n)$, $A_{Mn}(\xi^n)$) multiplying an updated Kelly[3] type parameterization of one of the proton form factors. The Kelly parameterization is:

$$G^{Kelly}(Q^2) = \frac{\sum_{k=0}^{m} a_k \tau_p^k}{1 + \sum_{k=1}^{m+2} b_k \tau_p^k},$$

where $a_0 = 1$ and $m = 1$. In our analysis, we use all the datasets used by Kelly[3], updated to include the recent BLAST[5] results, to fit G_{Ep}, G_{En}, G_{Mp}/μ_p, and G_{Mn}/μ_n ($\mu_p = 2.7928$, $\mu_n = -1.9130$). Our parameterization employs the published Kelly functional form to G_{Ep}^{Kelly}, and an updated set of parameters for $G_{MP}^{Kelly-upd}(Q^2)$. The parameters used for G_{Ep}^{Kelly} and $G_{Mp}^{Kelly-upd}$ are listed in Table I, and $A_N(\xi)$ is given by

$$A_N(\xi) = \sum_{j=1}^{n} P_j(\xi)$$
$$P_j(\xi) = p_j \prod_{k=1,k\neq j}^{n} \frac{\xi - \xi_k}{\xi_j - \xi_k}.$$

Each P_j is a LaGrange polynomial in ξ. The ξ_j are equidistant "nodes" on an interval $[0, 1]$ and p_j are the fit parameters that have an additional property $A_N(\xi_j) = p_j$. The functional form $A_N(\xi)$ (for G_{Ep}, G_{Mp}, G_{En},

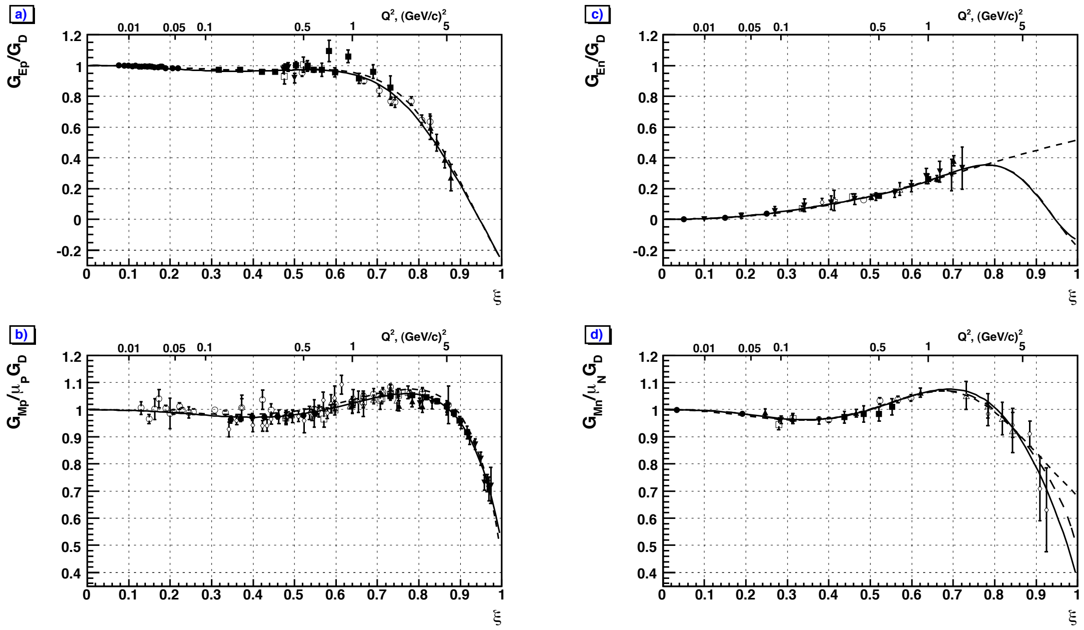

FIG. 1: Ratios of G_{Ep} (a), G_{Mp}/μ_p (b), G_{En} (c) and G_{Mn}/μ_n (d) to G_D. The short-dashed line in each plot is the old Kelly parameterizations (old Galster for G_{En}). The solid line is our new $BBBA07_{25}$ parameterization for $\frac{d}{u}=0.0$, and the long-dashed line is $BBBA07_{43}$ for $\frac{d}{u}=0.2$. The values of ξ and the corresponding values of Q^2 are shown on the bottom and top axis.

and G_{Mn}) is used with seven p_j parameters at ξ_j=0, 1/6, 1/3, 1/2, 2/3, 5/6, and 1.0. In the fitting procedure described below, the parameters of $A_N(\xi)$ are constrained to give the same vector form factors as the recent low Q^2 fit of Arrington and Sick [4] for $Q^2 < 0.64(GeV/c)^2$ (as that analysis includes coulombs corrections which modify G_{Ep}, and two photon exchange corrections which modify G_{Mp} and G_{Mn}). Since the published form factor data do not have these corrections, this constraint is implemented by including additional $''fake''$ data points for $Q^2 < 0.64(GeV/c)^2$.

Our fits to the form factors are:

$$\begin{aligned} G_{Mp}(Q^2)/\mu_p &= A_{Mp}(\xi^p) \times G_{Mp}^{Kelly-upd}(Q^2) \\ G_{Ep}(Q^2) &= A_{Ep}(\xi^p) \times G_{Ep}^{Kelly}(Q^2) \\ G_{Mn}(Q^2)/\mu_n &= A_{Mn}^{25,43}(\xi^n) \times G_{Mp}(Q^2)/\mu_p \\ G_{En}(Q^2) &= A_{En}^{25,43}(\xi^n) \times G_{Ep}(Q^2) \times \left(\frac{a\tau_n}{1+b\tau_n}\right), \end{aligned}$$

where we use our updated parameters in the Kelly parameterizations. For G_{En} the parameters a=1.7 and b=3.3 are the same as in the Galster[6] parametrization and ensure that dG_{En}/dQ^2 at for $Q^2 = 0$ is in agreement with measurements. For convenience, we also provide fits for the form factors G_{Ep} and G_{Mp}/μ_p that give very close to the same values, but use the dipole form instead:

$$\begin{aligned} G_{Ep}(Q^2) &= A_{Ep-dipole}(\xi^p) \times G_D^V(Q^2) \\ G_{Mp}(Q^2)/\mu_p &= A_{Mp-dipole}(\xi^p) \times G_D^V(Q^2) \end{aligned}$$

The values $A(\xi)$=p_1 at ξ_1=0 ($Q^2 = 0$) for G_{Mp}, G_{Ep}, G_{En}, G_{Mn} are set to to 1.0. The value $A(\xi)$=p_7 at ξ_j=1 ($Q^2 \to \infty$) for G_{Mp} and G_{Ep} is set to 1.0.

The value $A(\xi)$=p_j at ξ_j=1 for G_{Mn} and G_{En} are fixed by constraints from quark-hadron duality. Quark-hadron duality implies that the ratio of neutron and proton magnetic form factors should be the same as the ratio of the corresponding inelastic structure functions $\frac{F_{2n}}{F_{2p}}$ in the ξ=1 limit. (Here $F_2 = \xi \sum_i e_i^2 q_i\,(\xi)$)

$$\frac{G_{Mn}^2}{G_{Mp}^2} = \frac{F_{2n}}{F_{2p}} = \frac{1+4\frac{d}{u}}{4+\frac{d}{u}} = \left(\frac{\mu_n^2}{\mu_p^2}\right) A_{Mn}^2(\xi=1)$$

We ran fits with two different values of $\frac{d}{u}$ at the ξ=1 limit: $\frac{d}{u}$ =0 and 0.2 (corresponding to $\frac{F_{2n}}{F_{2p}}$ = 0.25 and 0.4286). The fit utilizing $\frac{d}{u}$ = 0 is A_{Mn}^{25}, and the fit utilizing $\frac{d}{u}$ = 0.2 is A_{Mn}^{43}. The final parameters are given in Table II (or download computer code[14]).

The value $A(\xi)$=p_j at ξ_j=1 for G_{En} is set by another duality-motivated constraint. R is defined as the ratio of deep-inelastic longitudinal and transverse structure functions. For inelastic scattering, as $Q^2 \to \infty$, $R_n = R_p$. If we assume quark-hadron duality, the same should be true for the elastic form factors at ξ=1 ($Q^2 \to \infty$) limit:

$$\begin{aligned} R_n\,(x=1;Q^2) &= \frac{4M_n^2}{Q^2}\left(\frac{G_{En}^2}{G_{Mn}^2}\right) \\ G_{En}^2/G_{Mn}^2 &= G_{Ep}^2/G_{Mp}^2 \end{aligned}$$

	a_1	b_1	b_2	b_3	χ^2/ndf
G_{Ep}^{Kelly}	−0.24	10.98	12.82	21.97	0.78
$G_{Mp}^{Kelly-upd}$	0.1717	11.26	19.32	8.33	1.03

TABLE I: Parameters for G_{Ep}^{Kelly} and $G_{Mp}^{Kelly-upd}$. Our parameterization employs the as-published Kelly parameterization to G_{Ep}^{Kelly} and an updated set of parameters for $G_{MP}^{Kelly-upd}(Q^2)$ that includes the recent BLAST[5] results.

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
ξ, Q^2	$0, 0$	$0.167, 0.029$	$0.333, 0.147$	$0.500, 0.440$	$0.667, 1.174,$	$0.833, 3.668$	$1.0, \infty$
A_{Ep}	1.	0.9927	0.9898	0.9975	0.9812	0.9340	1.
A_{Mp}	1.	1.0011	0.9992	0.9974	1.0010	1.0003	1.
$A_{Ep-dipole}$	1.	0.9839	0.9632	0.9748	0.9136	0.5447	−0.2682
$A_{Mp-dipole}$	1.	0.9916	0.9771	0.9801	1.0321	1.0429	0.5084
A_{Mn}^{25}	1.	0.9958	0.9877	1.0193	1.0350	0.9164	0.7300
A_{Mn}^{43}	1.	0.9958	0.9851	1.0187	1.0307	0.9080	0.9557
A_{En}^{25}	1.	1.1011	1.1392	1.0203	1.1093	1.5429	0.9706
A_{En}^{43}	1.	1.1019	1.1387	1.0234	1.1046	1.5395	1.2708
$A_{FA}^{25-dipole}$	1.0000	0.9207	0.9795	1.0480	1.0516	1.2874	0.7707

TABLE II: Fit parameters for $A_N(\xi)$, the LaGrange portion of the new parameterization. Note A_{Mn}^{25}, A_{En}^{25}, and A_{FA}^{25} are constrained to have $\frac{d}{u} = 0$ at $\xi = 1$, and A_{Mn}^{43}, A_{En}^{43}, are constrained to have $\frac{d}{u} = 0.2$.

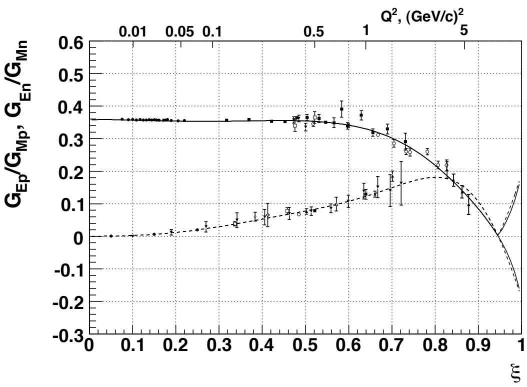

FIG. 2: The constraint used in fitting G_{En} stipulates that $G_{En}^2/G_{Mn}^2 = G_{Ep}^2/G_{Mp}^2$ at high ξ. The solid line is $\frac{G_{Ep}}{|G_{Mp}|}$ and $\frac{|G_{Ep}|}{|G_{Mp}|}$, and the short-dashed line is $\frac{G_{En}}{|G_{Mn}|}$ and $\frac{|G_{En}|}{|G_{Mn}|}$.

In order to constrain the fits to G_{En} at high Q^2 we have assumed that the values of $\frac{G_{En}^2}{G_{Mn}^2}$ are the same as the measured $\frac{G_{Ep}^2}{G_{Mp}^2}$ for the three highest Q^2 data points for G_{Ep}, and included these three $''fake''$ data points in the G_{En} fits. In addition, the $R_n = R_p$ condition yields the following constraint at $\xi = 1$:

$$A_{En}^{25,43}(\xi = 1) = P_7 = \left(\frac{b}{a}\right) \times \left(\frac{1 + 4\frac{d}{u}}{4 + \frac{d}{u}}\right)^{1/2}$$

where $b/a = 1.7/3.3$. As there are two parameter sets $A_{Mn}^{25,43}(\xi)$, we have produced two parameter sets $A_{En}^{25,43}$ as shown in Table II. The new form factors G_{Ep}, G_{Mp}/μ_p, G_{Mn}/μ_n, and G_{En} are plotted in Figure 1 as ratios to the dipole form G_D^V.

$Experiment$	M_A (published)	ΔM_A new–old
$Miller - D - ANL_{82,77,73}$	1.00 ± 0.05	−0.030
$Baker - D - BNL_{81}$	1.07 ± 0.06	−0.028
$Kitagaki - D - FNAL_{83}$	$1.05^{+0.12}_{-0.16}$	−0.025
$Kitagaki - D - BNL_{90}$	$1.070^{+0.040}_{-0.045}$	−0.036

TABLE III: M_A (GeV/c^2) values published by neutrino-deuterium experiments[9] and corrections ΔM_A when re-extracted with updated $BBBA2007_{25}$ form factors.

As seen in Table II, $A_N(\xi)$ is not needed for G_{Mp} as it is very close to 1.0. For G_{Ep} it yields a correction of 1% at low Q^2 (because it is required to agree with the fits of Arrington and Sick[4] (which include two photon exchange and Coulomb corrections. For G_{En} and G_{Mn} it is used to impose quark-hadron duality asymptotic constraints. Figure 2 shows plots of the data and fits to $\frac{G_{En}}{|G_{Mn}|}$ and $\frac{G_{Ep}}{|G_{Mp}|}$ (for the $\frac{d}{u} = 0$ at $\xi = 1$ case).

Using our updated $BBBA2007_{25}$ form factors and an updated value g_A = -1.267, we perform a complete re-analysis of published ν quasielastic [9] (QE) data on deuterium (ν_μ n $\rightarrow \mu^-$p) using the procedure described in detail in ref. [7]. We extract new values of M_A (given in Table III), and updated values of $F_A(Q^2)$. The average of the corrected measurements of M_A from Table III is 1.0156±0.0278 GeV/c^2. This is to be compared to the average value of 1.0140±0.0160 GeV/c^2 extracted from pion electroproduction experiments (excluding $Bloom_{73}$)[8] after corrections for hadronic effects. The average of

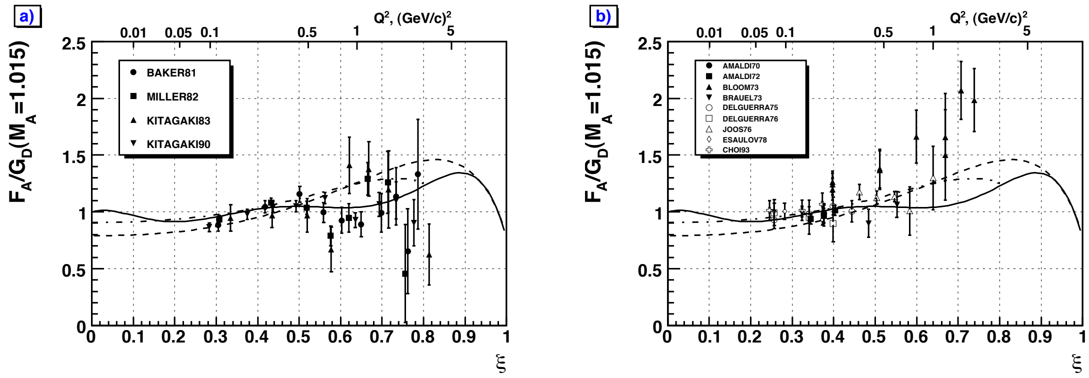

FIG. 3: (a) $F_A(Q^2)$ re-extracted from neutrino-deuterium data divided by $G_D^A(Q^2)$[15]. (b) $F_A(Q^2)$ from pion electroproduction divided by $G_D^A(Q^2)$[15], corrected for for hadronic effects[8]. Solid line - duality based fit; Short-dashed line - $F_A(Q^2)_{A2=V2}$. Dashed-dot line - constituent quark model[10].

the two average values is $1.0144 \pm 0.0136\ GeV/c^2$. This is smaller than the recent results[11] from MiniBoone on a carbon target ($M_A = 1.23 \pm 0.20\ GeV/c^2$) and $K2K$[12] on oxygen ($M_A = 1.20 \pm 0.12\ GeV/c^2$). Both collaborations use updated vector form factors. collaborations attribute a difference from deuterium to nuclear effects. However, there is experimental and theoretical evidence[13] that M_A in nuclear targets is the same (or smaller) than in deuterium. This discrepancy is important for ν oscillations experiments since it affects the normalization (at high energies the QE cross section is approximately proportional to M_A) and non-linearity of the QE cross section, which is relevant to the extraction of ν mass difference and mixing angle.

For deep-inelastic scattering, the vector and axial parts of F_2 are equal. Local quark-hadron duality at large Q^2 implies that the axial and vector components of $F_2^{elastic}$ are also equal, which yields:

$$[F_A(Q^2)_{A2=V2}]^2 \;=\; \frac{(G_E^V)^2(Q^2) + \tau_N (G_M^V(Q^2))^2}{(1+\tau_N)},$$

where $G_E^V(Q^2) = G_{Ep}(Q^2) - G_{En}(Q^2)$ and $G_M^V(Q^2) = G_{Mp}(Q^2) - G_{Mn}(Q^2)$.

We extract values of $F_A(Q^2)$ from the differential cross sections using the procedure of ref. [7]. The overall normalization is set by the theoretical QE cross section[15]. We then do a duality based fit to $F_A(Q^2)$ (including pion electroproduction data, excluding $Bloom_{73}$) of the form:

$$F_A(Q^2) = A_{FA}^{25}(\xi^N) \times G_D^A(Q^2).$$

We impose the constraint $A_{FA}^{25}(\xi_1 = 0) = p_1 = 1.0$. We also constrain the fit by requiring that $A_{FA}^{25}(\xi^N)$ yield $F_A(Q^2) = F_A(Q^2)_{A2=V2}$ by including additional $''fake''$ data points) for $\xi > 0.9$ ($Q^2 > 7.2(GeV/c)^2$). Figure 3(a) shows $F_A(Q^2)$ extracted from neutrino-deuterium experiments divided by $G_D^A(Q^2)$[15]. Figure 3(b) shows $F_A(Q^2)$ extracted from pion electroproduction experiments divided by $G_D^A(Q^2)$[15]. These pion electroproduction values can be directly compared to the neutrino results because they are multiplied by a factor $F_A(Q^2, M_A = 1.014\ GeV/c^2)/F_A(Q^2, M_A = 1.069\ GeV/c^2)$ to correct for $\Delta M_A = 0.055\ GeV/c^2$ originating from hadronic effects[8]. The solid line is our duality based fit. The short-dashed line is $F_A(Q^2)_{A2=V2}$. The dashed-dot line is a constituent-quark model[10] prediction.

In summary, our new parameterizations are useful in modeling ν interactions for oscillations experiments. Our predictions[16] for $G_{En}(Q^2)$ and $F_A(Q^2)$ can be tested in future $e-N$ and ν-N experiments.

[1] O. Gayou *et al*, Phys. Rev. Lett. 88, 092301 (2002).
[2] R. Bradford, H. Budd, A. Bodek and J. Arrington, Nucl. Phys. B 159, 127 (2006).
[3] J.J. Kelly, Phys. Rev. C 70, 068202 (2004).
[4] J. Arrington and I.Sick nucl-th/0612079
[5] C.B. Crawford *et al*, Phys. Rev. Lett 98, 052301 (2007).
[6] S. Galster *et al*, Nucl. Phys. B32, 221 (1971).
[7] H. Budd, A. Bodek, J. Arrington, Nucl.Phys.Proc.Suppl. 139, 90 (2005).
[8] V. Bernard *et al*, J. Phys. G28, R1 (2002).
[9] N.J. Baker *et al.*, Phys. Rev. D23 (1981) 2499; K.L. Miller *et al.*, Phys. Rev. D26 (1982) 537; T. Kitagaki *et al.*, Phys. Rev. D28 (1983) 436; T. Kitagaki *et al.*, Phys. Rev. D42 (1990) 1331
[10] R. F., Wegenbrunn, *et al*, hep-ph/0212190. Few Body Syst.Suppl.14:411-414,2003
[11] A. Aguilar-Areval *et al* (MiniBoone) hep-ex/0706.0926
[12] R. Gran *et al* (K2K), Phys. Rev. D74 (2006) 052002.
[13] S. K. Singh and E. Oset, Nucl.Phys A542 (1992) 587.
[14] *www.pas.rochester.edu/ ~ bodek/FF/*
[15] Evaluated with $M_A = 1.015\ GeV/c^2$.
[16] Local duality may not hold for the elastic peak alone, as we assumed, and the analysis may require using the sum of elastic and first resonance combined.