Predictions for Neutrino and Antineutrino Quasielastic Scattering Cross Sections and Q2 distributions with the Latest Elastic Form Factors

A Review of Weak and Electromagnetic Form Factors

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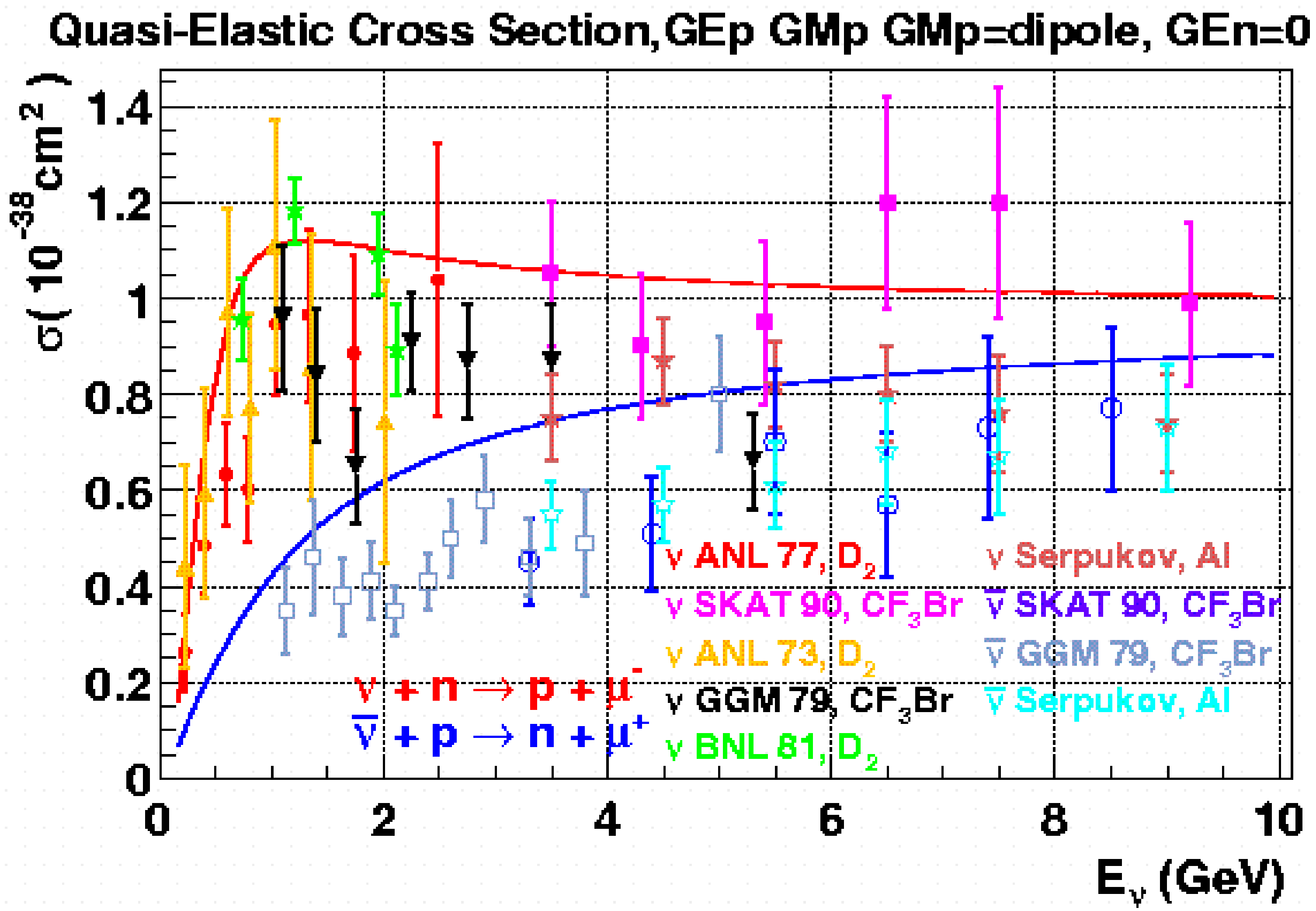
http://www.pas.rochester.edu/~bodek/FormFactors-FNAL.ppt

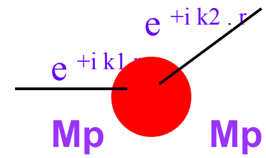
$\sigma_{\text{quasi-elastic neutrinos on Neutrons}}$-Dipole

$\sigma_{\text{quasi-elastic Antineutrinos on}}$ Protons -Dipole

DATA - FLUX ERRORS ARE 10%. ***Note: Nuclear Effects are large- data on nuclear Targets is lower***

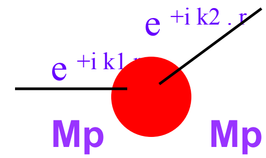


fixed W scattering - form factors



- **Electron Scattering:**
- **Elastic Scattering, Electric and Magnetic Form Factors (G_E and G_M) versus Q^2 measure size of object (the electric charge and magnetization distributions). Final State W = M^p = M**
- **(G_E and G_M) TWO Form factor measures Matrix element squared $| \langle p_f | V(r) | p_i \rangle |^2$ between initial and final state lepton plane waves. Which becomes:**
- **$| \langle e^{-i\,k2.\,r} | V(r) | e^{+i\,k1\,.\,r} \rangle |^2$ q = k1 - k2 = momentum transfer**
- **$G_E^{P.N}(Q^2) = \int \{e^{\,i\,q\,\cdot\,r} \rho(r)\, d^3r\}$ = Electric form factor is the Fourier transform of the charge distribution for Proton And Neutron**
- **The magnetization distribution $G_M^{P.N}((Q^2)$ Form factor is relates to structure functions by:**
- **$2xF_1(x, Q^2)_{elastic} = x^2 G_M^{2}{}_{elastic}\, \delta(x-1)$**
  - **Neutrino Quasi-Elastic (W=Mp)**
  - **ν_μ + N --> μ^- + P (x =1, W=Mp)**
  - **Anti-ν_μ + P --> μ^++ N (x =1, W=Mp)**
  - **$F_1^V(Q^2)$ and $F_2^V(Q^2)$ = Vector Form Factors which are related by CVC to**
  - **$G_E^{P.N}(Q^2)$ and $G_M^{P.N}((Q^2)$ *from Electron Scattering***
  - **$F_A(Q^2)$ = Axial Form Factor *need to be measured* in Neutrino Scattering.**
  - **Contributions proportional to Muon Mass (which is small)**
  - **$F_P(Q^2)$ = Pseudo-scalar Form Factor. estimated by relating to $F_A(Q^2)$ via PCAC, Also extracted from pion electro-production**
  - **$F_S(Q^2)$, $F_T(Q^2)$, = scalar, tensor form factors=0 if no second class currents.**

Need to update - Axial Form Factor extraction



1. **Need to account for Pauli Suppression, Fermi Motion/binding Energy effects in nucleus e.g. Bodek and Ritchie (Phys. Rev. D23, 1070 (1981), Re-scattering corrections etc (see talk by Sakuda NuInt02 for feed-down from single pion production)**
2. **Need to to account for muon mass effects and other structure functions besides $F_1^V(Q^2)$ and $F_2^V(Q^2)$ and $F_A(Q^2)$ (see talk by Kretzer NuInt02 for Fp + similar terms in DIS). This is more important in Tau neutrinos than for muon neutrinos [here use PCAC for Gp(Q2).]**

- **This Talk (What is the difference in the quasi-elastic cross sections if:**

1. **We use the most recent very precise value of $g_A = F_A(Q^2) = 1.263$ (instead of 1.23 used in earlier analyses.) Sensitivity to g_A and $m_{A,}$**
2. **Sensitivity to knowledge of $Gp(Q^2)$**
3. **Use the most recent Updated $G_E^{P.N}(Q^2)$ and $G_M^{P.N}((Q^2)$ *from Electron Scattering (instead of the dipole form assumed in earlier analyses)* In addition *There are new precise measurments of* $G_E^{P.N}(Q^2)$ Using polarization transfer experiments**
4. **How much does $m_{A,}$ measured in previous experiments change if current up to date form factors are used instead --- Begin updating m_A**

Neutrino Cross Sections

NuMI-112
PDK-626
Nov. 10, 1995

H. M. Gallagher and M. C. Goodman

They implemented
The Llewellyn-Smith
Formalism for NUMI

$$\frac{d\sigma}{d|q^2|}\begin{pmatrix}\nu n \to l^- p \\ \bar{\nu} p \to l^+ n\end{pmatrix} = \frac{M^2 G^2 cos^2\theta_c}{8\pi E_\nu^2}\left[A(q^2) \mp B(q^2)\frac{(s-u)}{M^2} + \frac{C(q^2)(s-u)^2}{M^4}\right]. \quad (2)$$

In this expression, G is the Fermi coupling constant and θ_c is the Cabibbo mixing angle ($G = 1.16639 \times 10^{-5}\mathrm{GeV}^{-2}$). The functions A, B, and C are convenient combinations of the nucleon form factors.

Contraction of the hadronic and leptonic currents yields:

Non zero

$$A = \frac{(m^2 - q^2)}{4M^2}\left[\left(4 - \frac{q^2}{M^2}\right)|F_A|^2 - \left(4 + \frac{q^2}{M^2}\right)|F_V^1|^2 - \frac{q^2}{M^2}|\xi F_V^2|^2\left(1 + \frac{q^2}{4M^2}\right) - \frac{4q^2 ReF_V^{1*}\xi F_V^2}{M^2} \quad (3)\right.$$

$$\left. + \frac{q^2}{M^2}\left(4 - \frac{q^2}{M^2}\right)|F_T|^2 - \frac{m^2}{M^2}\left(|F_V^1 + \xi F_V^2|^2 + |F_A + 2F_P|^2 + \left(\frac{q^2}{M^2} - 4\right)\left(|F_S|^2 + |F_P|^2\right)\right)\right]$$

$$B = -\frac{q^2}{M^2}ReF_A^*(F_V^1 + \xi F_V^2) - \frac{m^2}{M^2}Re\left[\left(F_V^1 + \frac{q^2}{4M^2}\xi F_V^2\right)^* F_S - \left(F_A + \frac{q^2 F_P}{2M^2}\right)^* F_T\right] \quad (4)$$

$$C = \frac{1}{4}\left(|F_A|^2 + |F_V^1|^2 - \frac{q^2}{M^2}\left|\frac{\xi F_V^2}{2}\right|^2 - \frac{q^2}{M^2}|F_T|^2\right), \quad (5)$$

where m is the final state lepton mass. Ignoring second-class currents (those which violate G-parity) allows us to set the scalar and tensor form factors to zero. According to the CVC

$$F_V^1(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1}\left[G_E^V(q^2) - \frac{q^2}{4M^2}G_M^V(q^2)\right] \quad (6)$$

$$\xi F_V^2(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{1}\left[G_M^V(q^2) - G_E^V(q^2)\right]. \quad (7)$$

The electromagnetic form factors are determined from electron scattering experiments:

UPDATE: Replace by $G_E^V = G_E^P - G_E^N$

$$G_E^V(1^2) = \frac{1}{(1 - \frac{q^2}{M_V^2})^2} \qquad G_M^V(q^2) = \frac{1 + \mu_p - \mu_n}{(1 - \frac{q^2}{M_V^2})^2}.$$

UPATE: Replace by $G_M^V = G_M^P - G_M^N$

The situation is slightly more complicated for the hadronic axial current. $F_A(q^2 = 0) = -1.261 \pm .004$ is known from neutron beta decay. The q^2 dependence has to be inferred or measured. By analogy with the vector case we assume the same dipole form:

$M_A = 1.032 \pm .036$ GeV [7].

$$F_A(q^2) = \frac{-1.23}{(1 - \frac{q^2}{M_A^2})^2}. \quad (9)$$

$Q^2 = -q^2$

g_A, M_A need to Be updated

$$F_P(q^2) = \frac{2M^2 F_A(q^2)}{M_\pi^2 - q^2}. \quad (10)$$

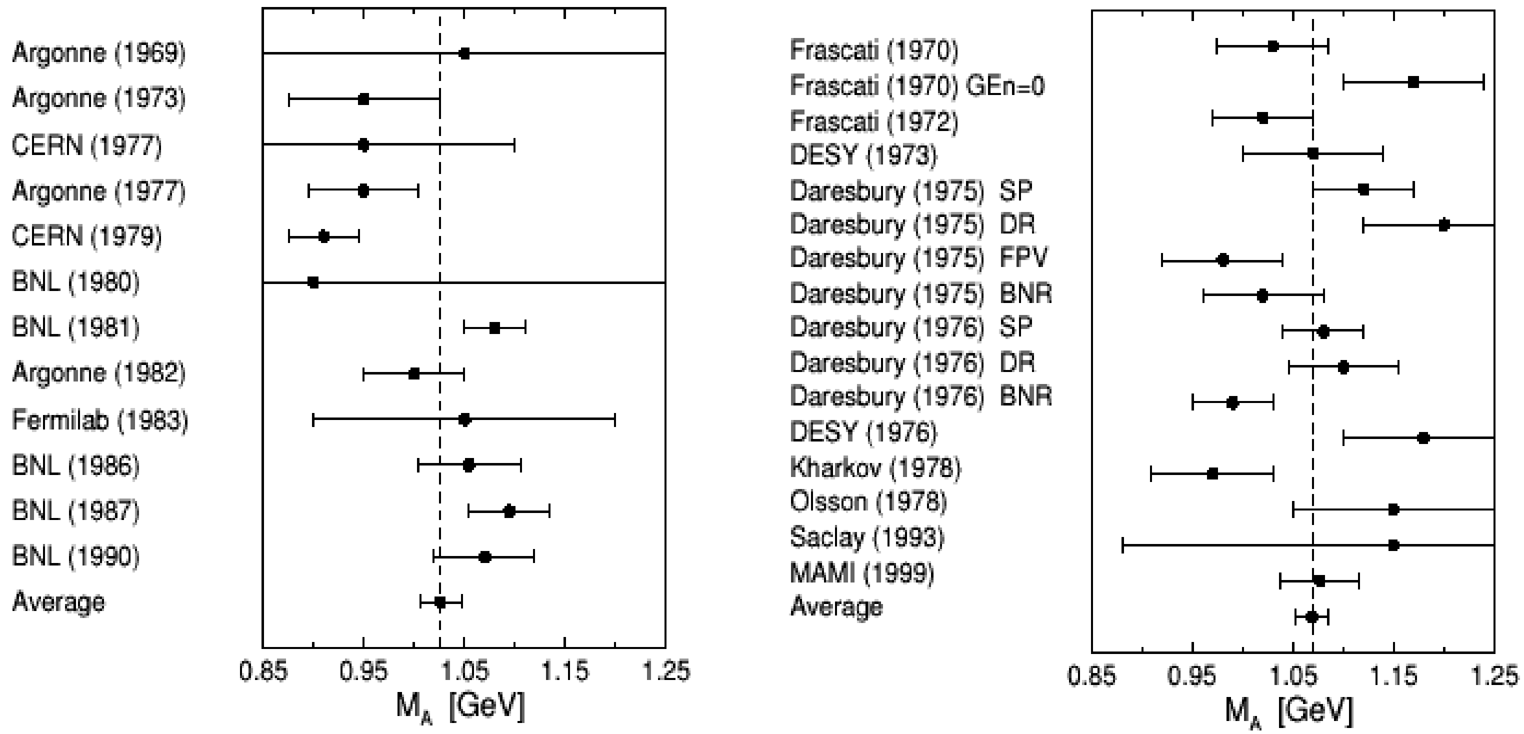
Fp important for Muon neutrinos only at Very Low Energy

The inclusion of F_P leads to an approximately 5% reduction in both the ν_τ and ν_τ quasi-elastic cross sections. The only remaining parameters needed to describe the quasi-elastic cross section are thus M_V and M_A. $M_V = .71$ GeV, as determined with high accuracy

From C.H. Llewellyn Smith (SLAC). SLAC-PUB-0958 Phys.Rept.3:261,1972

Axial structure of the nucleon   Hep-ph/0107088 (2001)

Véronique Bernard†, Latifa Elouadrhiri‡, Ulf-G Meißner§



For updated M_A expt: need to be reanalyzed with new g_A, and G_E^N

Difference
In Ma between
Electroproduction
And neutrino
Is understood

Figure 1. Axial mass M_A extractions. Left panel: From (quasi)elastic neutrino and antineutrino scattering experiments. The weighted average is $M_A = (1.026 \pm 0.021)$ GeV. Right panel: From charged pion electroproduction experiments. The weighted average is $M_A = (1.069 \pm 0.016)$ GeV. Note that value for the MAMI experiment contains both the statistical and systematical uncertainty; for other values the systematical errors were not explicitly given. The labels SP, DR, FPV and BNR refer to different methods evaluating the corrections beyond the soft pion limit as explained in the text.

M_A from neutrino expt. No theory corrections needed

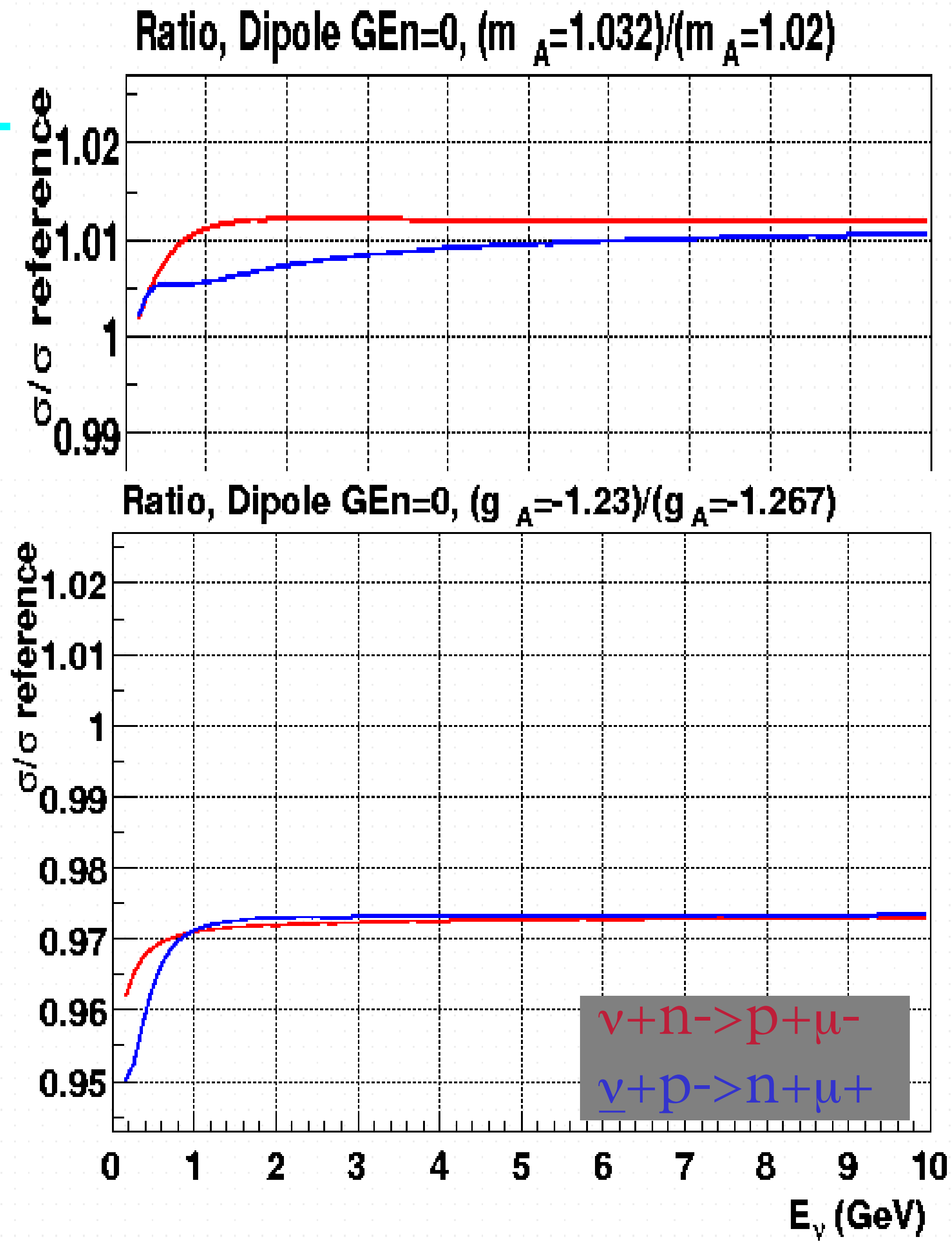
Use: N. Nakamura et al. Nucl-th/0201062 April 2002 as default DIPOLE Form factors

For the weak coupling constant, instead of $G_F = 1.16637 \times 10^{-5}$ GeV^{-2} employed in NSGK, we adopt here $G_F' = 1.1803 \times 10^{-5}$ GeV^{-2} obtained from $0^+ \to 0^+$ nuclear β-decays [26].[4] G_F' subsumes the bulk of the *inner* radiative corrections.[5] The K-M matrix element is taken to be $V_{ud} = 0.9740$[26] instead of $V_{ud} = 0.9749$ used in NSGK.

$$G_D(q_\mu^2) = \left(1 - \frac{q_\mu^2}{0.71\text{GeV}^2}\right)^{-2}, \qquad (19)$$

$$G_A(q_\mu^2) = \left(1 - \frac{q_\mu^2}{1.04\text{GeV}^2}\right)^{-2}, \qquad (20)$$

where $\mu_p = 2.793$, $\mu_n = -1.913$, $\eta = -\frac{q_\mu^2}{4m^2}$ and m_π is the pion mass. For g_A, we adopt the current standard value, g_A=1.267[29], instead of g_A=1.254 used in NSGK. In addition, as the axial-vector mass in Eq.(20), we use the value which was obtained in the latest analysis[28] of (anti)neutrino scattering and charged-pion electroproduction. The change in $G_A(q_\mu^2)$ is in fact not consequential for $\sigma_{\nu d}$ in the solar-ν energy region. Regarding f_P, we assume PCAC and pion-pole dominance. A contribution from this term is known to be proportional to the lepton mass, which leads to very small contribution from the induced pseudoscalar term in our case. Although deviations from the naive pion-pole dominance of f_P have been carefully studied[30], we can safely neglect those



Effect of g_A and M_A

Compare sensitivity to M_A e.g. =1.02 (Nakamura 2002) to M_A =1.032 (NuMI 112 Gallagher and Goodman (1995) while- K2K uses : **M_A** =1.1 (factor of 10 larger difference)

Note : **M_A Should be re-extracted with the new the value of g_A =1.267 and new vector form factors.**

Compare the new precise Value **g_A =1.267** from beta Decay- to **g_A =1.23 (used by MINOS and previous analyses.**

ratio_ma1032_D0DD.pict

ratio_ga123_D0DD.pict

Parametrization of Fits to Form Factors

GEP, GMP: - Simultaneous fit to 1/(1+p1*q+p2*q**2+...) and mu_p/(1+...) - Fit to cross sections (rather than the Ge/Gm tables). Added 5 cross section points from Simon to help constrain Q^2<0.1 GeV^2 - Fit normalization factor for each data set (break up data sets from different detectors).
- Up to p6 for both electric and magnetic
- Fits with and without the polarization transfer data. Allow systematic error to 'float' for each polarization experiment.

GEP, GMP : CROSS SECTION DATA ONLY FIT:

p1= -0.53916 **!p1-p6 are parameters for GMP**
p2= 6.88174
p3= -7.59353
p4= 7.63581
p5= -2.11479
p6= 0.33256

q1= -0.04441 **!q1-q6 are parameters for GEP**
q2= 4.12640
q3= -3.66197
q4= 5.68686
q5= -1.23696
q6= 0.08346
chi2_dof= 0.81473

GEP, GMP:
CROSS SECTION AND POLARIZATION DATA Fit:
GMP
p1= -0.43584
p2= 6.18608
p3= -6.25097
p4= 6.52819
p5= -1.75359
p6= 0.28736
q1= -0.21867
GEP
q2= 5.89885
q3= -9.96209
q4= 16.23405
q5= -9.63712
q6= 2.90093
chi2_dof= 0.95652

GMN: - Fit to -1.913/(1+p1*q+p2*q**2+...)
- NO normalization uncertainties included.
- Added 2% error (in quadruture) to all data points.
Typically has small effect, but a few points had <1% errors.

PARAMETER	VALUE						
P1	-0.40468,	P2	5.6569 ,	P3	-4.664, 5	P4	3.8811

GEN: Use Krutov parameters for Galster form see below

[21] M. Garcon and J.W. Van Orden, Adv.Nucl. Phys. 26 (2001) 293.

Krutov-> (a = 0.942, b=4.61)
Hep-ph/0202183(2002)

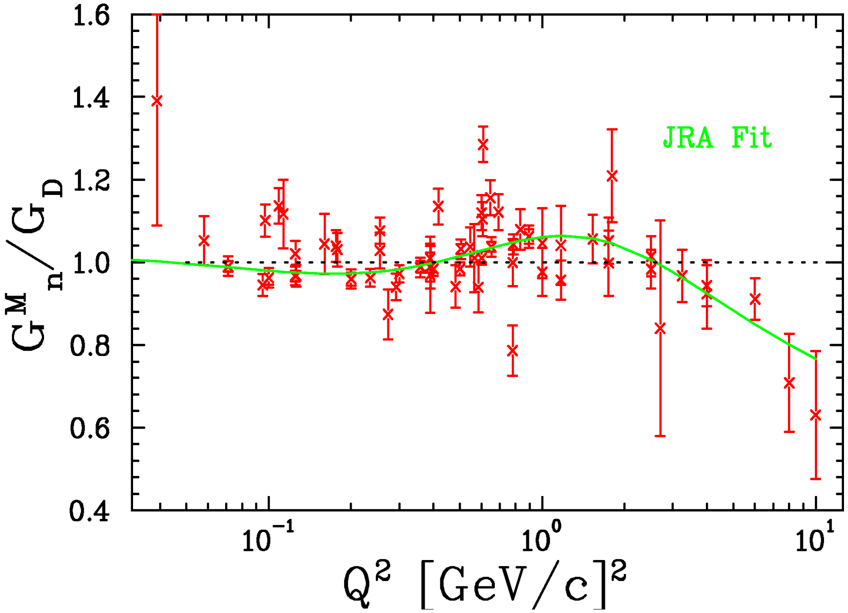
vs. Galster ->(a=1 and b=5.6)
[15] S. Galster *et al.*, Nucl.Phys. B 32 (1971) 221.

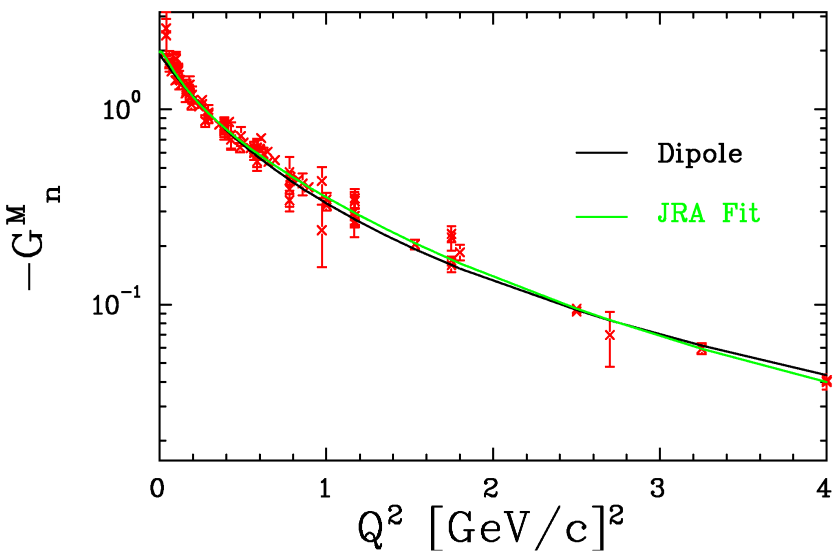
$$G_E^n(Q^2) = -\mu_n \frac{a\tau}{1+b\tau} G_D(Q^2), \quad G_D(Q^2) = \left(1 + \frac{Q^2}{0.71}\right)^{-2}, \quad \tau = \frac{Q^2}{4M^2}. \qquad (13)$$

The neutron magnetic moment μ_n = -1.91304270(5) [49]. Q^2 in $G_D(Q^2)$ is given in (GeV^2).

Neutron G_M^N is negative

Neutron (G_M^N / $G_M^{N\,dipole}$)





Neutron (G_M^N / $G_M^{N\,dipole}$)

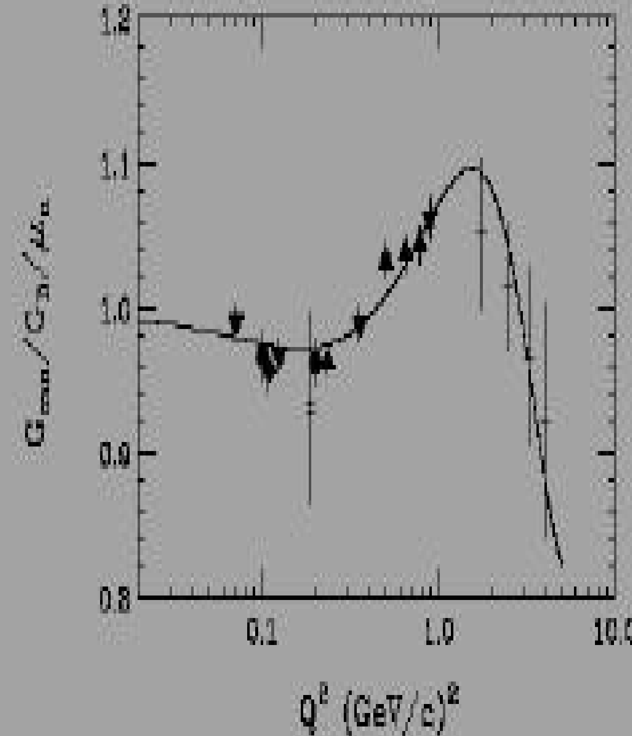
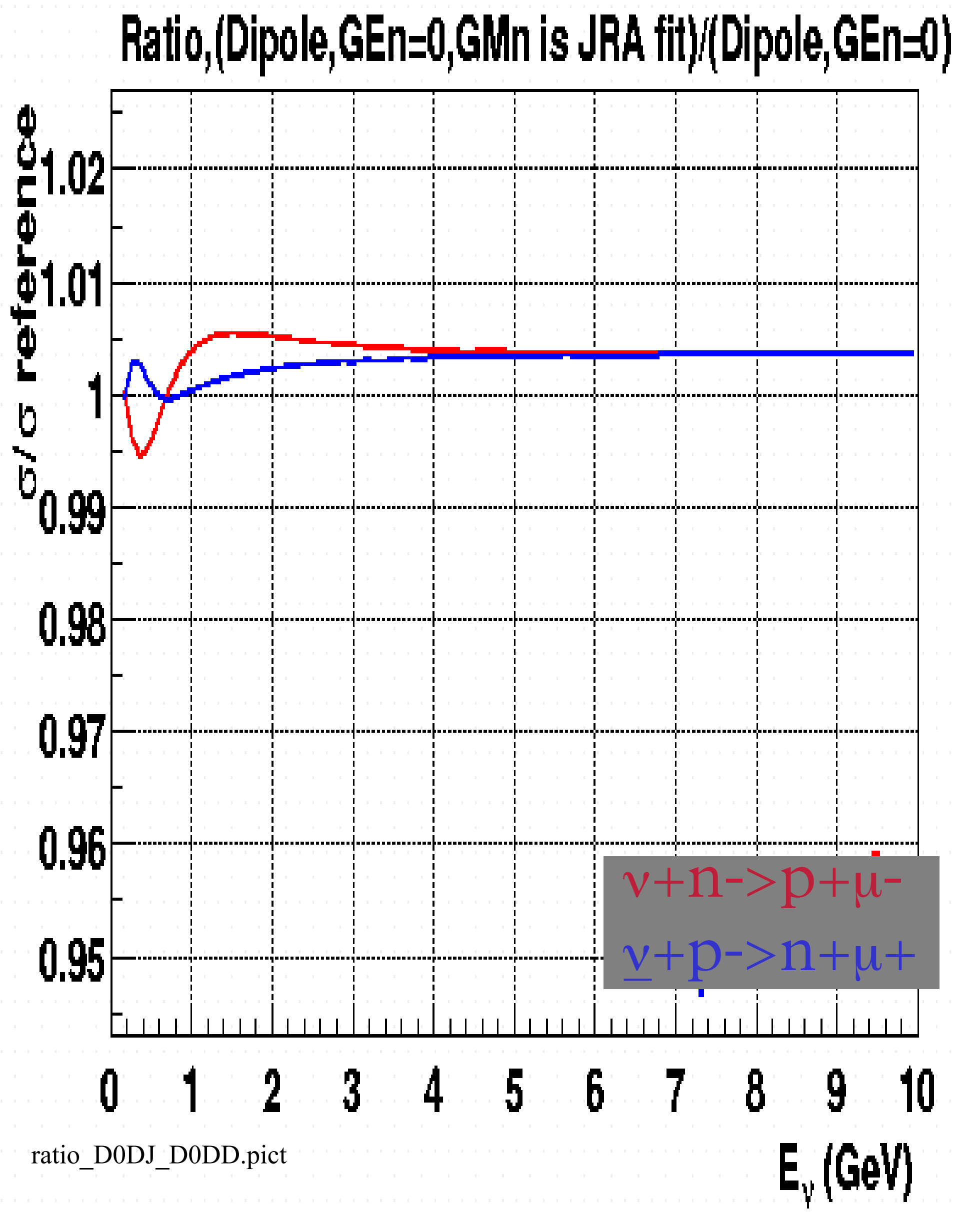


Fig. 2. The figure shows the continued fraction fit to the data. Symbols for the data as in figure 1) plus the data by Lung et al. (+) [23].

At low Q2 Our Ratio to Dipole similar to that nucl-ex/0107016 G. Kubon, et al Phys.Lett. B524 (2002) 26-32

$$G_{mn}(Q^2) = \frac{\mu_n}{1+\frac{Q^2 b_1}{1+\frac{Q^2 b_2}{1+\cdots}}} \qquad (2)$$

Effect **of using Fit to G_M^N** versus using **G_M^N** Dipole



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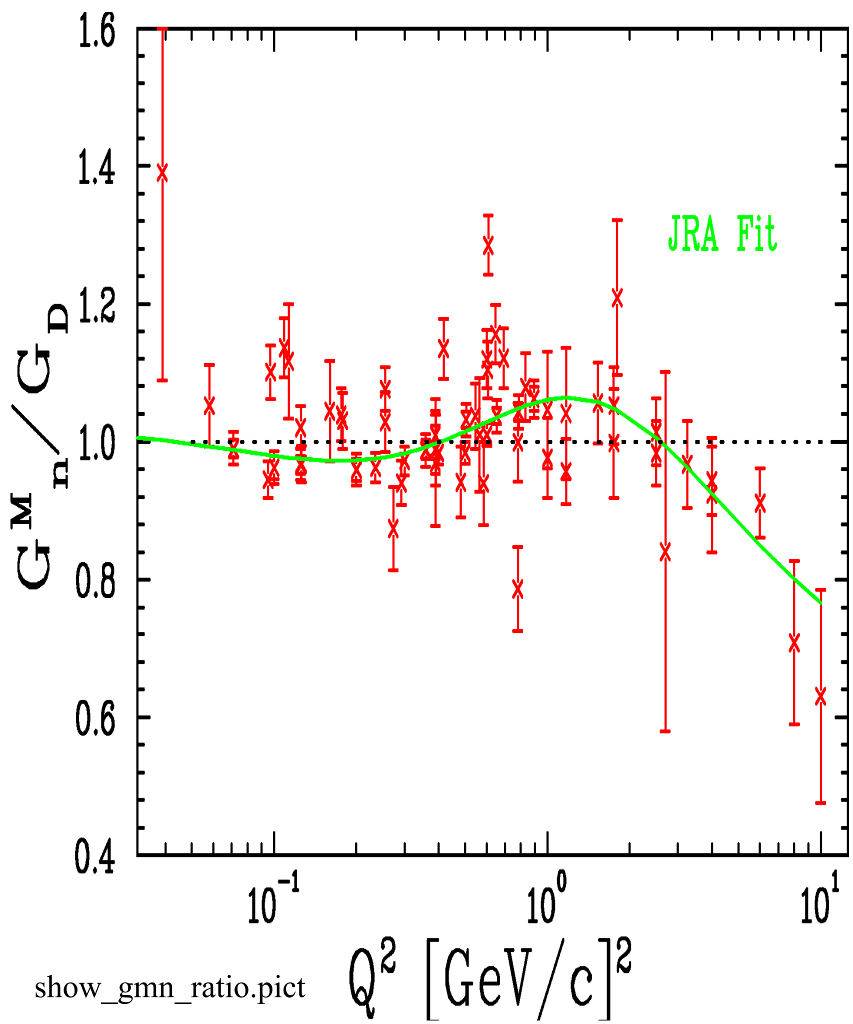
Neutron G_M^N is negative

Neutron (G_M^N / G_M^N dipole)



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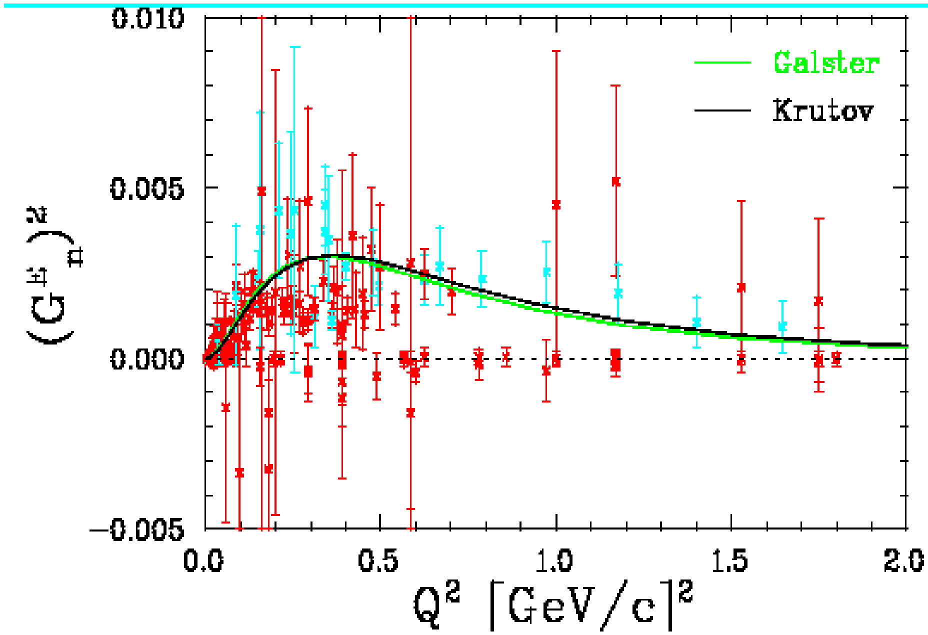
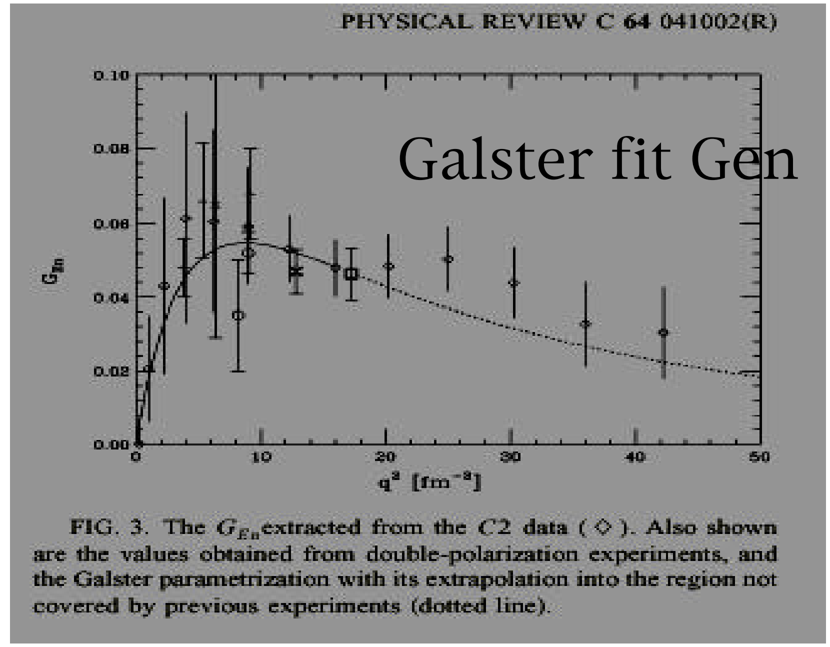
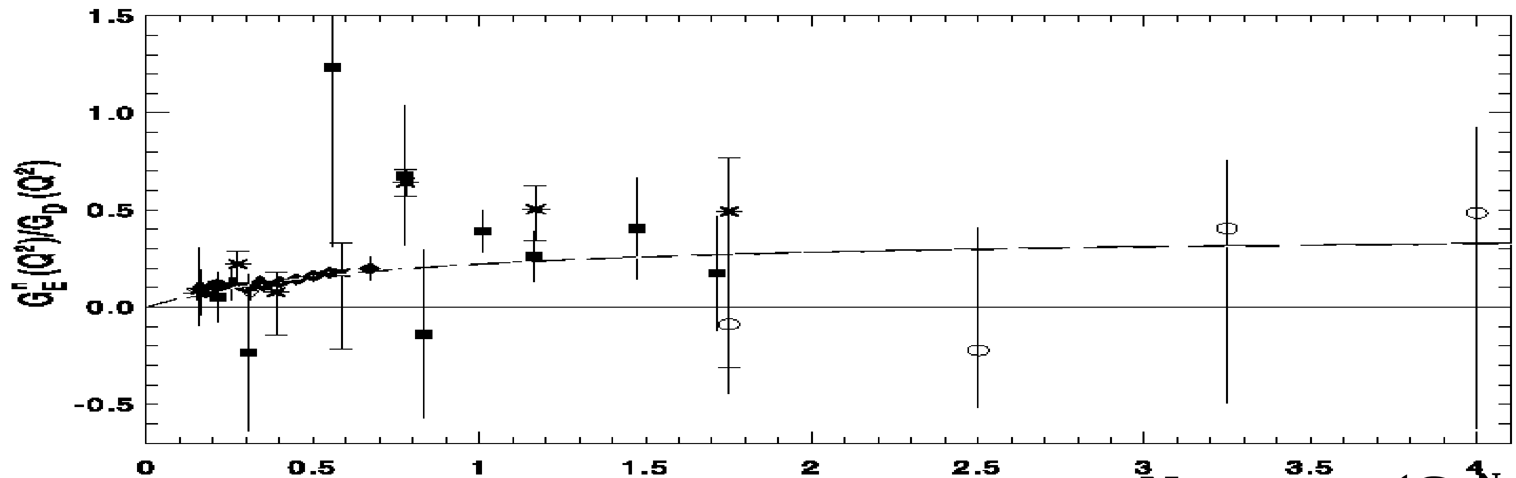
Neutron, G_E^N is positive - Imagine N=P+pion cloud

Neutron G_E^N is positive New Polarization data gives Precise non zero G^E_N hep-ph/0202183(2002)







Krutov

Neutron (G_E^N / $G_E^{P\ dipole}$)

[21] M. Garcon and J.W. Van Orden, Adv.Nucl. Phys. 26 (2001) 293.

Krutov-> (a = 0.942, b=4.61)
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$$G_E^n(Q^2) = -\mu_n \frac{a\,\tau}{1+b\,\tau} G_D(Q^2)\,, \quad G_D(Q^2) = \left(1+\frac{Q^2}{0.71}\right)^{-2}, \quad \tau = \frac{Q^2}{4\,M^2}\,. \qquad (13)$$

The neutron magnetic moment μ_n = -1.91304270(5) [49]. Q^2 in $G_D(Q^2)$ is given in (GeV^2).

[14, 39]:

$$\left.\frac{dG_E^n}{dQ^2}\right|_{Q^2=0} = 0.0199 \pm 0.0003\ \mathrm{fm}^2\,. \qquad (14)$$

The fitting of the slope (14) gives a=0.942 with the accuracy ≈ 1.5%.

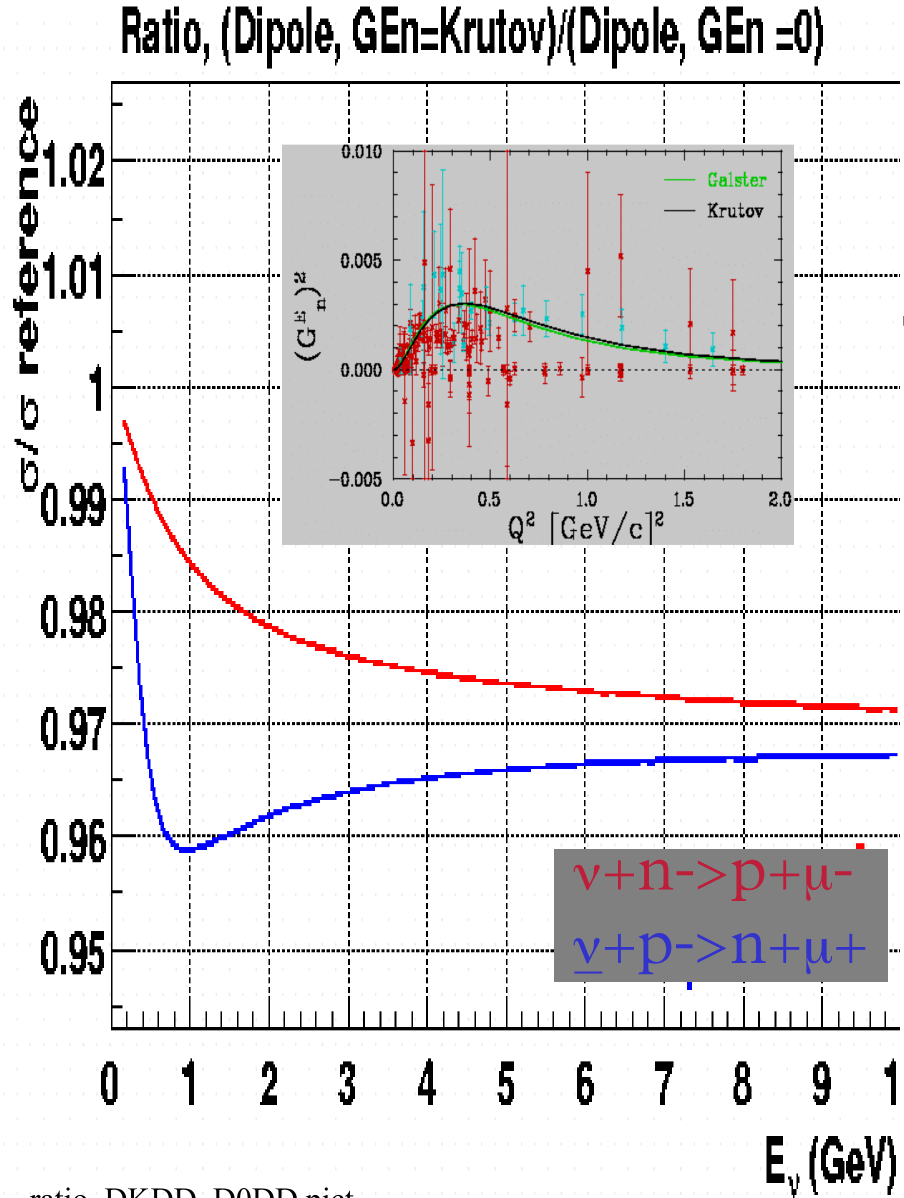
This value of a gives the slope of $G_E^n(Q^2)$ at $Q^2 = 0$ which is measured directly in the experiment.

The parameter b is fitted using the χ^2 criterion. If we use all the 35 points we obtain b = 4.61 with $\chi^2 = 69.0$. Note that the fit DRN–GK(3) [39] of 23 points has $\chi^2 = 63.9$.

If we exclude the points # 4–8 then the 30–point fitting gives $b = 4.62$ with $\chi^2 = 61.5$.

show_gen_new.pict

Effect **of** using G_E^N (Krutov) or (Galster) versus using $G_E^N=0$ (Dipole Assumption) Krutov and Galster very similar



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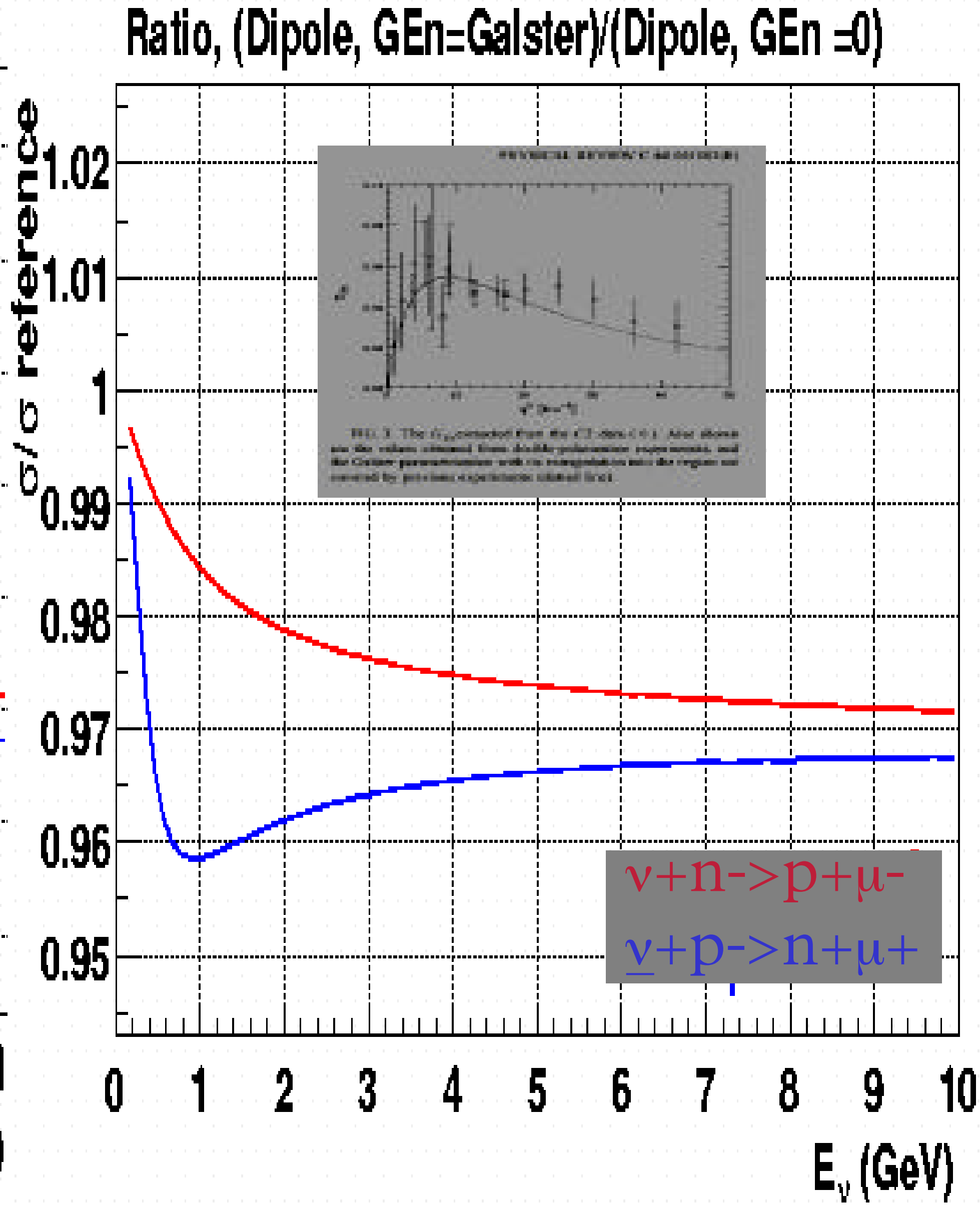
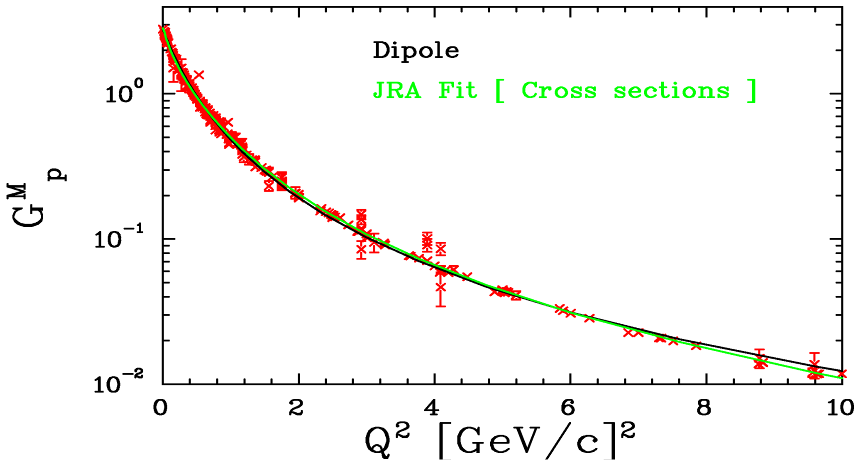
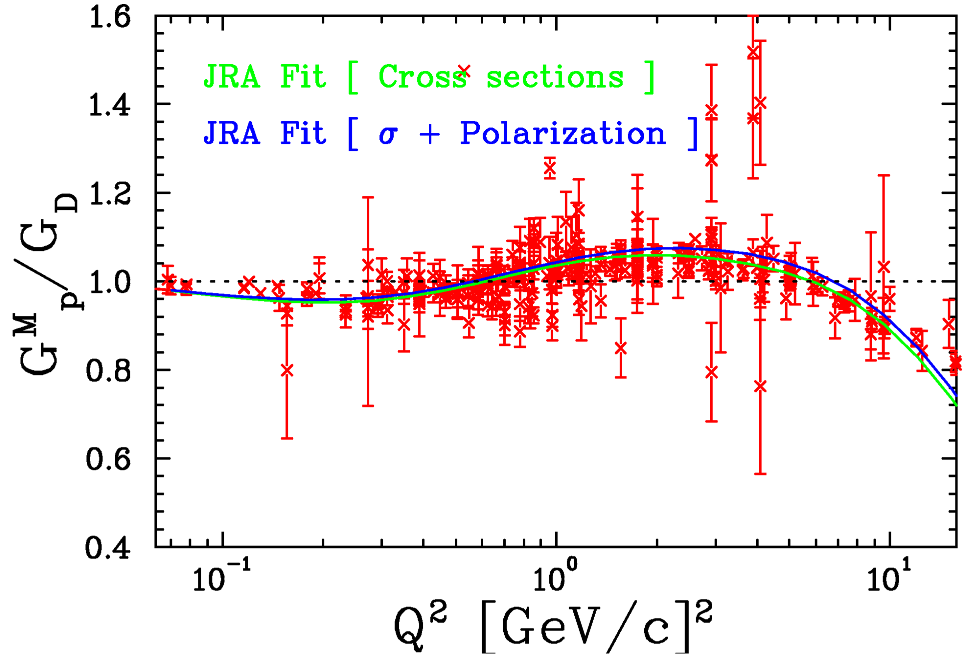
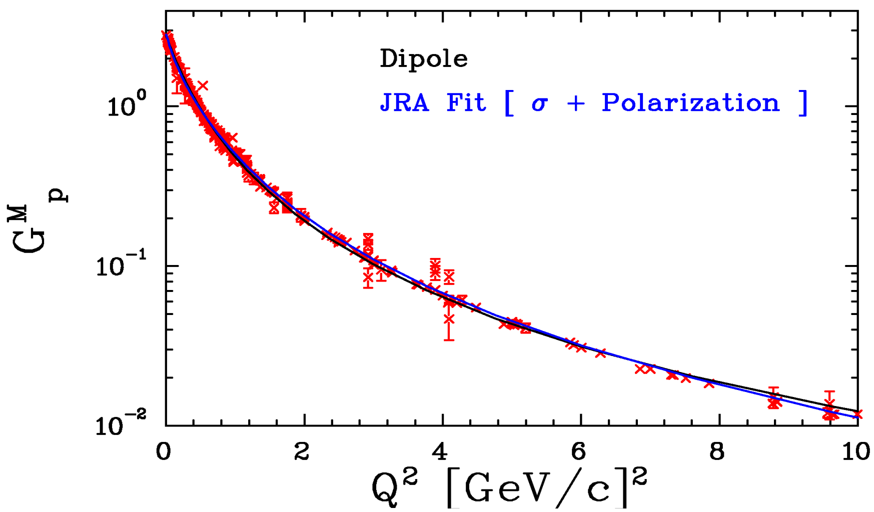


Extract Correlated Proton G_M^P, G_E^P simultaneously from e-p Cross Section Data with and without Polarization Data



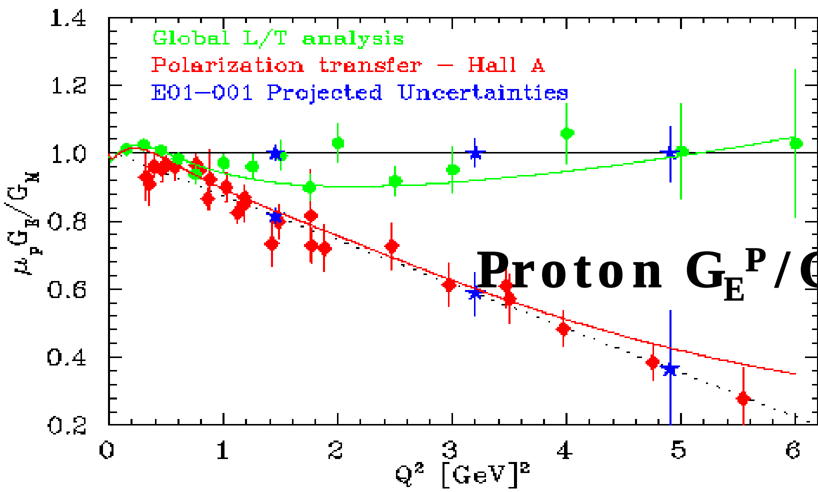
Proton G_M^P / G_M^P -DIPOLE





Proton G_M^P

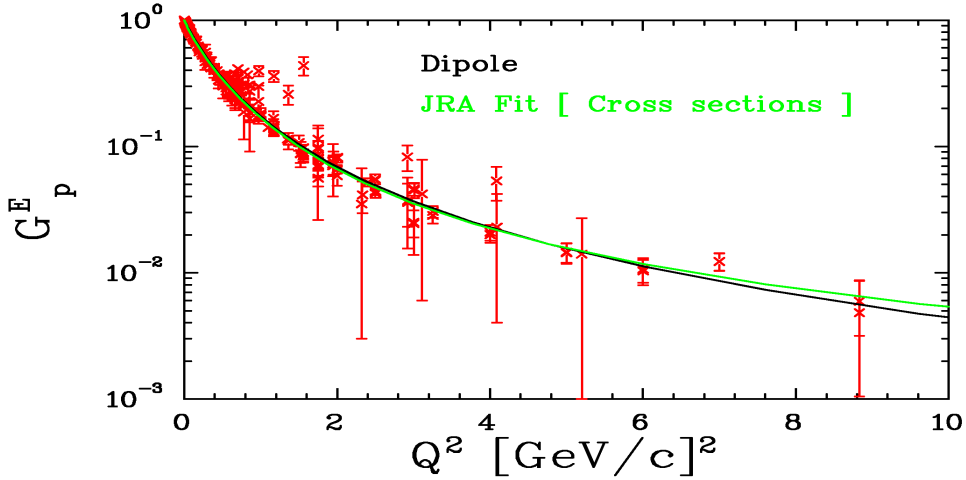
Compare Rosenbluth Cross section Form Factor Separation Versus new Hall A Polarization measurements

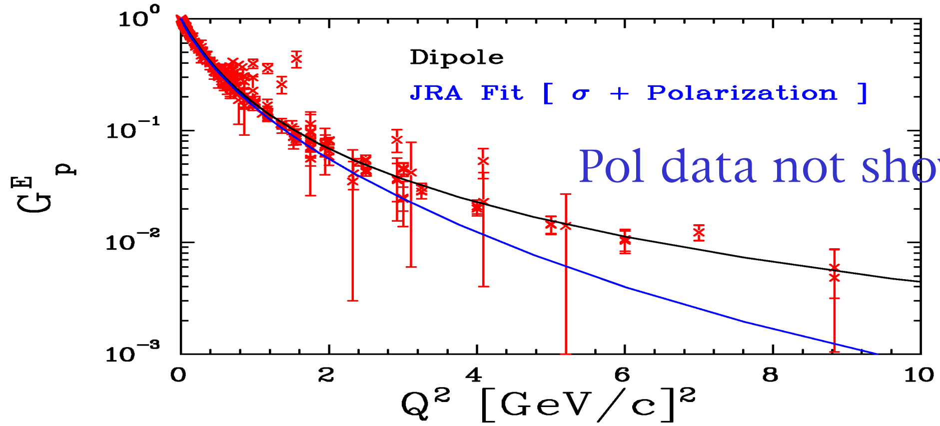


Proton G_E^P/G_M^P

Arie Bodek, Un

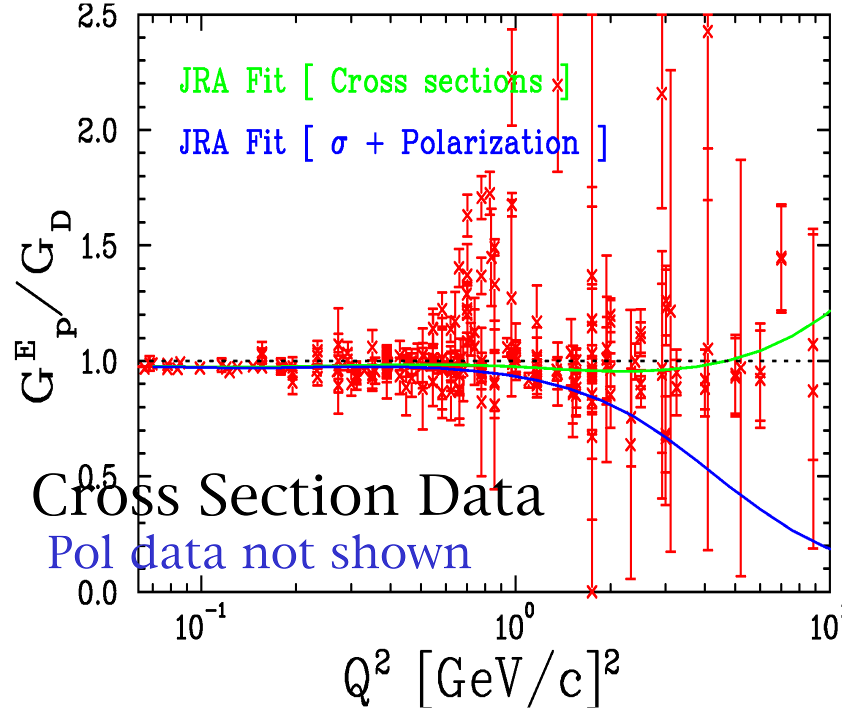
Proton G_E^P





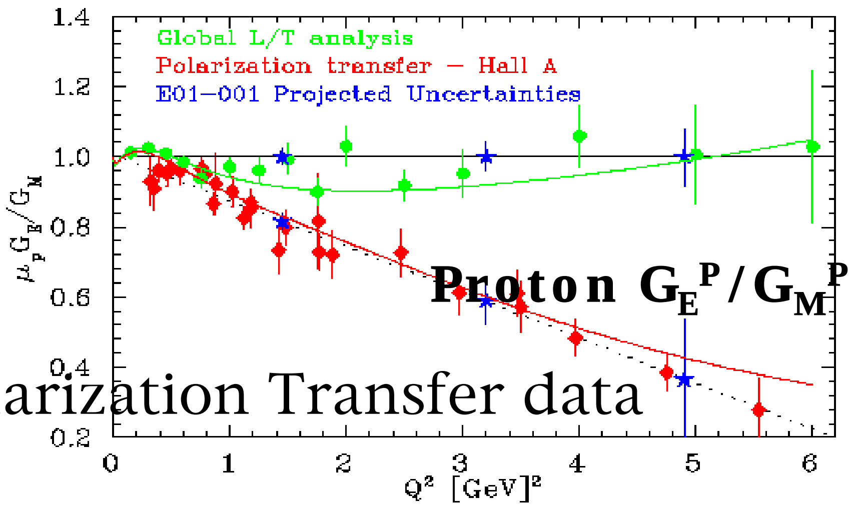
Pol data not shown

Proton G_E^P / G_E^P -DIPOLE



Cross Section Data

Pol data not shown



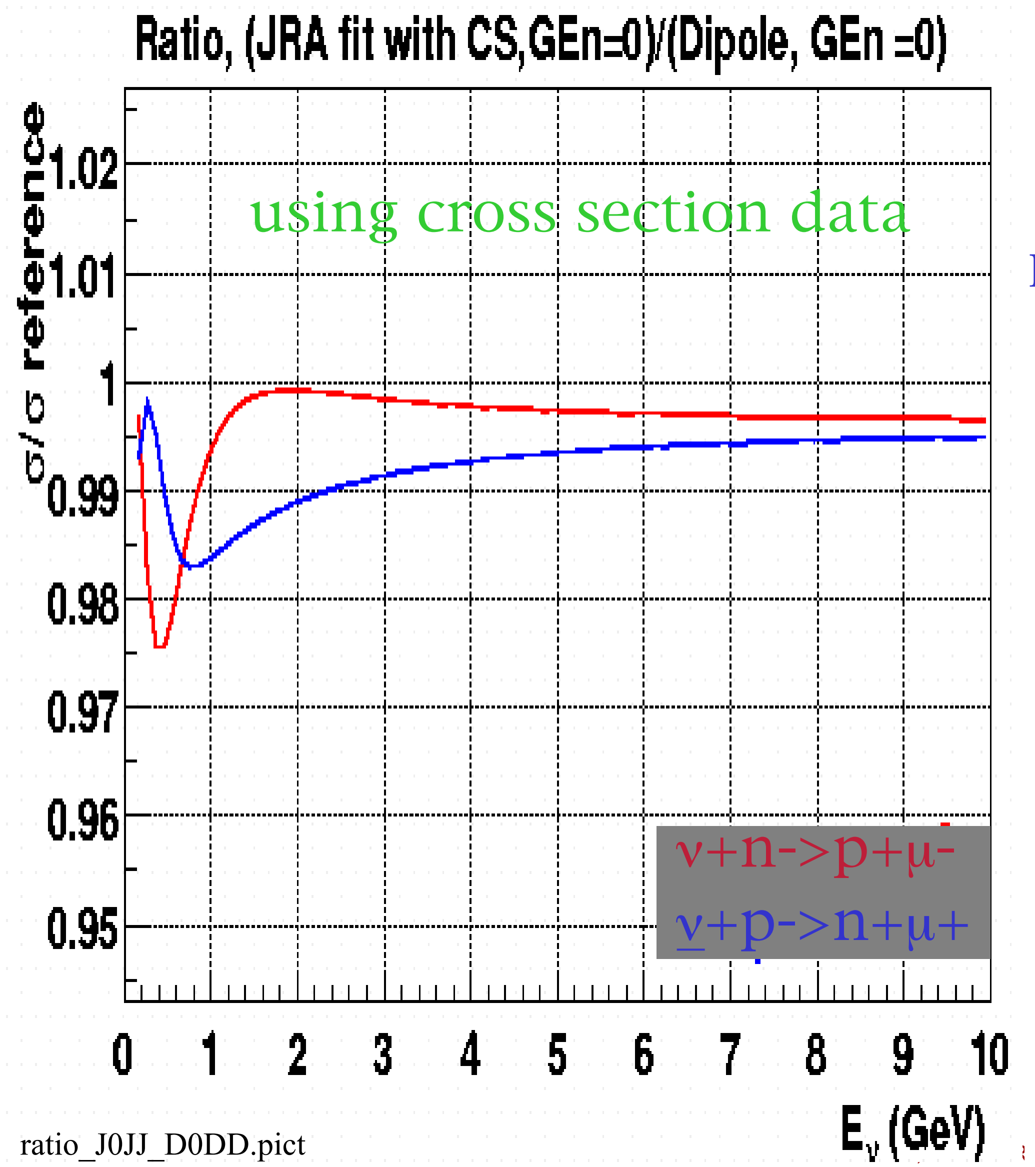
Proton G_E^P/G_M^P

Polarization Transfer data

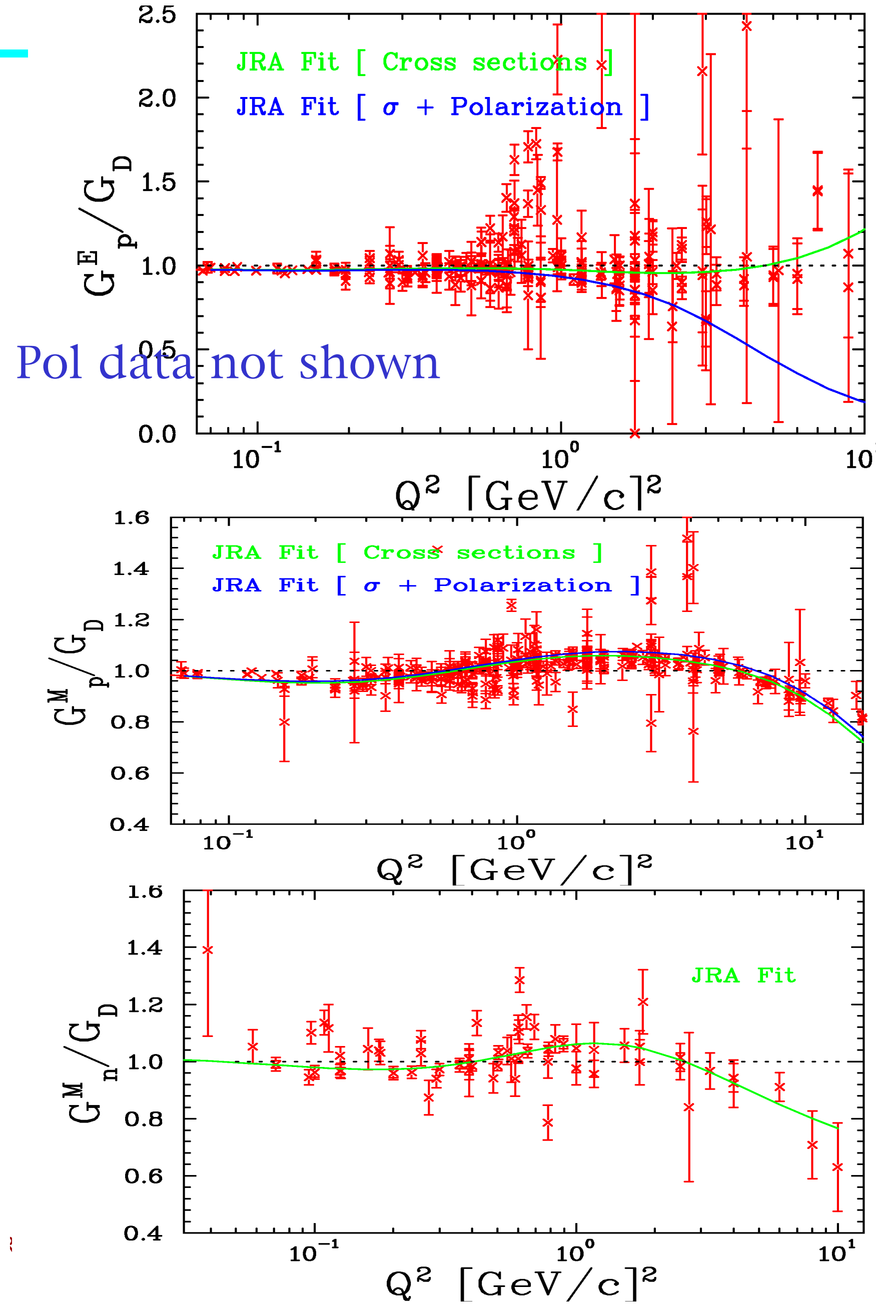
Proton G^E_P

Compare Rosenbluth Cross section Form Factor Separation Versus new Hall A Polarization measurements

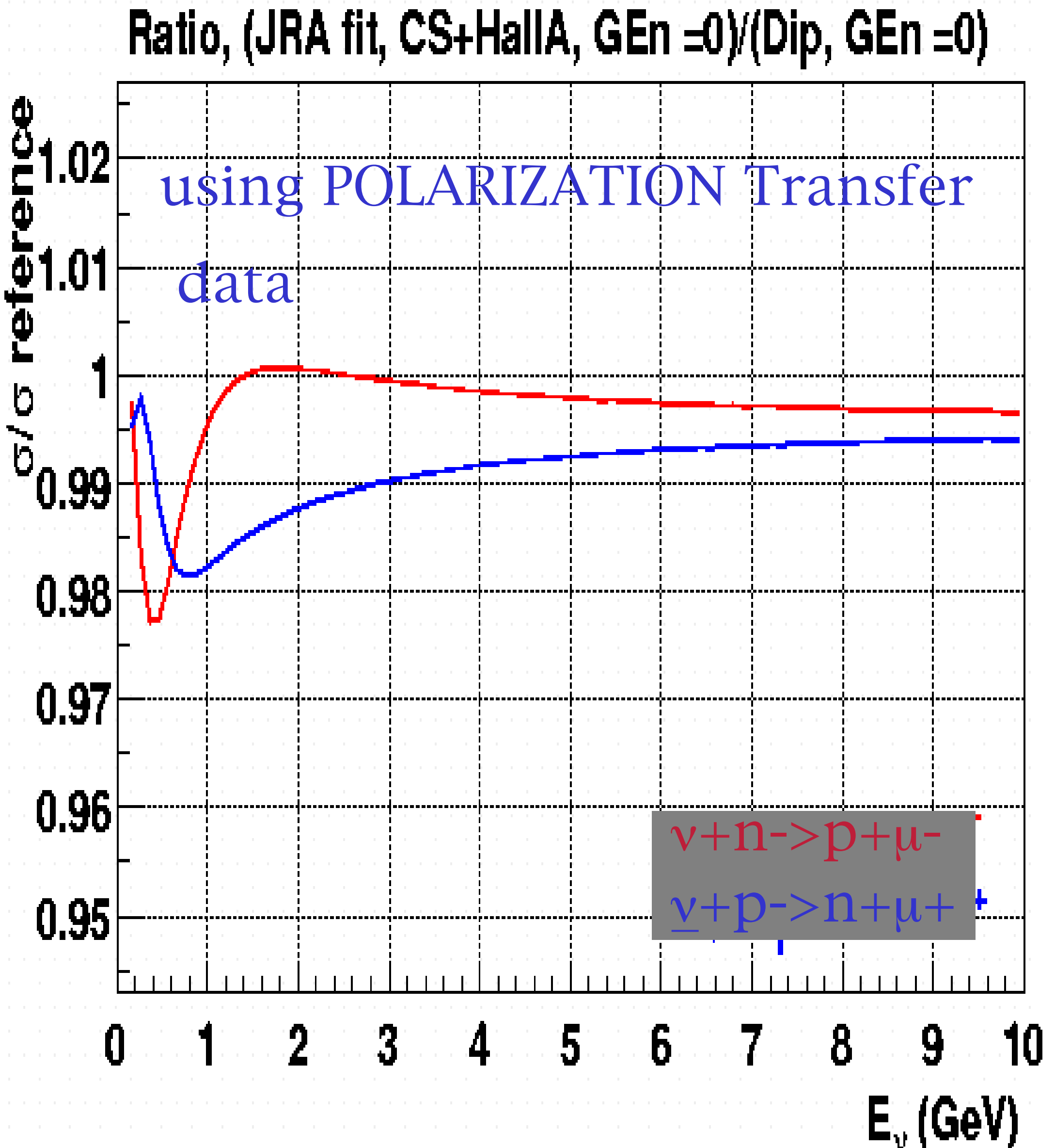
Effect of G_M^N, and G_M^P, G_E^P (using cross section data)(with G_E^N =0) Versus Dipole Form factor

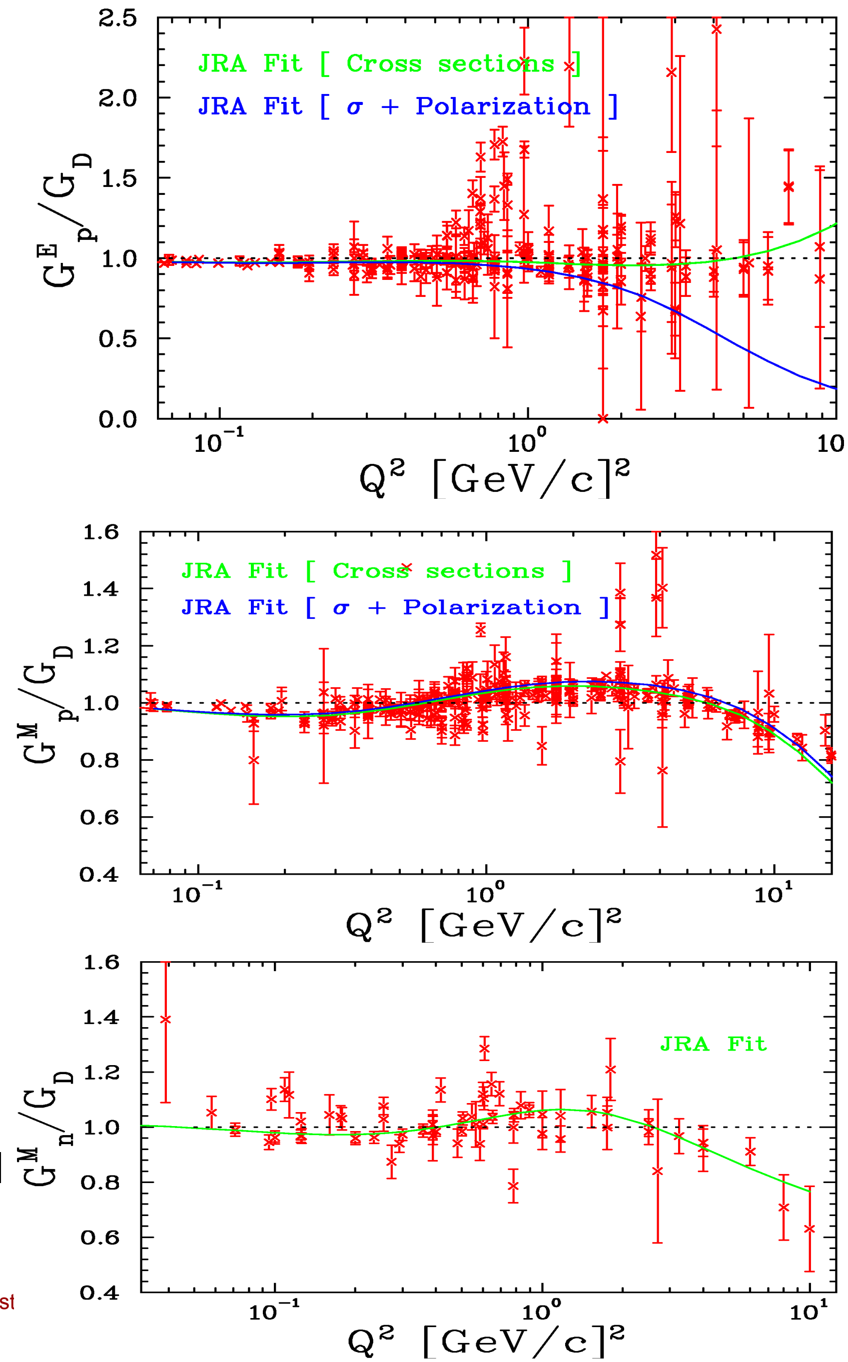


ratio_J0JJ_D0DD.pict



Effect of G_M^N, G_M^P, G_E^P (using POLARIZATION data) (with $G_E^N = 0$) Versus Dipole Form Factor

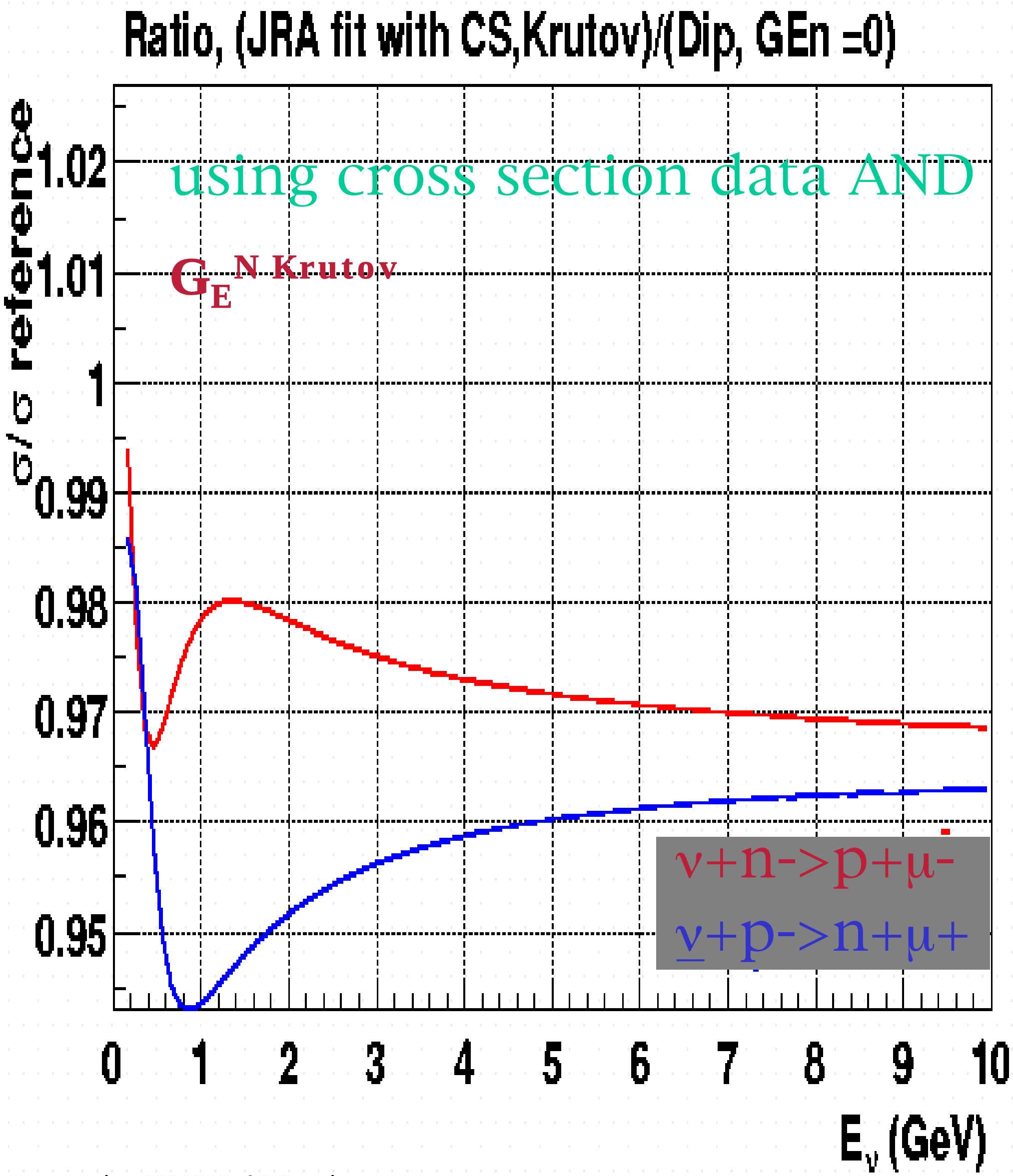




ratio_Jha0JhaJ_D0DD.pict

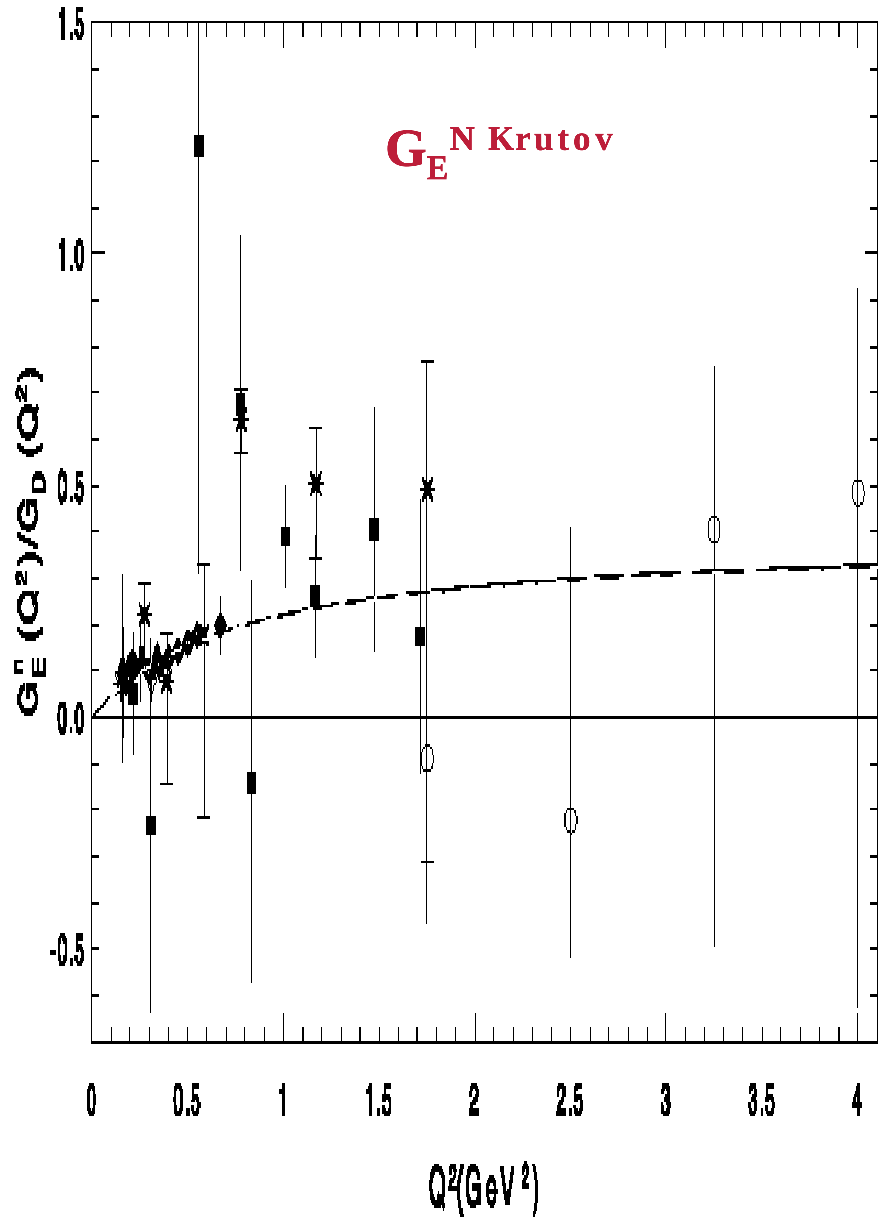
Arie Bodek, Univ. of Rochest

Effect of G_M^N, G_M^P, G_E^P (using cross section data AND non zero $G_E^{N\ Krutov}$) Versus Dipole Form

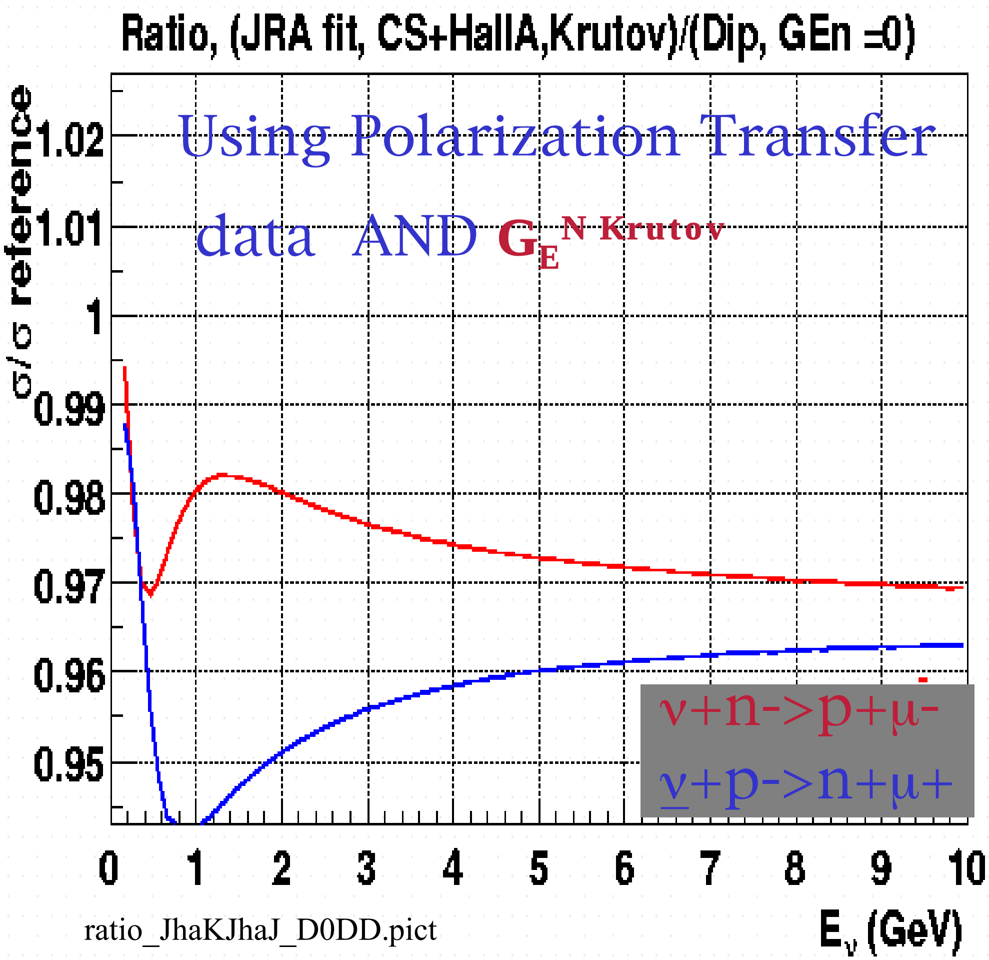


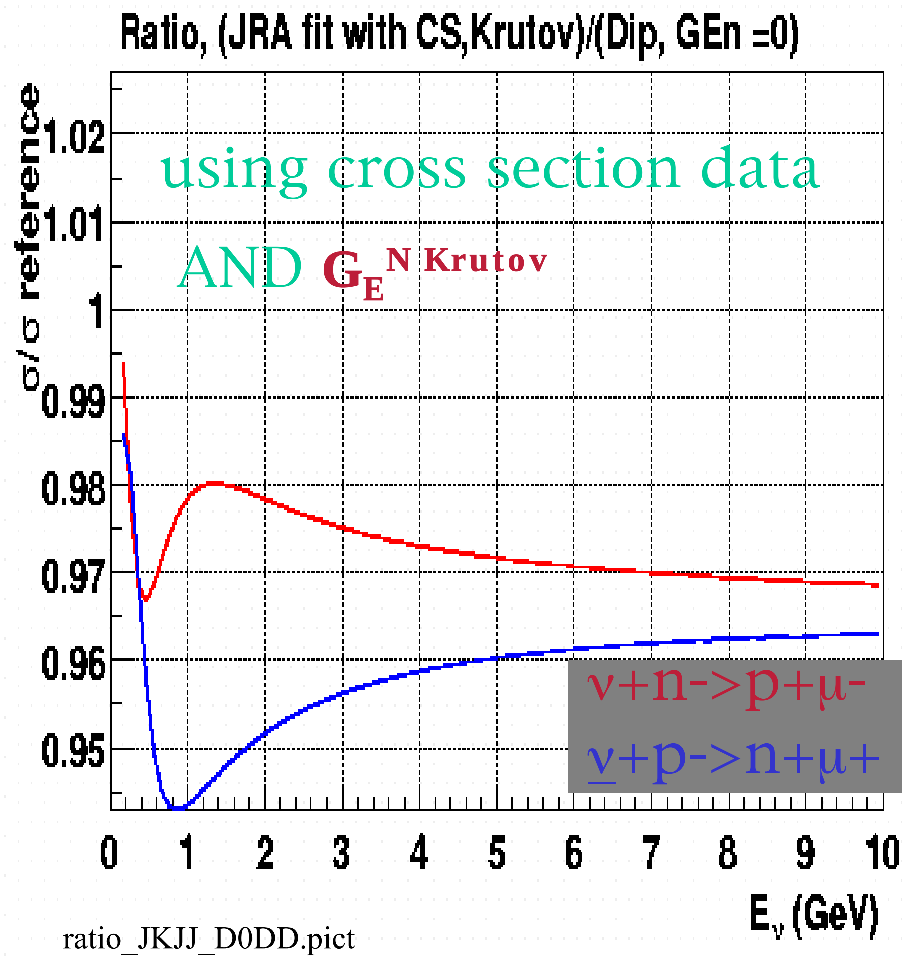
ratio_JKJJ_D0DD.pict

Arie Bodek, Univ. of



Effect of G_M^N + (G_M^P ,G_E^P using POLARIZATION data
AND non zero G_E^N Krutov) - Versus Dipole Form
-> Discrepancy between G_E^P Cross Section and Polarization
Data Not significant for Neutrino Cross Sections





G_M^P ,G_E^P extracted with both e-p Cross section and Polarization data

G_M^P ,G_E^P extracted With e-p Cross Section data only

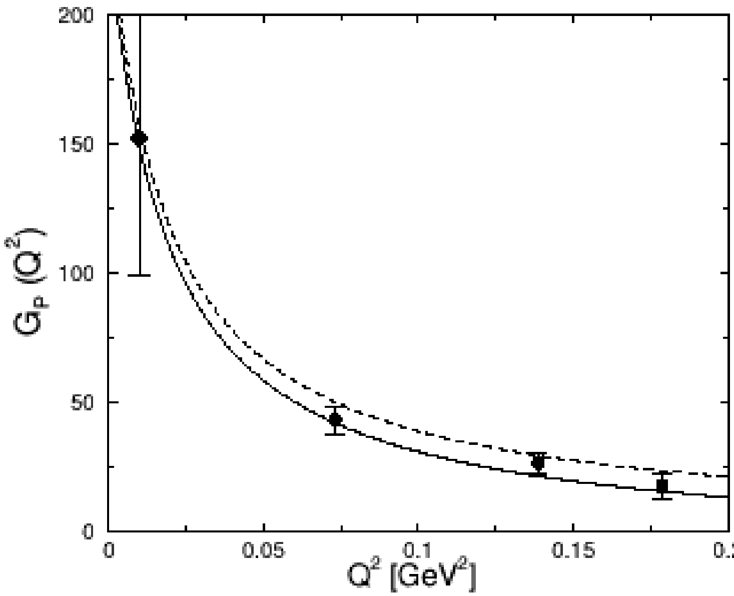
Axial structure of the nucleon

Hep-ph/0107088 (2001)

Véronique Bernard†, Latifa Elouadrhiri‡, Ulf-G Meißner§

induced pseudoscalar form factor is the least well known of all six electroweak nucleon form factors.

Seonho Choi, et al PRL V71 page 3927 (1993) Near Threshold Pion Electro-production and lowest Q2 point from Ordinary Muon Capture (**OMC**) both **agree with PCAC**



Current algebra Assumption for for Fp is OK. 5% effect For Tau neutrinos For muon neutrino Only needed near E=0

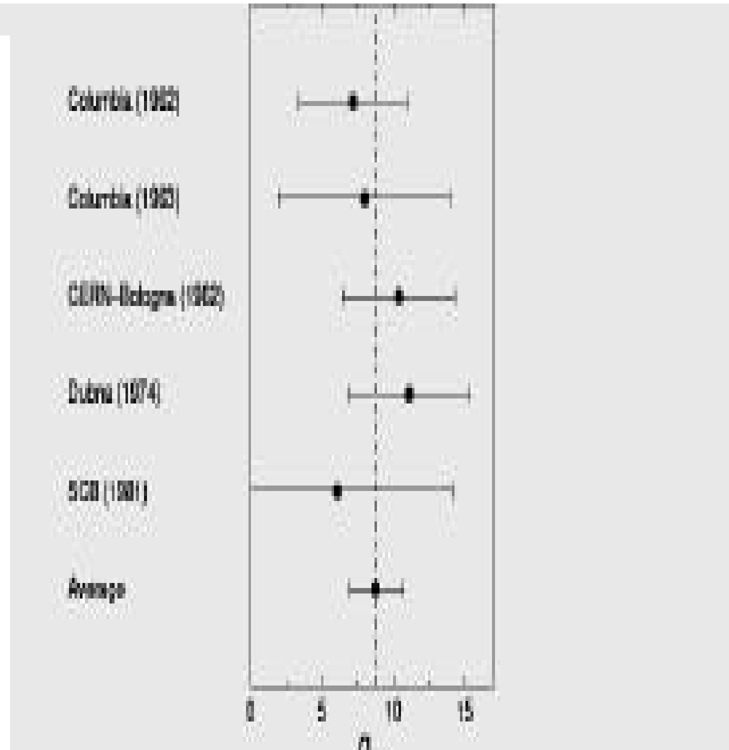
A third way to measure gp. is from Radiative Muon Capture (**RMC**), but the first measurement is factor of 1.4 larger

Figure 5. The "world data" for the induced pseudoscalar form factor $G_P(Q^2)$. The pion electroproduction data (filled circles) are from reference [65]. Also shown is the world average for ordinary muon capture at $Q^2 = 0.88 M_\mu^2$ (diamond). For orientation, we also show the theoretical predictions discussed later. Dashed curve: Pion–pole (current algebra) prediction. Solid curve: Next–to–leading order chiral perturbation theory prediction.

Table 1. Pseudoscalar coupling constant determined from OMC in light nuclei.

Nucleus	g_P	Reference
^{3}He (capture to triton)	8.6 ± 1.5	[58]
^{12}C (capture to ground state)	8.3 ± 2.5	[59]
^{16}O (capture to ^{16}N(0^-))	10.0 ± 1.2	[60] [61]

From OMC



backgrounds. Precisely for this reason only very recently a first measurement of RMC on the proton has been published [62, 63]. The resulting number for g_P, which was obtained using a relativistic tree model including the Δ–isobar [64] to fit the measured photon spectrum, came out significantly larger than expected from OMC,

$$g_P^{\rm RMC} = 12.35 \pm 0.88 \pm 0.38 \simeq 1.4\, g_P^{\rm OMC} \,, \qquad (15)$$

From RMC

and thus also about 40% above all theoretical expectations (see section 4.1). It should

$$g_P = (8.74 \pm 0.23) - (0.48 \pm 0.02) = 8.26 \pm 0.16 \,.$$

From PCAC

Axial structure of the nucleon

Hep-ph/0107088 (2001)

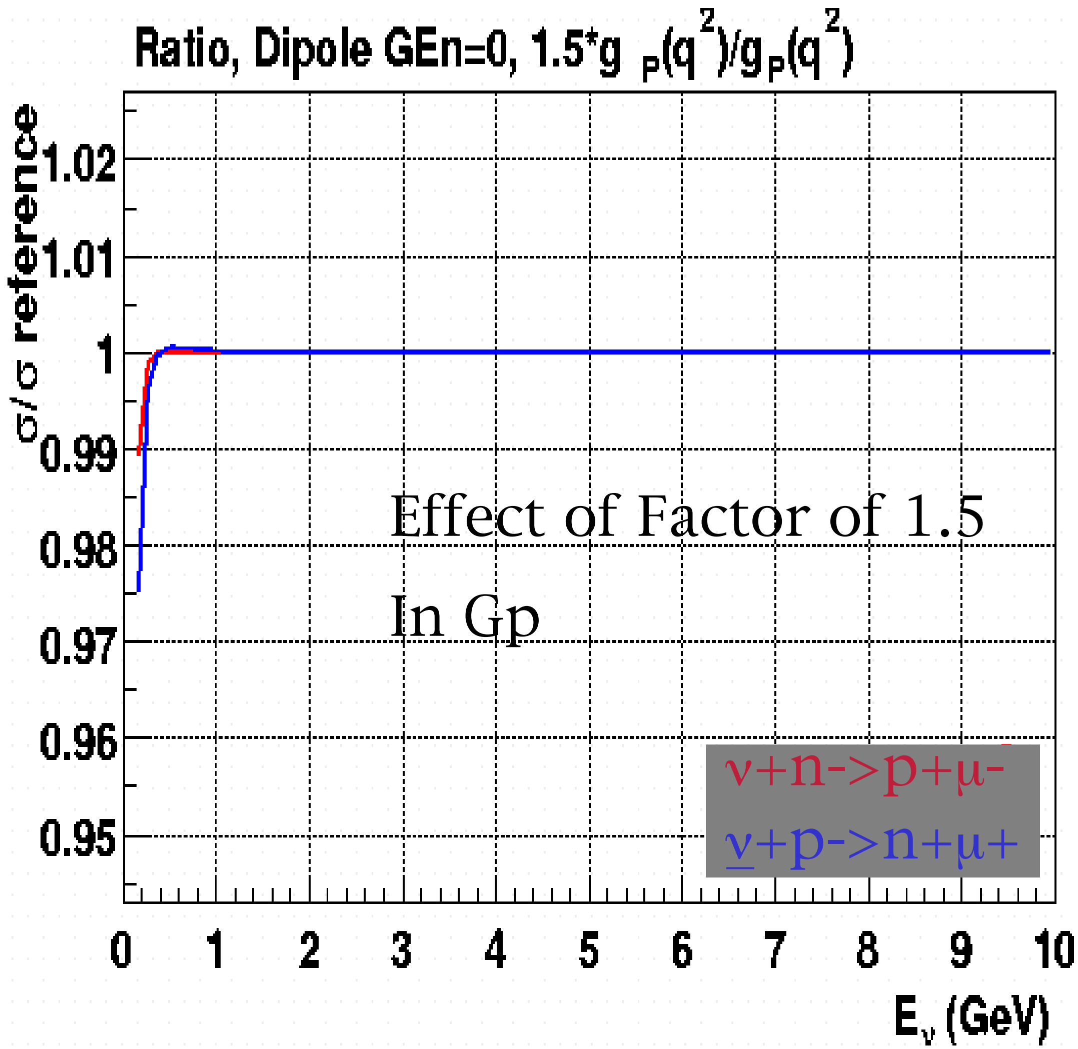
Véronique Bernard†, Latifa Elouadrhiri‡, Ulf-G Meißner§

Note , one measurement of gp from Radiative Muon Capture (RMC) at Q=Mmuon quoted in the above Review disagrees with PCAC By factor of 1.4. PRL V77 page 4512 (1996) .

In contrast Seonho Choi, et al PRL V71 page 3927 (1993) from OMC, agrees with PCAC.

The plot (ratio_gp15_D0DD.pict) shows the sensitivity of the cross section to a factor of 1.5 increase in Gp.

IT IS ONLY IMPORTANT FOR the lowest energies.



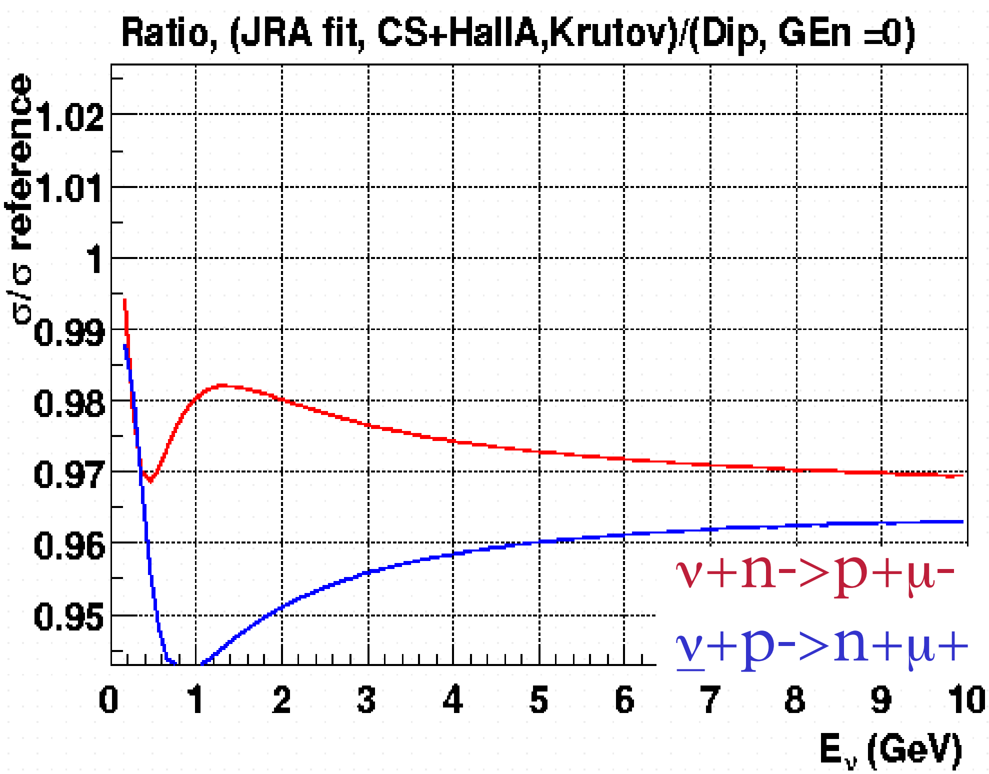
Conclusions -1

1. Non Zero Value of $\mathbf{G_E^N}$ is the most important (5% effect)

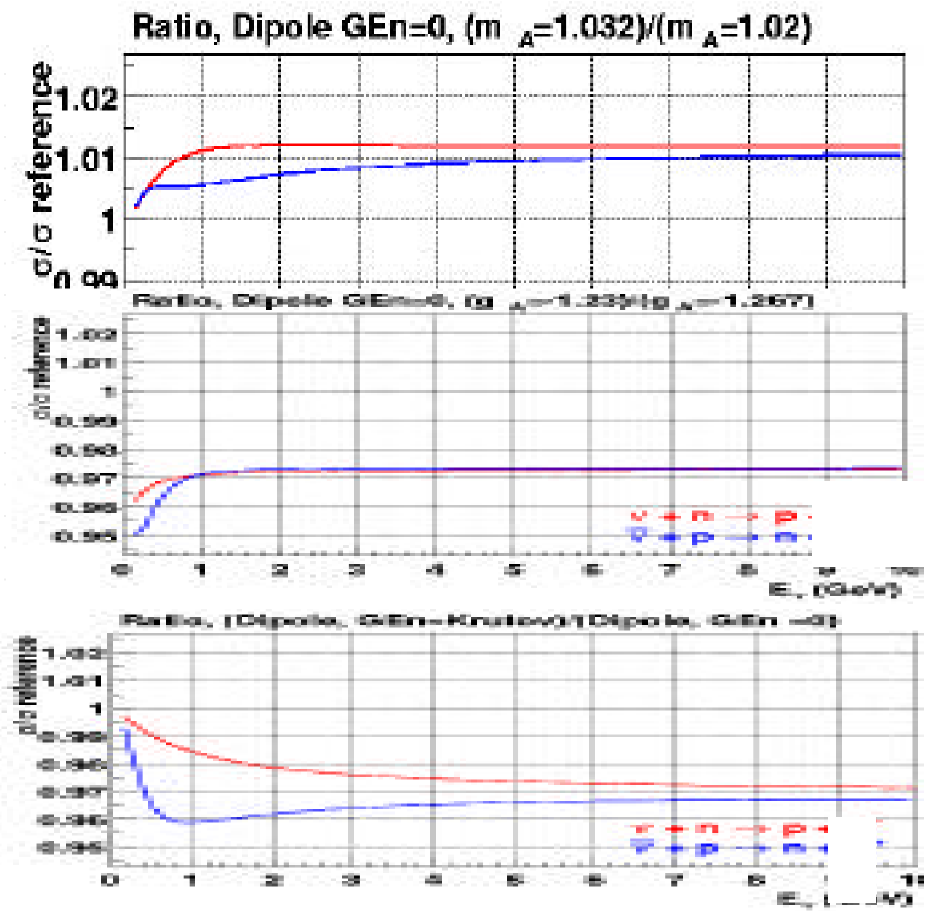
2. We have begun a re-analysis of neutrino quasielastic data for $d\sigma/dQ^2$ to obtain update values of M_A *with*

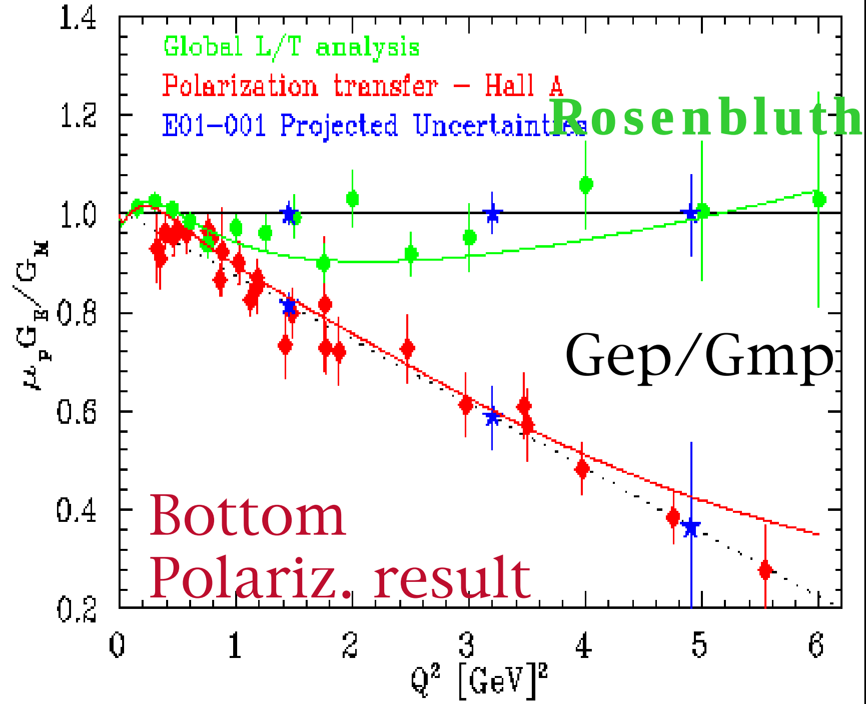
- Latest values of $\mathbf{G_E^N}$, $\mathbf{G_M^N}$, $\mathbf{G_M^P}$, $\mathbf{G_E^P}$ which affect the shape.

- Latest value of g_A (not important if normalization is not used in $d\sigma/dQ^2$ Flux errors are about 10%).



ratio_JhaKJhaJ_D0DD.pict





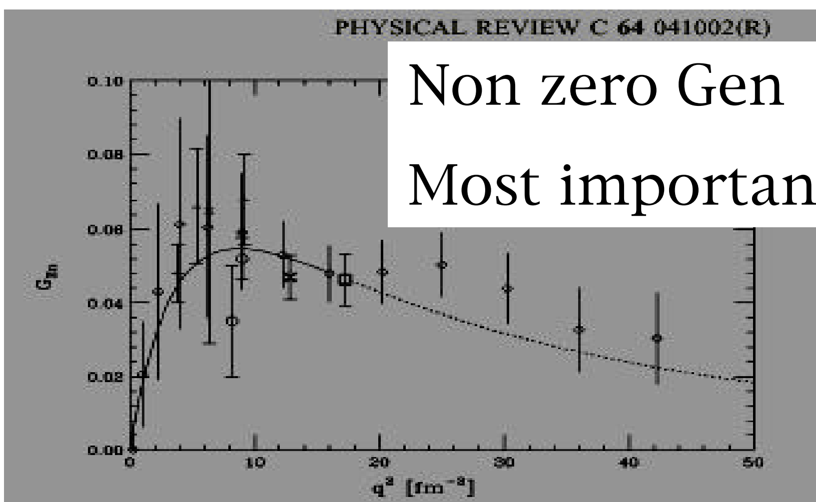


FIG. 3. The G_{En} extracted from the C2 data (◇). Also shown are the values obtained from double-polarization experiments, and the Galster parametrization with its extrapolation into the region not covered by previous experiments (dotted line).

For $Q^2 < 1$ GeV^2 ONLY

New precision polarization Transfer measurements on Gep/Gmp agree with Standard Rosenbluth technique.

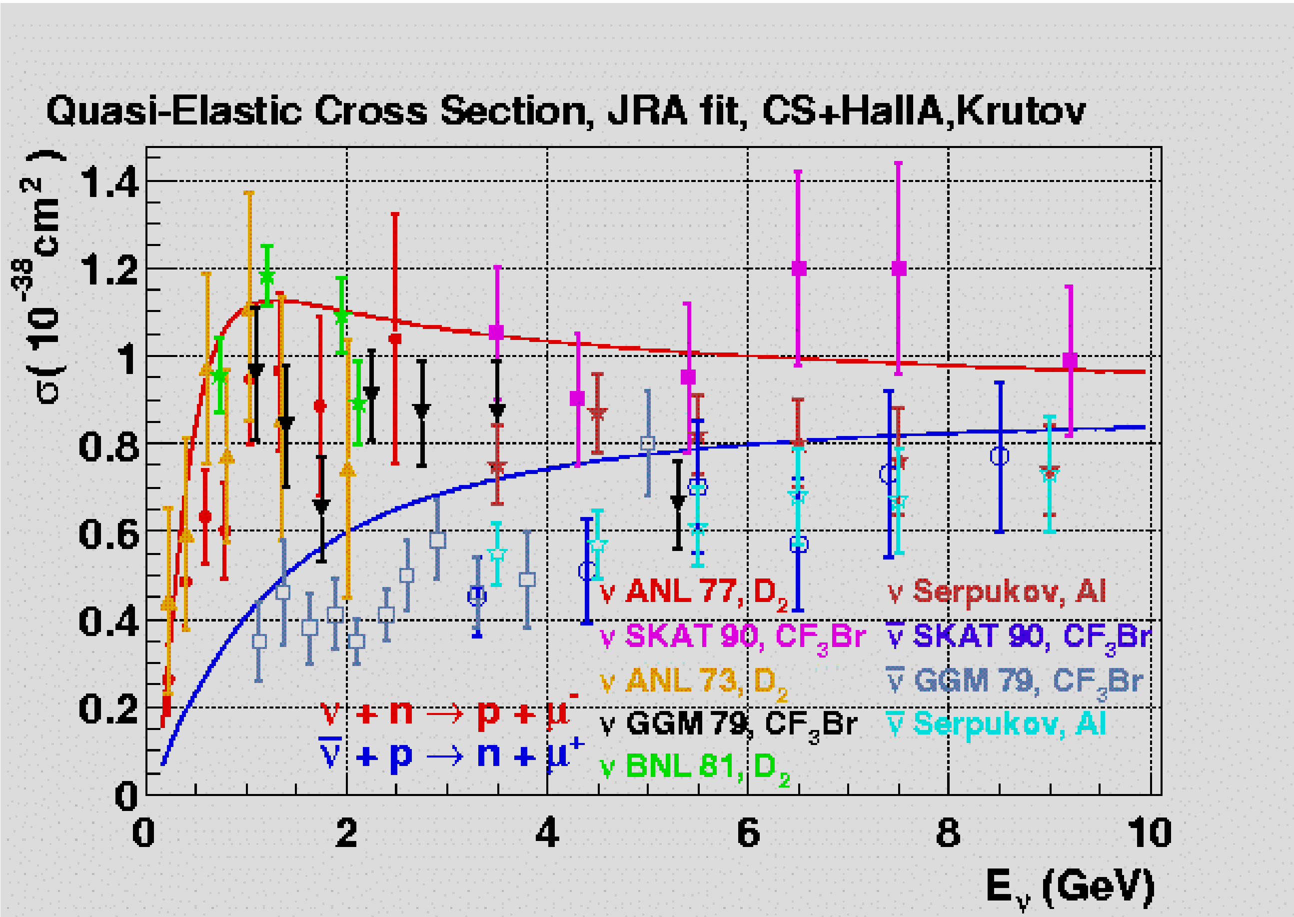
HOWEVER: Above $Q^2>1$ GeV^2 There is disagreement.

Note, this high Q^2 region Is not relevant to neutrino Experiments . So use latest Gen, Gep, Gmn, Gmp form factors As new input Vector form Factors for quasi-elastic Neutrino scattering.

$\sigma_{\text{quasi-elastic neutrinos on Neutrons}}$-(- Calculated

$\sigma_{\text{quasi-elastic Antineutrinos on}}$ Protons - Calculated

From H. Budd -U of Rochester (NuInt02) (with Bodek and Arrington) DATA - FLUX ERRORS ARE 10%

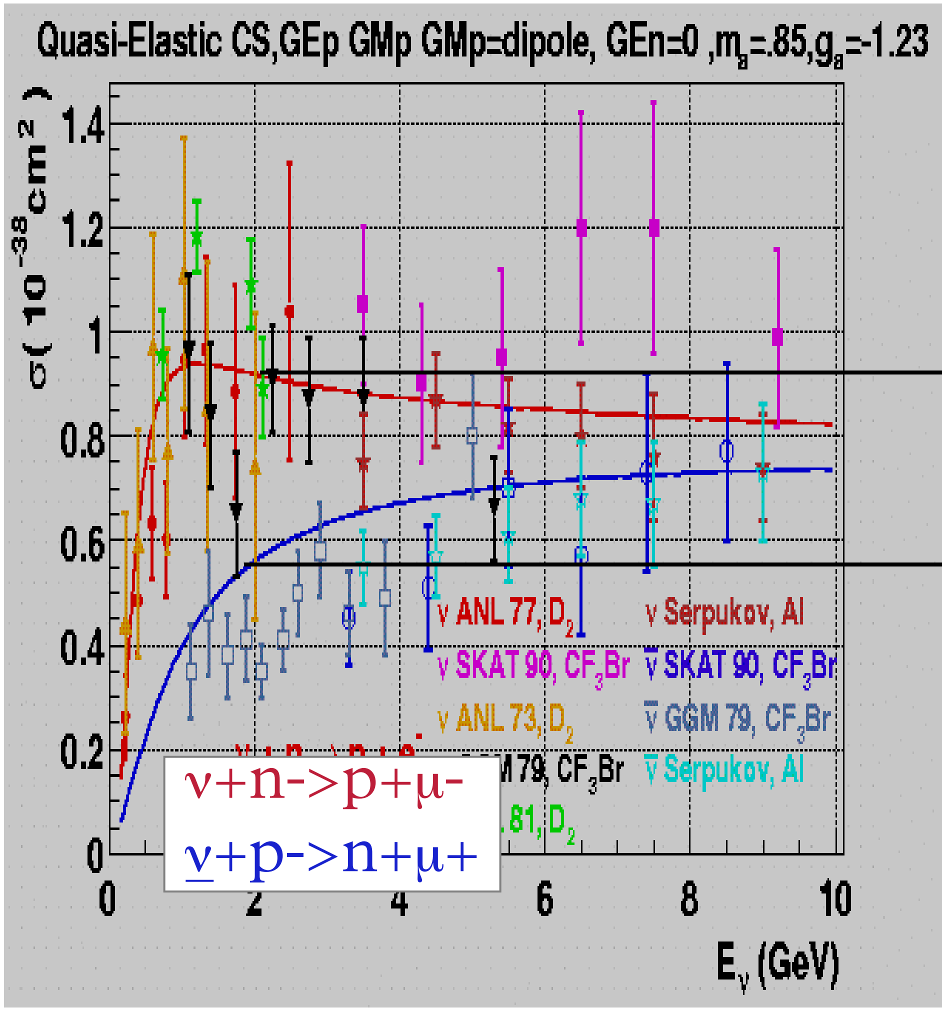


With the most Up to date Form Factors The agreement With data is ***not spectacular***

Antineutrino data mostly on nuclear targets

Compare to Original Llewellyn Smith Prediction (H. Budd)

Antineutrino data on nuclear targets



Old LS results with

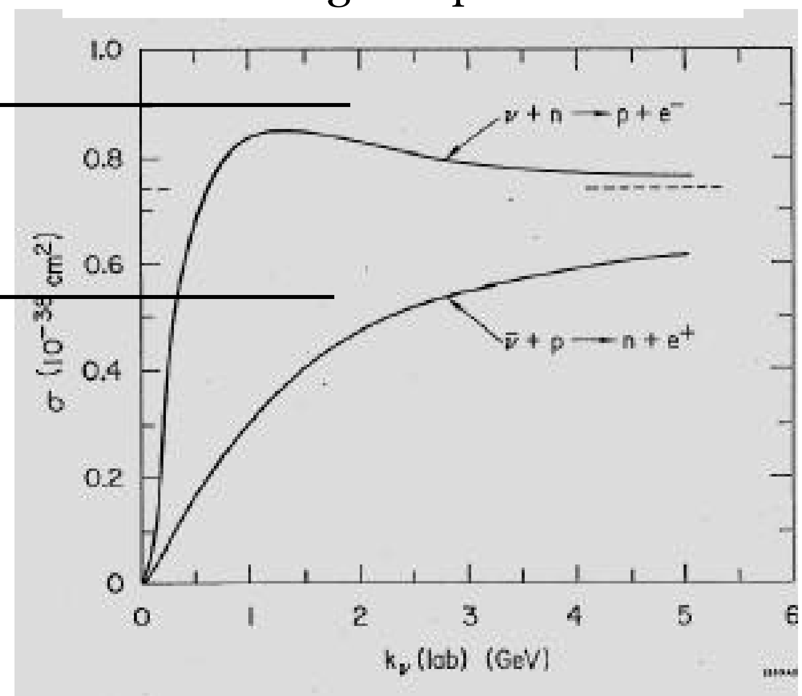
Old ga=-1.23 and

Ma below. Plot in LS paper

Is 10% lower than the cross

Section we calculate with the

Same wrong old parameters.



But we are not sure what was used in this old paper

Neutrino Cross Sections

H. M. Gallagher and M. C. Goodman

NuMI-112
PDK-626
Nov. 10, 1995

They implemented
The Llewellyn-Smith
Formalism for NUMI

$$\frac{d\sigma}{d|q^2|}\begin{pmatrix}\nu n \to l^- p \\ \bar{\nu} p \to l^+ n\end{pmatrix} = \frac{M^2 G^2 \cos^2\theta_c}{8\pi E_\nu^2}\left[A(q^2) \mp B(q^2)\frac{(s-u)}{M^2} + \frac{C(q^2)(s-u)^2}{M^4}\right]. \quad (2)$$

In this expression, G is the Fermi coupling constant and θ_c is the Cabibbo mixing angle ($G = 1.16639 \times 10^{-5}\mathrm{GeV}^{-2}$). The functions A, B, and C are convenient combinations of the nucleon form factors.

Contraction of the hadronic and leptonic currents yields: Non zero

$$A = \frac{(m^2 - q^2)}{4M^2}\left[\left(4 - \frac{q^2}{M^2}\right)|F_A|^2 - \left(4 + \frac{q^2}{M^2}\right)|F_V^1|^2 - \frac{q^2}{M^2}|\xi F_V^2|^2\left(1 + \frac{q^2}{4M^2}\right) - \frac{4q^2 ReF_V^{1*}\xi F_V^2}{M^2} \quad (3)\right.$$

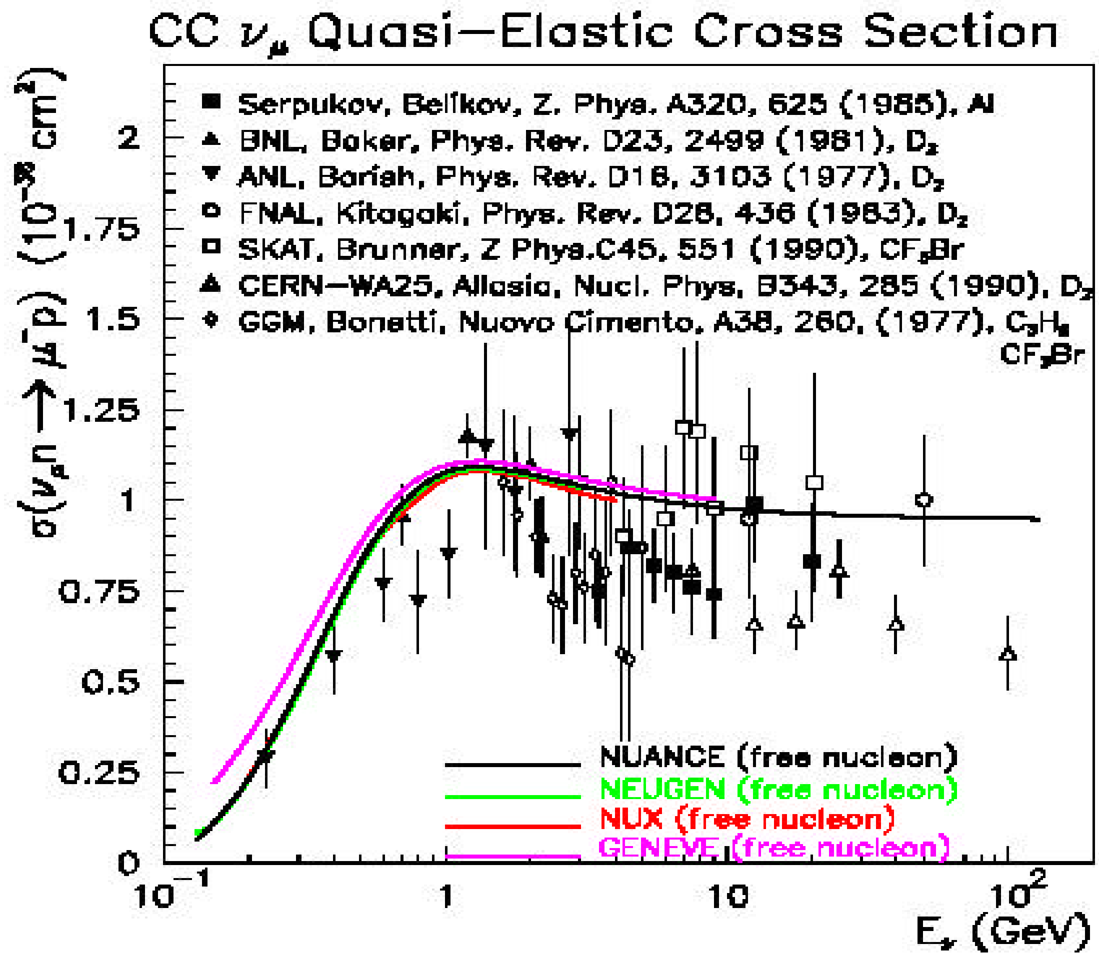
$$\left. + \frac{q^2}{M^2}\left(4 - \frac{q^2}{M^2}\right)|F_T|^2 - \frac{m^2}{M^2}\left(|F_V^1 + \xi F_V^2|^2 + |F_A + 2F_P|^2 + \left(\frac{q^2}{M^2} - 4\right)\left(|F_S|^2 + |F_P|^2\right)\right)\right]$$

$$B = -\frac{q^2}{M^2} ReF_A^*(F_V^1 + \xi F_V^2) - \frac{m^2}{M^2} Re\left[\left(F_V^1 + \frac{q^2}{4M^2}\xi F_V^2\right)^* F_S - \left(F_A + \frac{q^2 F_P}{2M^2}\right)^* F_T\right] \quad (4)$$

$$C = \frac{1}{4}\left(|F_A|^2 + |F_V^1|^2 - \frac{q^2}{M^2}\left|\frac{\xi F_V^2}{2}\right|^2 - \frac{q^2}{M^2}|F_T|^2\right), \quad (5)$$

where m is the final state lepton mass. Ignoring second-class currents (those which violate G-parity) allows us to set the scalar and tensor form factors to zero. According to the CVC

$\nu_\mu n \rightarrow \mu^- p$



- Selecting a consistent set of parameters:
  - $\leftrightarrow$ $M_A = 1.032$ GeV
  - $\leftrightarrow$ $M_V = 0.84$ GeV
  - $\leftrightarrow$ $F_A(q^2) = \frac{F_A(0)}{(1-q^2/M_A^2)^2}$; $F_A(0) = -1.25$

Monte Carlo Session. **Sam Zeller@NuInt02 Talk compares Various Monte Carlos for Quasi Elastic scattering**

NOTE: Budd-Bodek-Arrington code **Gives same results With the same Input form factors**

Also Much Thanks to Zeller, Hawker, etc for All the Physics Archeology.

$$F_V^1(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1}\left[G_E^V(q^2) - \frac{q^2}{4M^2}G_M^V(q^2)\right] \quad (6)$$

$$\xi F_V^2(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{1}\left[G_M^V(q^2) - G_E^V(q^2)\right]. \quad (7)$$

The electromagnetic form factors are determined from electron scattering experiments:

UPDATE: Replace by $G_E^V = G_E^P - G_E^N$

$$G_E^V(1^2) = \frac{1}{\left(1 - \frac{q^2}{M_V^2}\right)^2} \qquad G_M^V(q^2) = \frac{1 + \mu_p - \mu_n}{\left(1 - \frac{q^2}{M_V^2}\right)^2}.$$

UPATE: Replace by $G_M^V = G_M^P - G_M^N$

The situation is slightly more complicated for the hadronic axial current. $F_A(q^2 = 0) = -1.261 \pm .004$ is known from neutron beta decay. The q^2 dependence has to be inferred or

$M_A = 1.032 \pm .036$ GeV [7]. ector case we assume the same dipole form:

g_A, M_A need to Be updated

$$F_A(q^2) = \frac{-1.23}{\left(1 - \frac{q^2}{M_A^2}\right)^2}. \quad (9)$$

$Q^2 = -q^2$

$$F_P(q^2) = \frac{2M^2 F_A(q^2)}{M_\pi^2 - q^2}. \quad (10)$$

Fp important for Muon neutrinos only at Very Low Energy

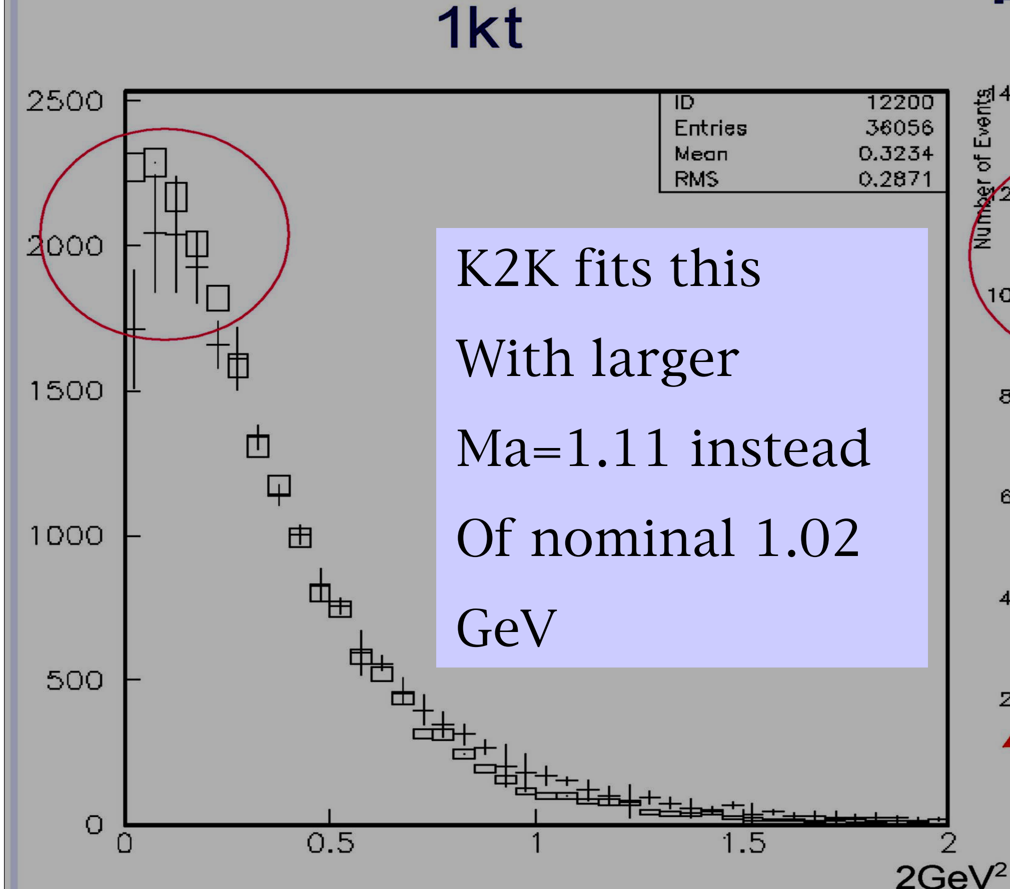
The inclusion of F_P leads to an approximately 5% reduction in both the ν_τ and ν_τ quasi-elastic cross sections. The only remaining parameters needed to describe the quasi-elastic cross section are thus M_V and M_A. $M_V = .71$ GeV, as determined with high accuracy

From C.H. Llewellyn Smith (SLAC). SLAC-PUB-0958 Phys.Rept.3:261,1972

First result done at NuInt02

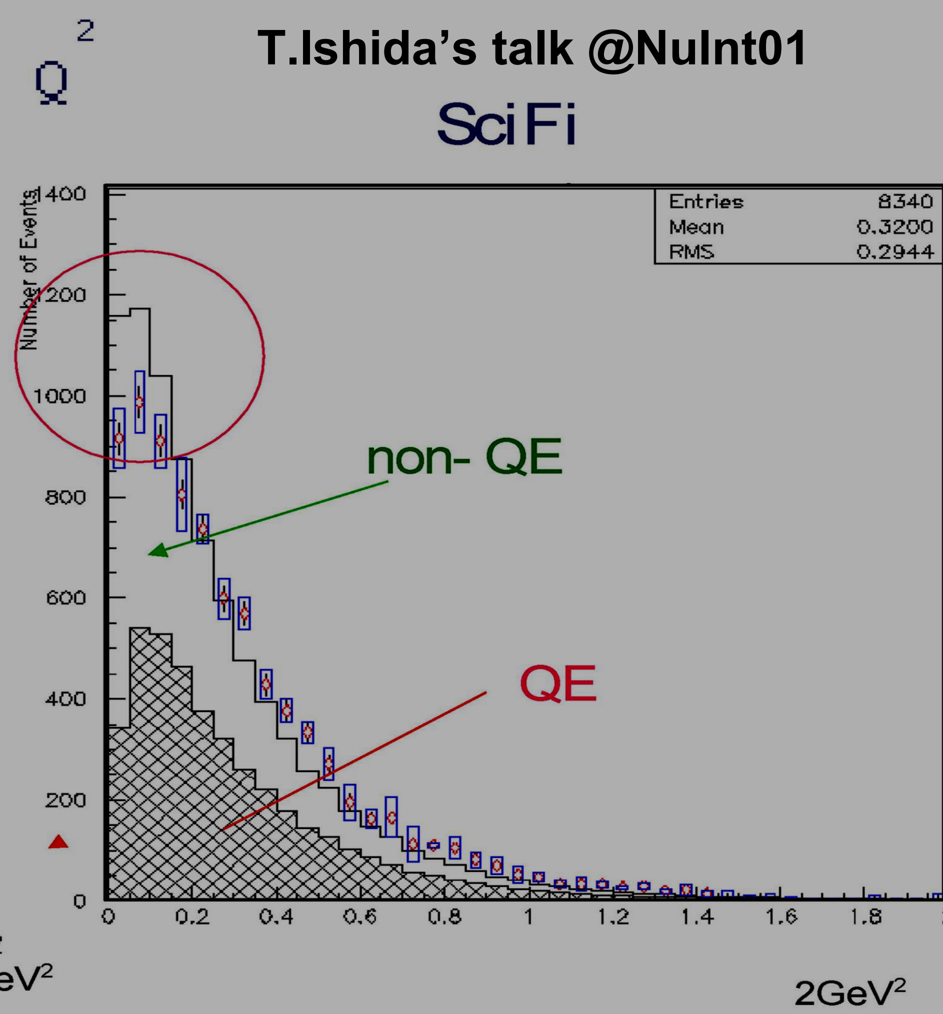
Low-Q2 suppression or Larger M_A?

From Ito NuInt02



* Errors shown here is an energy scale error (±5%)
* Nuclear binding energy is not taken into account..

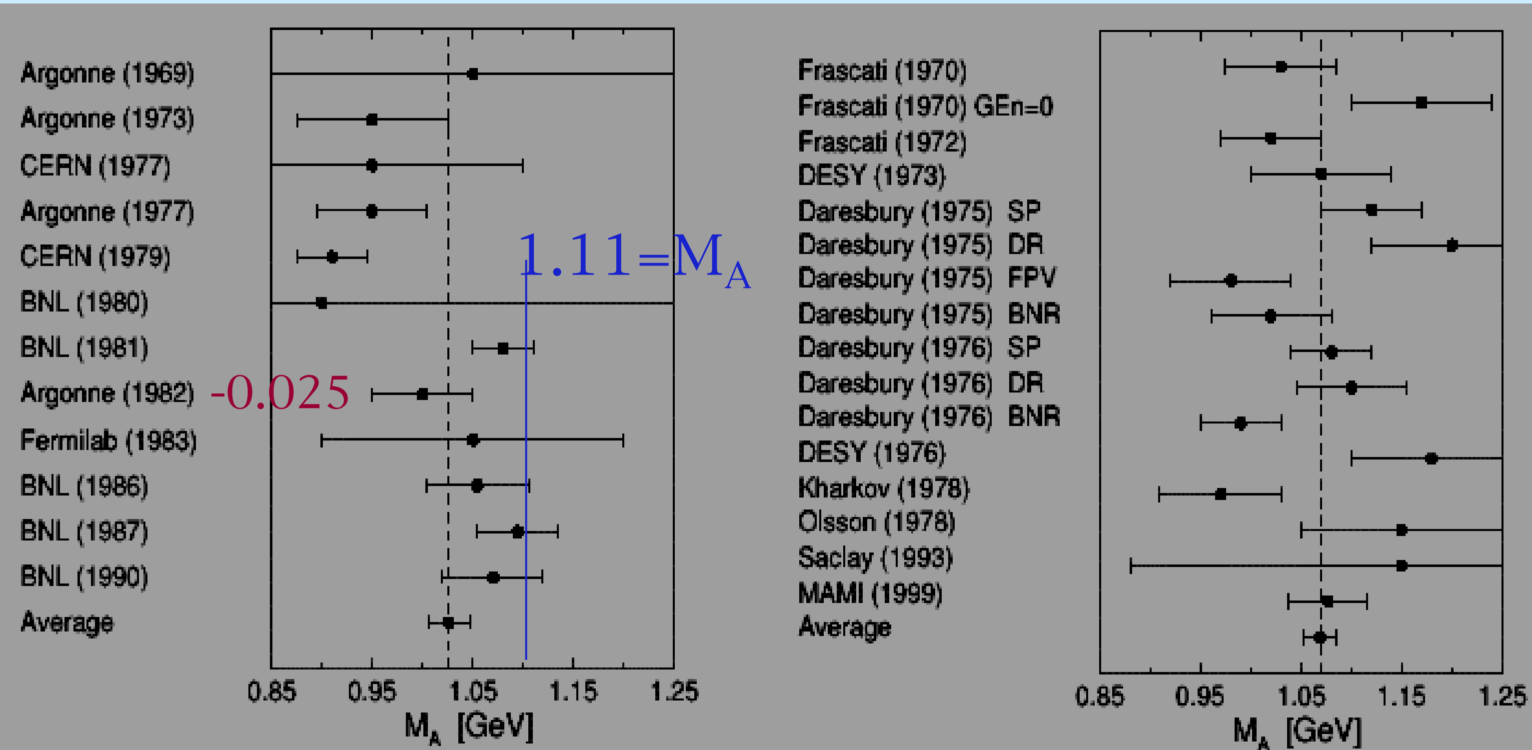
T.Ishida's talk @NuInt01



* Errors shown here is a typical energy scale error (±3%).
* Nuclear binding energy B = - 30MeV (for Oxygen) is taken into account.

Axial structure of the nucleon   Hep-ph/0107088 (2001)

Véronique Bernard†, Latifa Elouadrhiri‡, Ulf-G Meißner§



For updated M_A expt. need to be reanalyzed with new g_A, and G_E^N

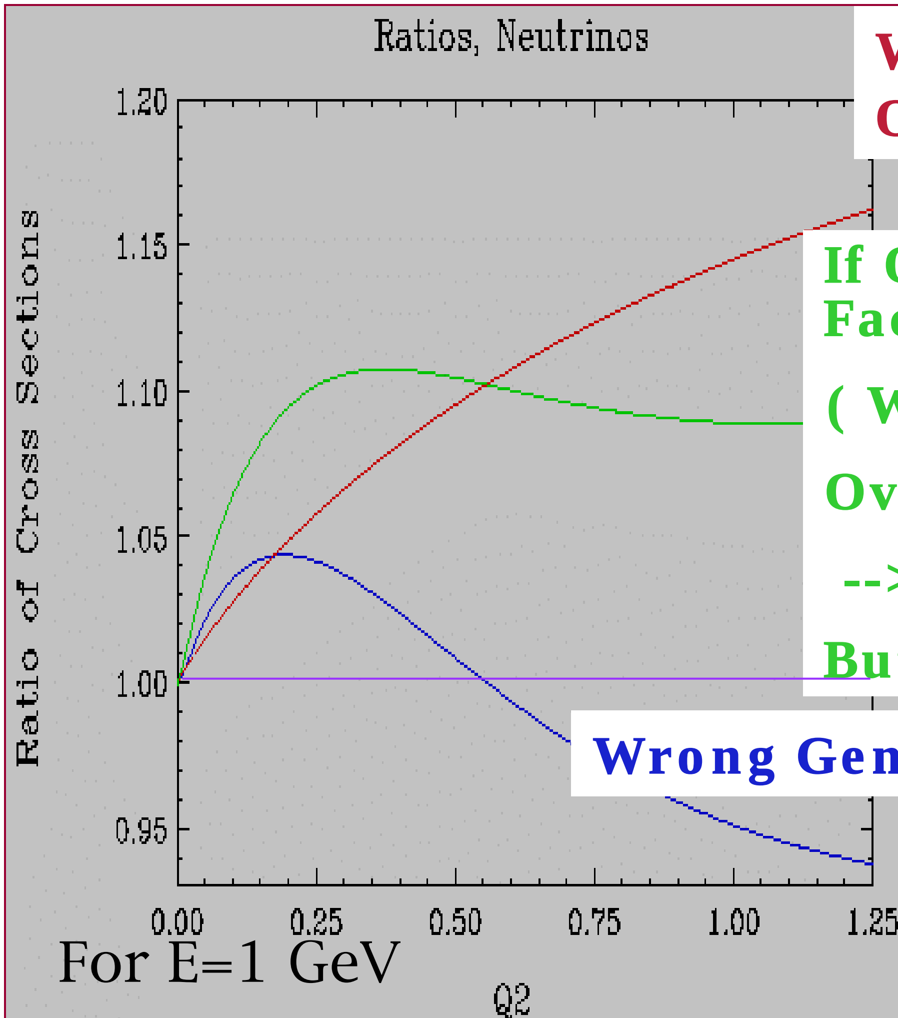
Difference In Ma between Electroproduction And neutrino Is understood

Figure 1. Axial mass M_A extractions. Left panel: From (quasi)elastic neutrino and antineutrino scattering experiments. The weighted average is $M_A = (1.026 \pm 0.021)$ GeV. Right panel: From charged pion electroproduction experiments. The weighted average is $M_A = (1.069 \pm 0.016)$ GeV. Note that value for the MAMI experiment contains both the statistical and systematical uncertainty; for other values the systematical errors were not explicitly given. The labels SP, DR, FPV and BNR refer to different methods evaluating the corrections beyond the soft pion limit as explained in the text.

M_A from neutrino expt. No theory corrections needed

ANSWER - Neutrino Community Using Outdated Form Factors

Effect is Low Q2 suppression from non Zero Gen



Wrong Ma=1.1 (used by K2K)
Over Ma=1.02 (Ratio)

If One Uses Both wrong Form Factors (used in K2K MC)

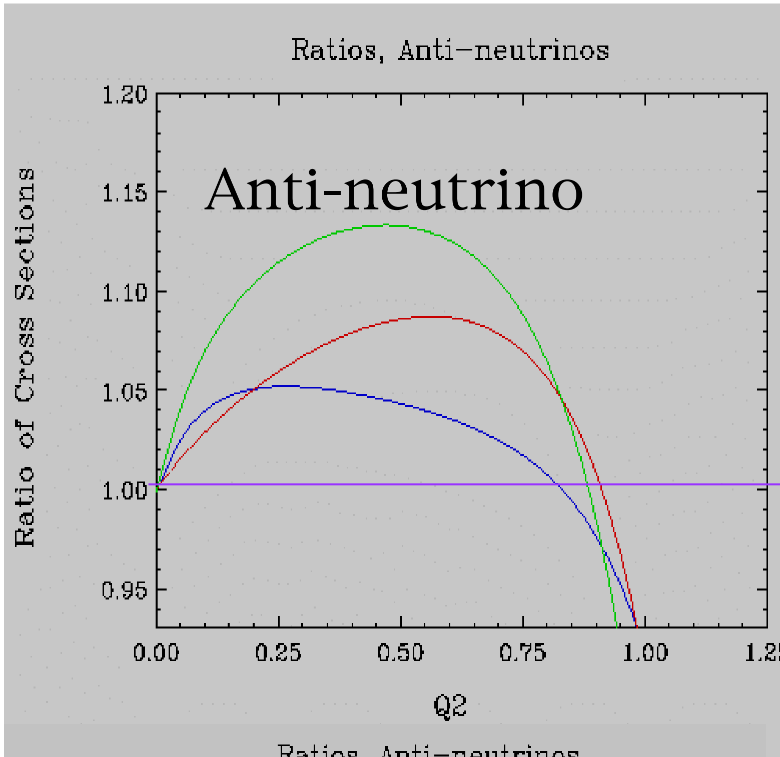
(Wrong Gen =0 +Wrong Ma=1.1)

Over Best Form Factors (Ratio)

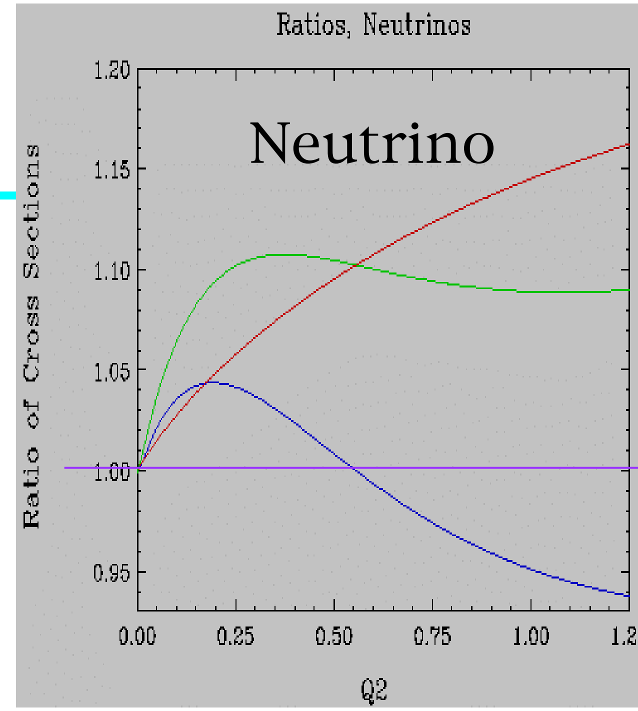
--> Get right shape

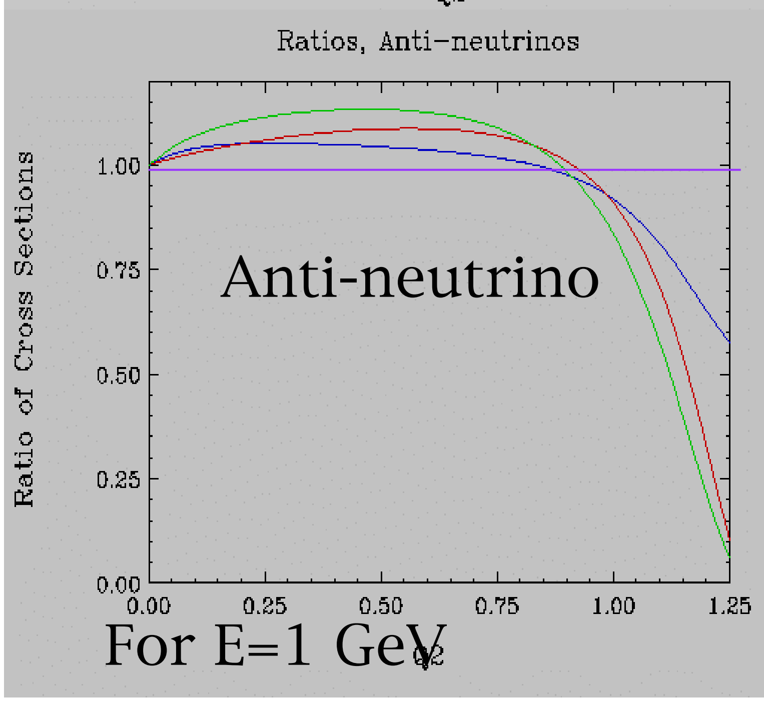
But wrong normalization of 10%

Wrong Gen /Best Form Factors (Ratio)



What about in Anti-Neutrino running (e.g. MiniBoone) Is there greater sensitivity to Gen or Ma -Greater sensitivity to Gen





Red Ma=1.1 (used by K2K)

/ Ma=1.02 (Ratio)

Green: If One Uses Two wrong Form Factors (used in K2K MC)

(Wrong Gen =0 +Wrong Ma=1.1) / Best Form Factors (Ratio)

Blue:

Wrong Gen /Best Form Factors (Ratio)

Updating Old Measurements of MA

Current Project - Howard Budd, Arie Bodek

Do a re-analysis of all previous neutrino differential Cross section data - versus Q2 (first focus on P and D data, where nuclear effects are small) and Re-extract Ma using the latest form factors as input.

Note that if one has perfect knowledge of all Vector Form Factors from Electron Scattering, one Can in principle fit these form factors within a Specific model. --> But

---> the Axial form factor CANNOT be Predicted reliably and must be extracted from data.

Reanalysis of

PARTICLES AND FIELDS

F

HIRD SERIES, VOLUME 26, NUMBER 3 1 AUGUST 1982

Study of the reaction $\nu_\mu d \rightarrow \mu^- p p_s$

K. L. Miller,* S. J. Barish,† A. Engl
Carnegie-Mellon University, P

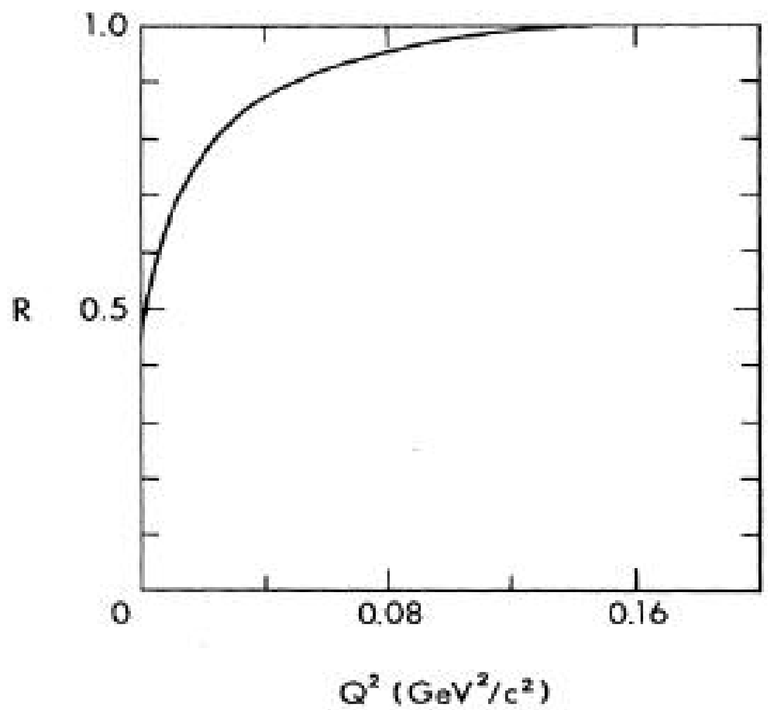


FIG. 3. Deuterium correction factor $R(Q^2)$.

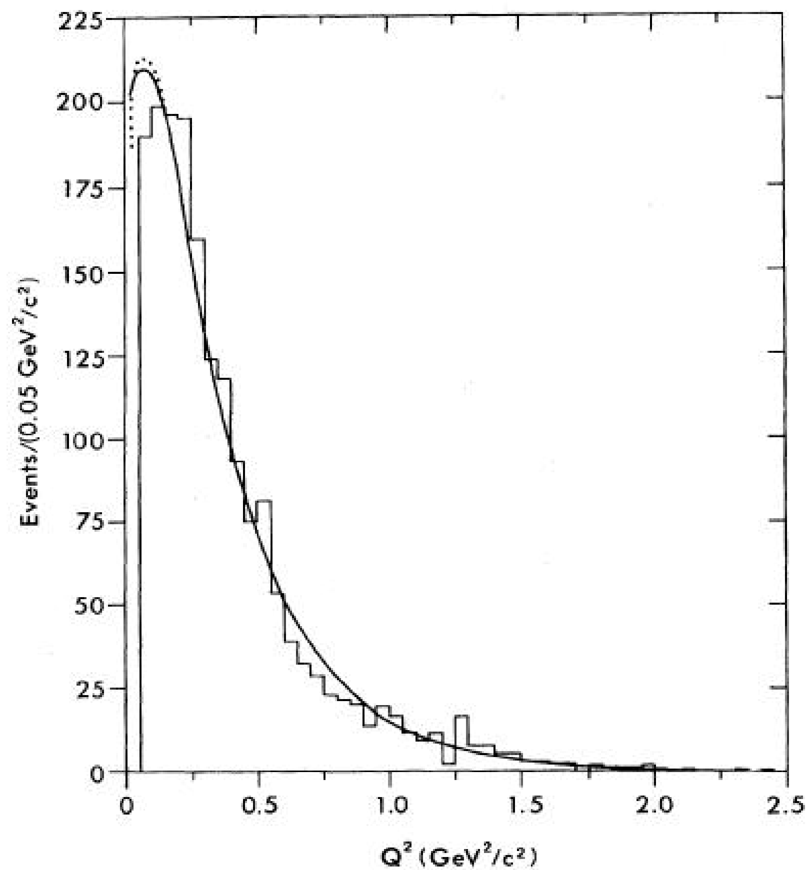


FIG. 4. Weighted Q^2 distribution. The solid curve is from a maximum-likelihood fit to the dipole model ($M_A = 1.00$ GeV/c^2). The dotted curve is from a fit to the AVMD model ($M_A = 1.11$ GeV/c^2).

Arie

STUDY OF THE REACTION $\nu_\mu d \to \mu^- p p_s$

TABLE I. Maximum-likelihood values of M_A (GeV/c^2) for each model.

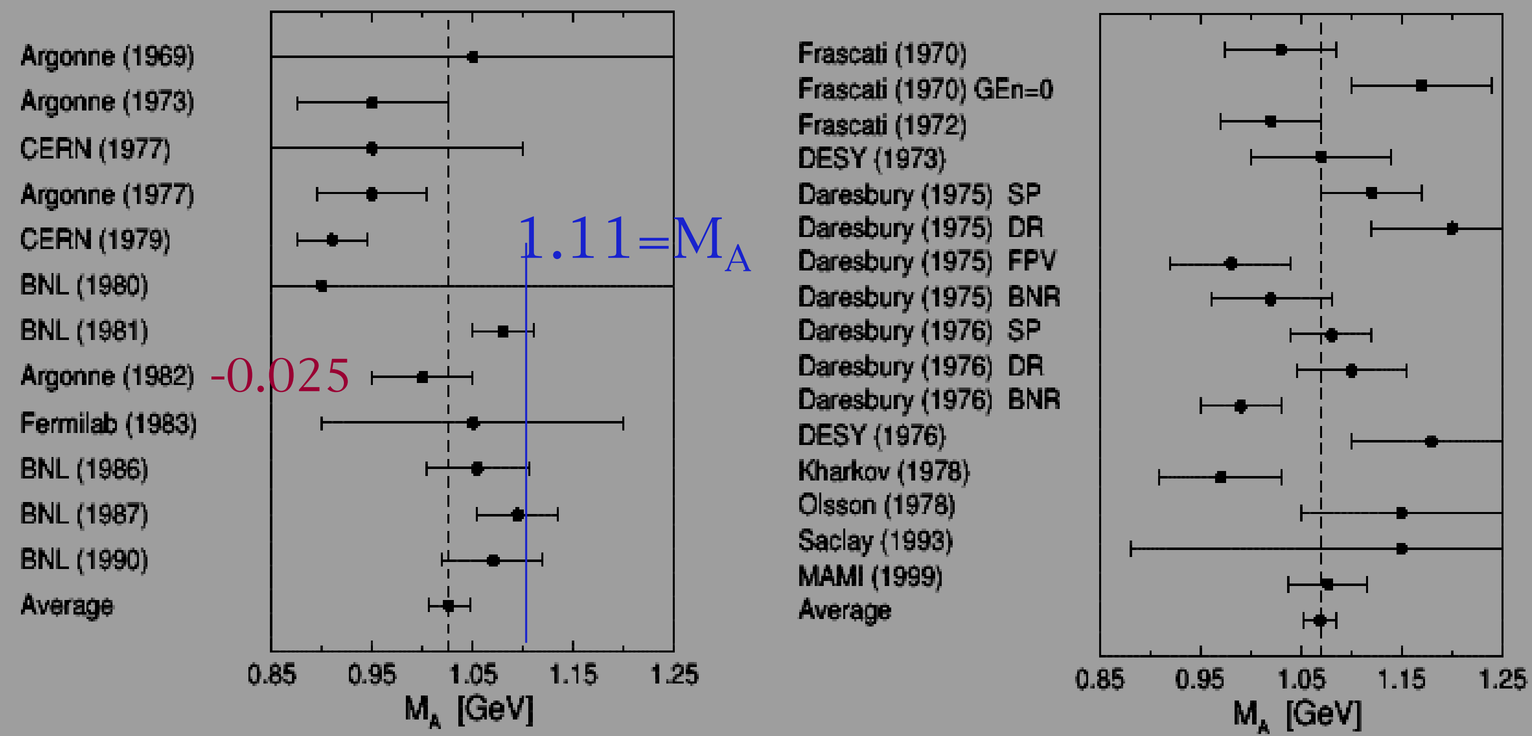
	Monopole	Dipole	Tripole	QM-AVMD
Rate	0.45±0.11	0.74±0.12	0.95±0.16	0.69±0.26
Shape	0.57±0.05	1.05±0.05	1.38±0.06	1.25±0.17
Total	0.55±0.05	1.03±0.05	1.35±0.07	1.20±0.17
Flux independent	0.54±0.05	1.00±0.05	1.31±0.07	1.11±0.16

Type in their dσ/dQ2 histogram. Fit with our best Knowledge of their parameters : Get M_A=1.11+-0.05 (A different central value, but they do event likelihood fit And we do not have their the event, just the histogram.

If we put is best knowledge of form factors, then we get M_A=1.085+-0.05 or ΔM_A= -0.025. So all their Values for M_A. should be reduced by 0.025

Axial structure of the nucleon   Hep-ph/0107088 (2001)

Véronique Bernard†, Latifa Elouadrhiri‡, Ulf-G Meißner§



For updated M_A expt. need to be reanalyzed with new g_A, and G_E^N

Difference

In Ma between

Electroproduction

And neutrino

Is understood

Figure 1. Axial mass M_A extractions. Left panel: From (quasi)elastic neutrino and antineutrino scattering experiments. The weighted average is $M_A = (1.026 \pm 0.021)$ GeV. Right panel: From charged pion electroproduction experiments. The weighted average is $M_A = (1.069 \pm 0.016)$ GeV. Note that value for the MAMI experiment contains both the statistical and systematical uncertainty; for other values the systematical errors were not explicitly given. The labels SP, DR, FPV and BNR refer to different methods evaluating the corrections beyond the soft pion limit as explained in the text.

M_A from neutrino expt. No theory corrections needed

Using these data we get ΔM_A to update to for latest ga+form factors.
(note different experiments have different neutrino energy Spectra, different fit region, different targets, so each experiment requires its own study).

A Pure Dipole analysis, with ga=1.23 (Shape analysis)

- if redone with best know form factors --> ΔM_A = -0.055

(I.e. results need to be reduced by 0.055)

for different experiments can get ΔM_A from -0.025 to -0.060

1.Change ga=1.23 to best known ga= 1.267 (shape analysis)

---> ΔM_A = + 0.005

2. Dipole-> better Gmn, Gep, Gmp --> ΔM_A = -0.025

3. Gen=0-> non zero Gen ---> ΔM_A = -0.035

Total -0.055

Acknowledgements -

Will Brooks, Jlab - Gmn Expert-----New jlab experiment for GMN is E94-017. It has much more sensitivity (in the sense of statistical information that influences a fit) than existing measurements, just not much more Q2 coverage. preliminary results this coming spring or summer, publication less than 1 year later.

And Andrei Semenov, - Kent State, Gen Expert----->New Jlab data on Gen are not yet available, but is important to confirm since non-zero Gen effect is large. The experiment is JLab E93-038. Data were taken in Jefferson Lab (Hall C) in October 2000/April 2001. Data analysis is in progress

The New Jlab Data on Gep/Gmp will help resolve the difference between the Cross Section and Polarization technique. However, it has little effect on the neutrino cross sections. For most recent results from Jlab see: hep-ph/0209243. Final results to be published soon,.

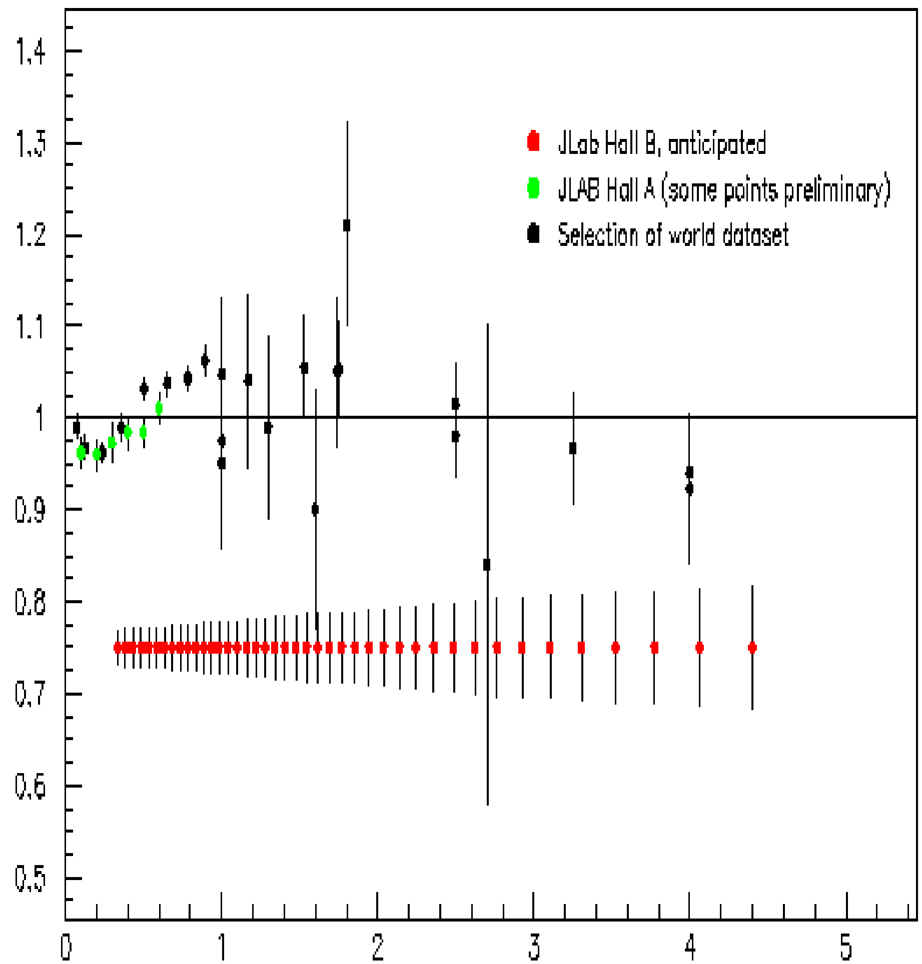
Thanks To: The following Experts (1)

Will Brooks, Jlab - Gmn brooksw@jlab.org

•High-precision low Q2 Gmn: nucl-ex/0107016 Precise Neutron Magnetic Form Factors; G. Kubon, et al , Phys.Lett. B524 (2002) 26-32

•Recent, moderate precision low Q2 data nucl-ex/0208007

•The best high Q2 data:
http://prola.aps.org/pdf/PRL/v70/i6/p718_1
http://prola.aps.org/pdf/PRL/v70/i6/p718_1
They will have a new Gmn measurement from Q2=0.2 or 0.3 out to Q2 approaching 5 GeV2. plot of the expected data quality versus old data (shown as Ratio to Dipole).



The new jlab experiment for GMN is E94-017. It has much more sensitivity (in the sense of statistical information that influences a fit) than existing measurements, just not much more Q2 coverage. The errors will be smaller and will be dominated by experimental systematic errors; previous measurements were dominated by theory errors that could only be estimated by trying different models (except for the new data below 1 GeV). The new experiment's data will dominate any chi-squared fit to previous data, except for the new high-precision data below 1 GeV2 where it will rival the new data. Time scale for results: preliminary results this coming spring or summer, publication less than 1 year later.

Thanks To: The following Experts (2)

Gen: Andrei Semenov, - Kent State, semenov@jlab.org

Who provided tables from (Dr. J.J.Kelly from Maryland U.) on Gen, Gmn, Gen, Gmp .

The new Jlab data on Gen are not yet available, but is important to confirm since non-zero Gen effect is large. The experiment is JLab E93-038. Data were taken in Jefferson Lab (Hall C) in October 2000/April 2001. Data analysis is in progress

The New Jlab Data on Gep/Gmp will help resolve the difference between the Cross Section and Polarization technique. However, it has little effect on the neutrino cross sections. For most recent results from Jlab see: hep-ph/0209243

Neutrino Cross Section Data

http://neutrino.kek.jp/~sakuda/nuint02/

charged current quasi-elastic neutrino
Gargamelle 79 ccqe.nu.ggm79.vec,
ccqe.nub.ggm79.vec -- CF3Br target

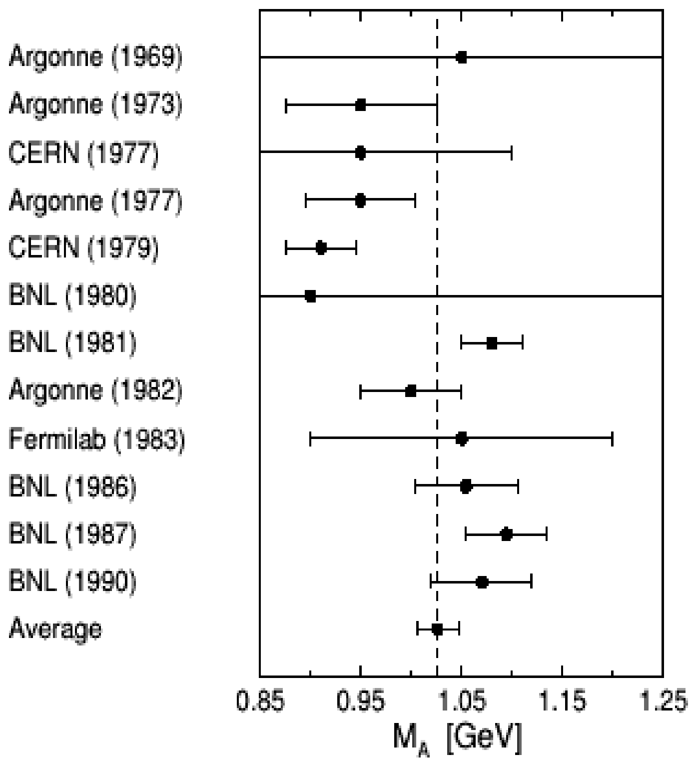
ccqe.serpukhov85.vec,
ccqe.nub.serpukov.vec -- Al. target

charged current quasi-elastic neutrino
Gargamelle 77 ccqe.ggm77.vec
- Propane-Freon

ccqe.nu.skat90.vec
ccqe.nub.skat90.vec -- CF3Br

ccqe.nu.bebc90.vec -- D2

Cross section in units of 10^(-38) cm^2.
E Xsection X +-DX Y +-DY or (x1, x2) y +-dy

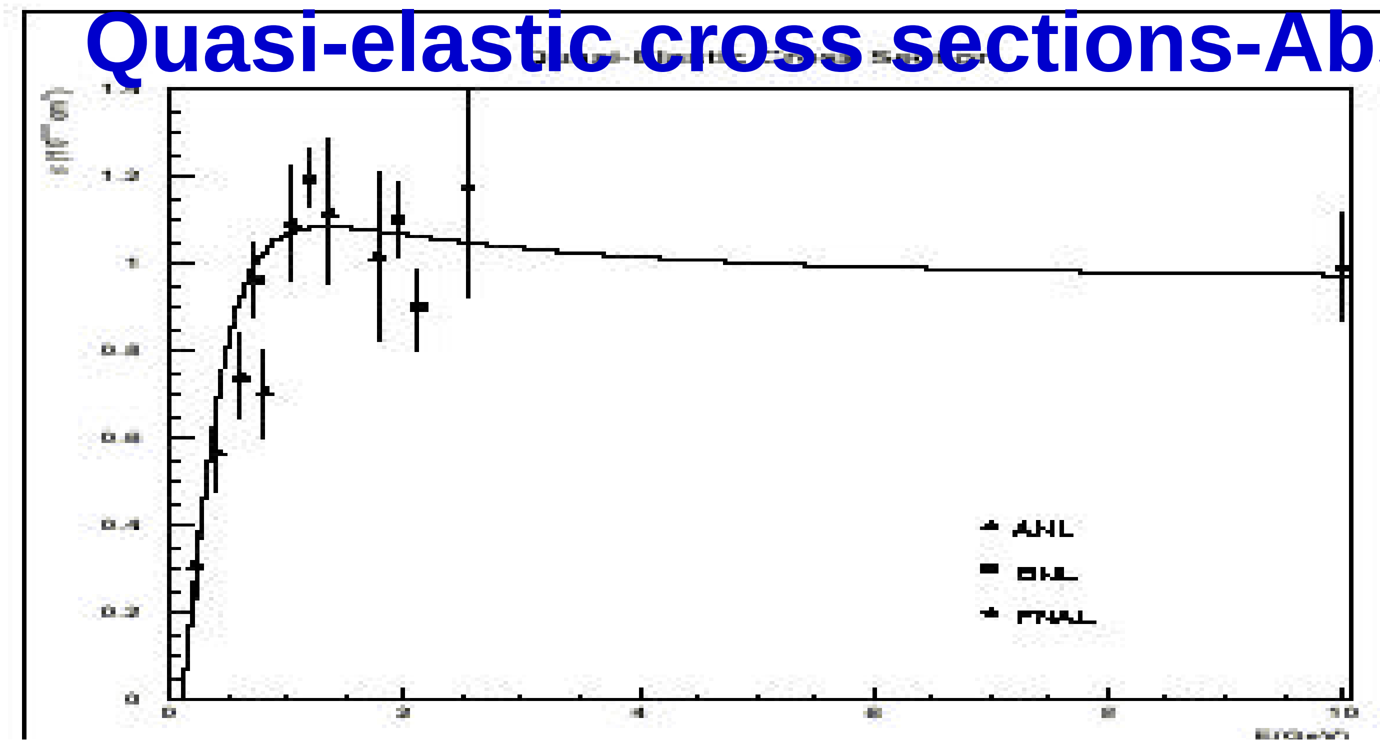


Note more recent

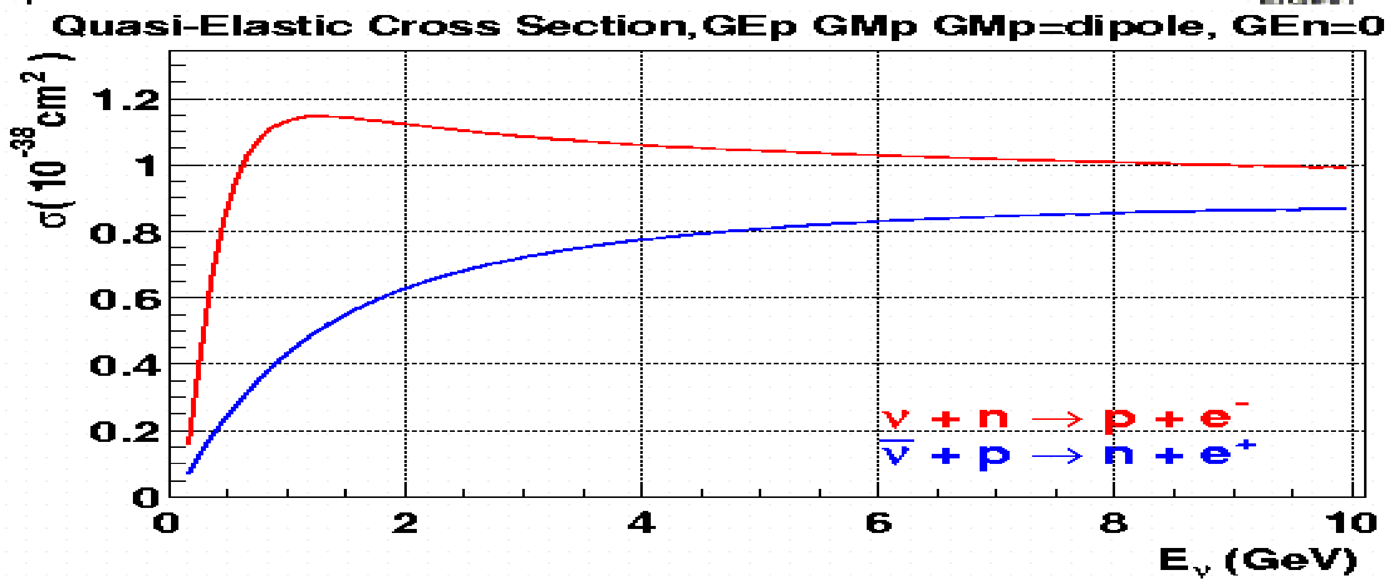
M_A is more reliable-

Better known flux

Examples of Low Energy Neutrino Data: Quasi-elastic cross sections-Absolute



$\sigma_{\text{quasi-elastic}}$ neutrinos On Neutrons From MINOS Paper and MINOS dipole MC



$\sigma_{\text{quasi-elastic}}$ neutrinos on Neutrons-Dipole

$\sigma_{\text{quasielasti}}$ Antineutrinos on Protons -Dipole

elas_D0DD.pict

By C.H. Llewellyn Smith (SLAC). SLAC-PUB-0958 Phys.Rept.3:261,1972

$$F_A(q^2) = -1.23/\left(1 - \frac{q^2}{M_A^2}\right)^n \quad (3.24)$$

Old g_A

Replace by New g_A

(5) Isotriplet current

$$F_V^1(q^2) = \left[F_1^p(q^2) - F_1^n(q^2)\right] = \text{Dirac electromagnetic isovector form factor.} \quad (3.15)$$

$$\xi = \mu_p - \mu_n = 3.71 \quad (\mu = \text{anomalous magnetic moment})$$

$$F_V^2(q^2) = \frac{\mu_p F_2^p(q^2) - \mu_n F_2^n(q^2)}{\mu_p - \mu_n} = \text{Pauli electromagnetic isovector form factor.}$$

In terms of the Sachs form factors

$$F_V^1(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1}\left[G_E^V(q^2) - \frac{q^2}{4M^2} G_M^V(q^2)\right]$$

$$\xi F_V^2(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1}\left[G_M^V(q^2) - G_E^V(q^2)\right] \quad (3.16)$$

UPDATES this talk

Experimentally, the G's are described to within ± 10% by:

This assumes

Dipole form factors

$G_E^N=0$

$$G_E^V(q^2) = \frac{1}{\left(1 - \frac{q^2}{0.71\ \text{GeV}^2}\right)^2}$$

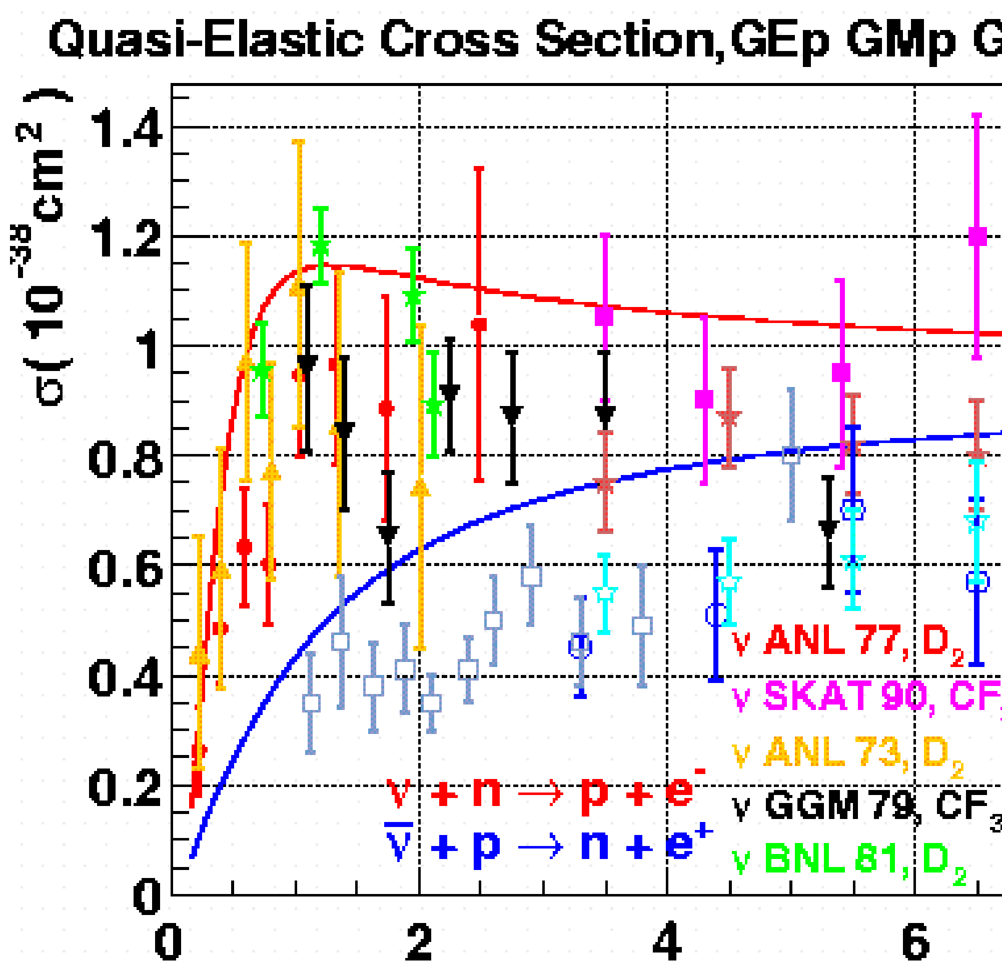
Replace by $G_E^V = G_E^P - G_E^N$

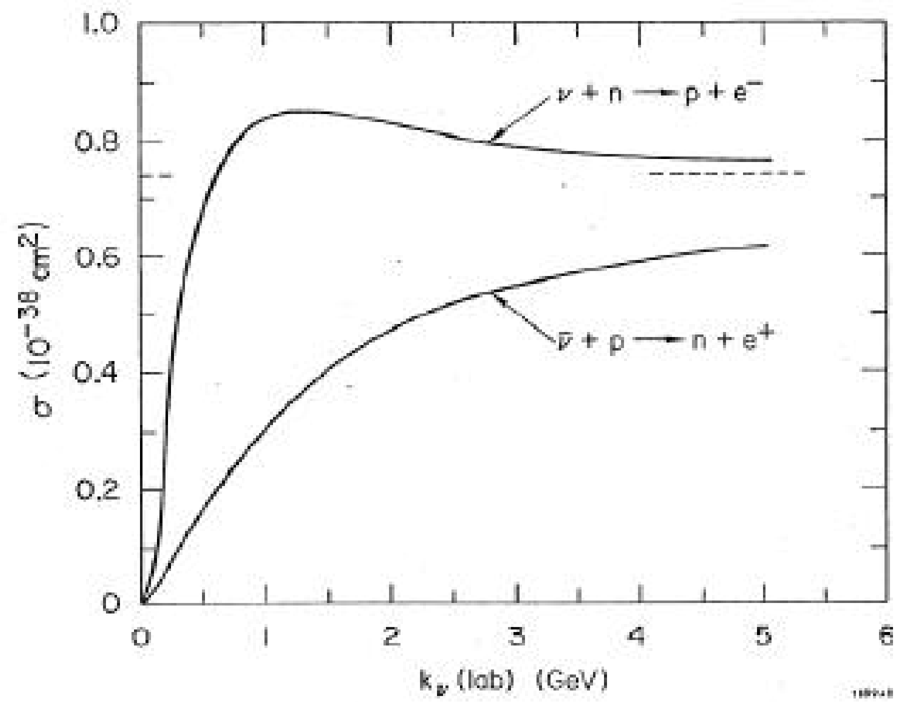
-->note G_E^N is POSITIVE

$$G_M^V(q^2) = \frac{1 + \mu_p - \mu_n}{\left(1 - \frac{q^2}{0.71\ \text{GeV}^2}\right)^2}$$

Replace by $G_M^V = G_M^P - G_M^N$

-->note G_M^N is NEGATIVE





10. Cross sections for the quasielastic process in the conventional theory with $m = 0$ and dipole forms

$$\frac{F(0)}{\left(1 - \frac{q^2}{0.73\ \mathrm{GeV}^2}\right)^2}$$

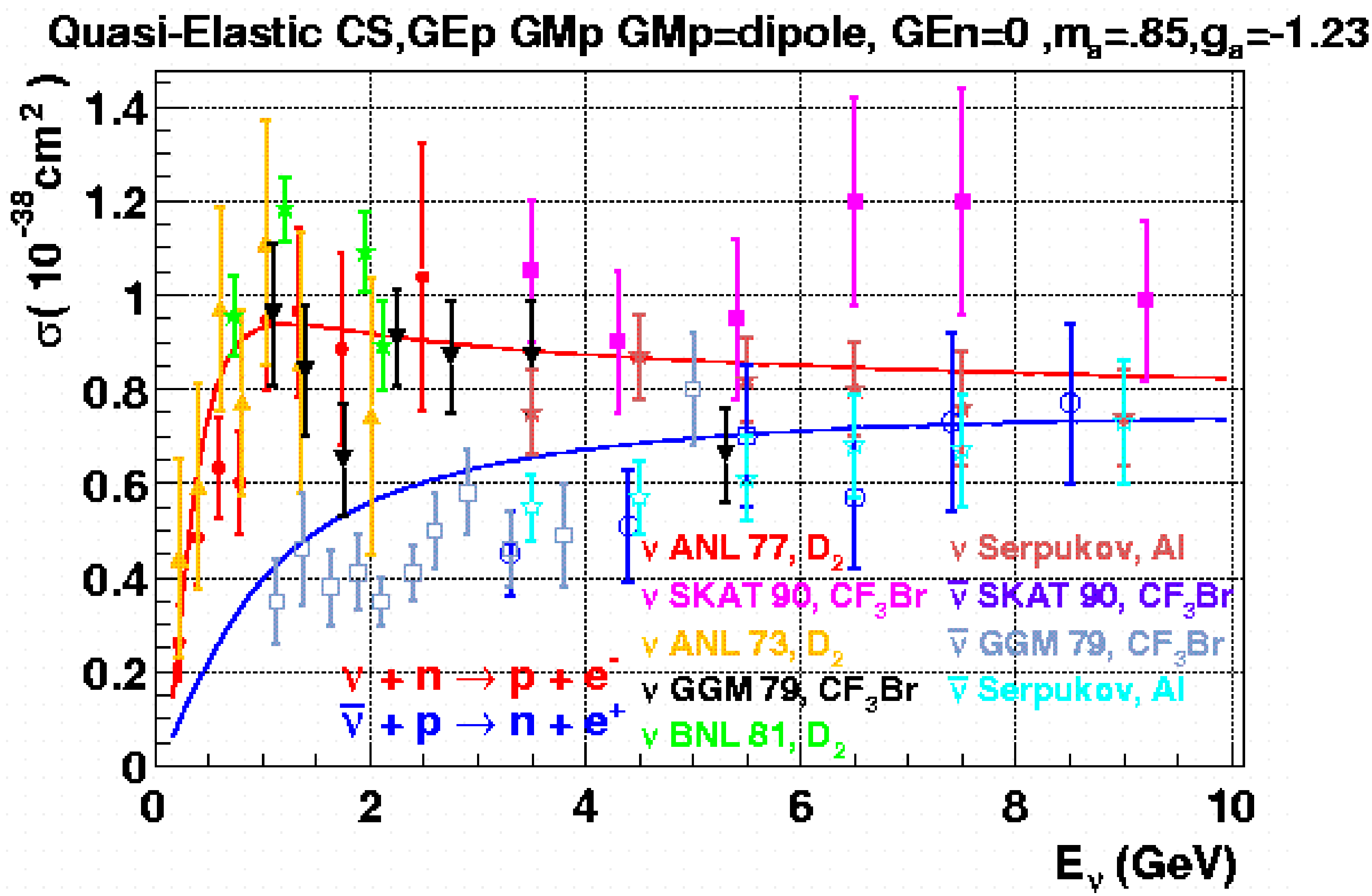
for the form factors F_A and $F_V^{1,2}$ LL12 (the dotted line is the limit for σ_ν and $\sigma_{\bar{\nu}}$ as $E \to \infty$).

Old LS results with

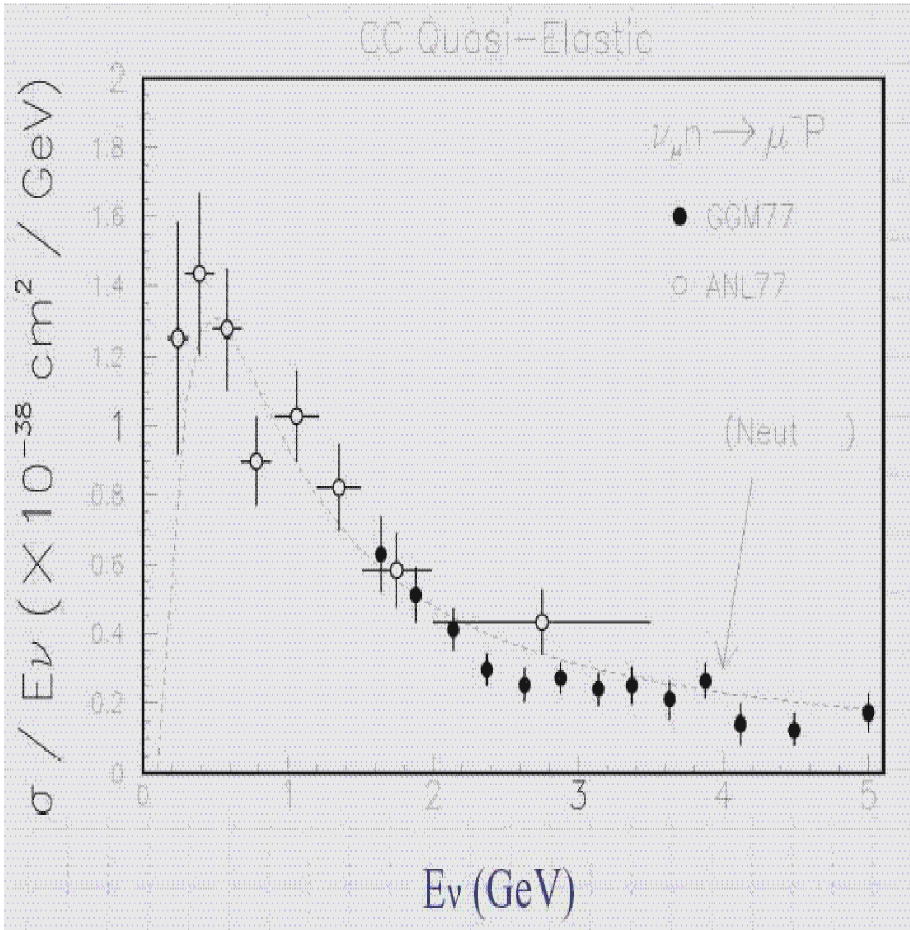
Old ga=-1.23 and

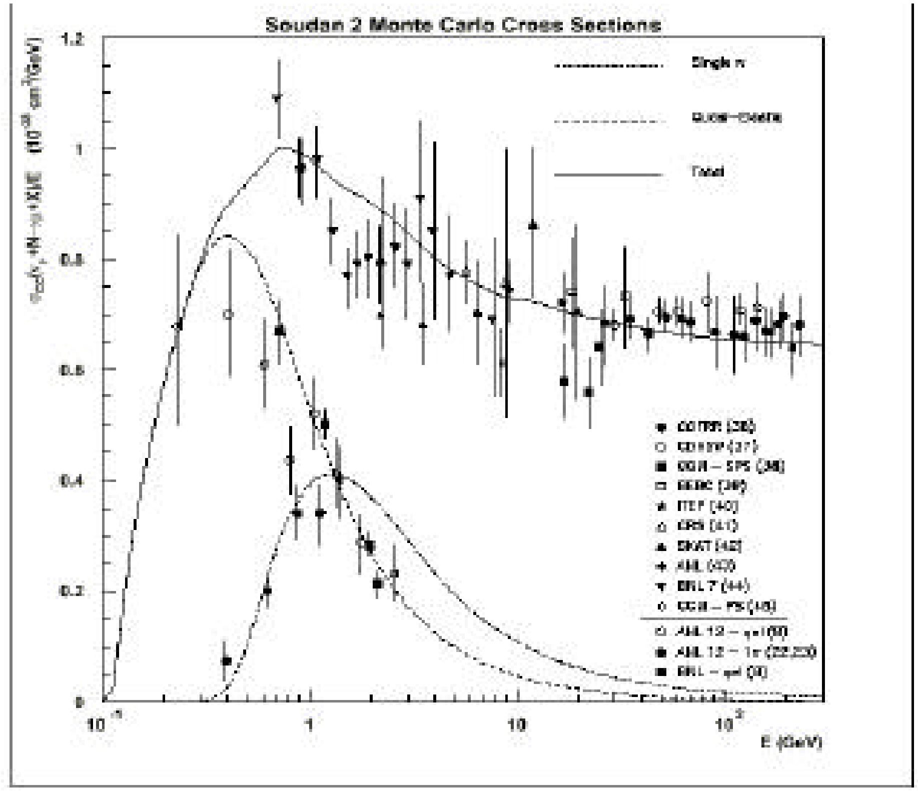
MA below)

Compare to Original Llewellyn Smith Prediction



Examples of Low Energy Neutrino Data: cross sections divided by Energy





σ_{quasit}/E on neutron target
Quasielastic only

σ_{tot}/E on Iso-scalar target, with
Different contributions
Quasi-elastic important in the
0-4 GeV region