The Adler Sum Rules, Quark-Hadron Duality, and Axial and Vector Nucleon Form Factors

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We investigate the Adler sum rules within the context of Quark-Hadron duality. We extract relationships between the axial and vector nucleon form factors in neutrino-nucleon and electron-nucleon scattering. Using these relationships, we make predictions for the electric and axial form factors of the neutron. These predictions are useful in explaining the Q^2 distributions observed in recent neutrino data at low energies.

INTRODUCTION

The Adler sum rules [1] for neutrino-nucleon scattering were first published in in 1966. These sum rules (which include both the elastic and inelastic contributions) are based on current-algebra. Therefore, they are expected to be exact and hold for all values of momentum transfer Q^2 (from zero to infinity). The Adler sum rules for the structure function F_2 are equivalent (at high Q^2) to the statement that the number of valence u quarks minus the number of valence d quarks is 2-1=1.

In contrast, Quantum Chromodynamics (QCD) sum rules break down at very low Q^2, and are expected to apply above $Q^2 \approx 1-2$ GeV/c^2. Even at these higher values of Q^2, QCD sum rules have Q^2 dependent logarithmic corrections. Example of QCD sum rules with Q^2 dependent logarithmic corrections include (a) the energy-momentum sum rule (integral of quark fractional momentum ≈ 0.5), (b) The Bjorken sum rule for polarized structure functions, (c) The Gottfried sum rule(for the difference in electron/muon scattering from neutrons and protons) and (d) the Gross-Llewyllyn-Smith sum rule in neutrino scattering (number of valence quarks equal to 3).

A successful model of structure functions and nucleon form factors in the Q^2 region of 0 to 5 GeV/c^2 should satisfy QCD sum rules at higher Q^2, and the Adler sum rules over the entire region of Q^2. This region, which spans the transition between the hadron and quark pictures of the nucleon, is of interest to neutrino oscillations experiments for which the neutrino energies are in the few GeV region. This energy region is also being investigated in electron-nucleon scattering experiments at Jefferson Laboratory.

Recently, in has been shown by Bodek and Yang [2] that with the inclusion of target mass corrections [3] and higher order NNLO QCD corrections, the standard Parton Distribution Functions (PDFs) provide a very good description of deep inelastic scattering above $Q^2 \approx 1$ GeV/c^2 over all regions of x. It was also shown that in NLO analyses, the missing NNLO higher order terms can be parametrized as small power corrections (higher twist) a formulated by renormalon models [2]. Within this framework it was shown that QCD predictions in NLO (with the addition of target mass corrections and the renormalon higher twist terms) provide a good description of the average value the structure functions in the resonance region [2]. Since the Adler and QCD sum rules include both elastic/quasielastic and inelastic contributions, they can be used to relate the inelastic and elastic vector and axial form factors of the nucleon.

ELECTRON-NUCLEON SCATTERING

The differential cross section for scattering of an unpolarized charged lepton with an incident energy E_0, final energy E' and scattering angle θ can be written in terms of the structure functions $F_1 = MW_1$ and $F_2 = \nu W_2$ as:

$$\frac{d^2\sigma}{d\Omega dE'}(E_0, E', \theta) = \frac{4\alpha^2 E'^2}{Q^4}\cos^2(\theta/2)\left[F_2(x,Q^2)/\nu + 2\tan^2(\theta/2)F_1(x,Q^2)/M\right]$$

where α is the fine structure constant, M is the nucleon mass, $\nu = E_0 - E'$ is energy of the virtual photon which mediates the interaction, $Q^2 = 4E_0E'\sin^2(\theta/2)$ is the invariant four-momentum transfer squared, and $x = Q^2/2M\nu$ is a measure of the longitudinal momentum carried by the struck partons.

Alternatively, one could view this scattering process as virtual photon absorption. Unlike the real photon, the virtual photon can have two modes of polarization. In terms of the cross section for the absorption of transverse (σ_T) and longitudinal (σ_L) virtual photons, the differential cross section can be written as,

$$\frac{d^2\sigma}{d\Omega dE'} = \Gamma\left[\sigma_T(x,Q^2) + \epsilon\sigma_L(x,Q^2)\right] \quad (1)$$

where,

$$\Gamma = \frac{\alpha K E'}{4\pi^2 Q^2 E_0}\left(\frac{2}{1-\epsilon}\right) \quad (2)$$

$$\epsilon = \left[1 + 2(1+\frac{Q^2}{4M^2x^2})tan^2\frac{\theta}{2}\right]^{-1} \quad (3)$$

$$K = \frac{2M\nu - Q^2}{2M}. \tag{4}$$

The quantities Γ and ϵ represent the flux and the degree of longitudinal polarization of the virtual photons respectively. The quantity R, is defined as the ratio σ_L/σ_T, and is related to the structure functions by,

$$R(x,Q^2) = \frac{\sigma_L}{\sigma_T} = \frac{F_2}{2xF_1}(1+\frac{4M^2x^2}{Q^2}) - 1 = \frac{F_L}{2xF_1} \tag{5}$$

where F_L is called the longitudinal structure function. The structure functions are expressed in terms of σ_L and σ_T as follows:

$$F_1 = \frac{MK}{4\pi^2\alpha}\sigma_T, \tag{6}$$

$$F_2 = \frac{\nu K(\sigma_L + \sigma_T)}{4\pi^2\alpha(1+\frac{Q^2}{4M^2x^2})} \tag{7}$$

$$F_L(x,Q^2) = F_2\left(1+\frac{4M^2x^2}{Q^2}\right) - 2xF_1. \tag{8}$$

or,

$$2xF_1 = F_2\left(1+\frac{4M^2x^2}{Q^2}\right) - F_L(x,Q^2). \tag{9}$$

In the elastic limit, i.e. $x=1$, these structure functions are related to the Sach's elastic form factors (G_M, G_E) by:

$$2xF_1^{em}(x,Q^2) = x^2 G_M^2(Q^2)\delta(x-1) \tag{10}$$

$$F_2^{em}(x,Q^2) = x\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau}\delta(x-1) \tag{11}$$

where $\tau = Q^2/4M_p^2$. The elastic form factors are normalized to $G_M^p(0) = \mu_p = \kappa_p + 1 = 2.7928$ (the magnetic moment of the proton), $G_E^p(0)$=1 (the charge of the proton), $G_M^n(0) = \mu_n = \kappa_n + 1 = -1.913$ (the magnetic moment of the neutron), $G_E^n(0)$=0 (the charge of the neutron)

The above relationships between the elastic form factors and the inelastic structure functions (equations 10 and 11) yield (using equation 9) a very simple expression for R at $x=1$.

$$R_{em}(x=1,Q^2) = \frac{4M^2}{Q^2}\left(\frac{G_E^2}{G_M^2}\right) \tag{12}$$

Integrating the differential cross section over E' gives:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\frac{1}{1+2E_0/M_p\sin^2(\theta/2)} \tag{13}$$

$$\left(\frac{G_E^2+\tau G_M^2}{1+\tau} + 2\tau G_M^2\tan^2(\theta/2)\right) \tag{14}$$

$$= \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\frac{1}{1+2E_0/M_p\sin^2(\theta/2)} \tag{15}$$

$$\left(\frac{1}{1+\tau}\right)\left(G_E^2 + \frac{\tau}{\epsilon}G_M^2\right) \tag{16}$$

and $\epsilon = \left(1+2(1+\tau)\tan^2(\theta/2)\right)^{-1}$ is the longitudinal polarization of the virtual photon.

At low Q^2 a reasonable description of the proton and neutron elastic form factors is given by the dipole approximation. The dipole approximation is a lowest-order attempt to incorporate the non-zero size of the proton into the form factors. If we also assume that the proton has a simple exponential spatial charge distribution:

$$\rho(r) = \rho_0 e^{-r/r_0} \tag{17}$$

where r_0 is the scale of the proton radius. The form factors are related, in the non-relativistic limit, to the Fourier transform of the charge and magnetic moment distribution. If it is also assumed that the magnetic moment distribution has the same spatial dependence as the charge distribution (*i.e.*, form factor scaling), we get the dipole approximation to the form factors:

$$\begin{aligned} G_D(Q^2) &\equiv \frac{1}{(1+Q^2r_0^2)^2} \\ &\equiv \frac{1}{(1+Q^2/M_V^2)^2} \\ &= G_E^p(Q^2) \\ &= G_M^p(Q^2)/\mu_p \\ &= G_M^n(Q^2)/\mu_n \end{aligned} \tag{18}$$

and $G_E^n = 0$. Previous measurements of e-p and e-n elastic scattering have indicated a best fit value of $r_0^2 = (0.24\text{ fm})^2 = 1/0.71\ (\text{GeV})^{-2}$, indicating an rms radius of $\sqrt{\langle r^2\rangle} \approx 0.81$ fm. Measurements of G_E^p, G_M^p, and G_M^n agree with the dipole approximation only for $Q^2 < 1\ \text{GeV}^2$. In this communication we use updated fits to the form factor by Bodek, Budd and Arrington [8] (BBA2003).

NEUTRINO-NUCLEON SCATTERING

We now expand the discussion to include neutrino quasielastic scattering. Here we use the notation of C.H. Llewellyn Smith [4]

$$q^2 = q_0^2 - \vec{q}_3^2 = -4E_0E'\sin^2\frac{\theta}{2} = -Q^2 \tag{19}$$

The hadronic current for QE neutrino scattering is given by [4]

$$\begin{aligned} &< p(p_2)|J_\lambda^+|n(p_1)> = \\ &\quad \overline{u}(p_2)\left[\gamma_\lambda F_V^1(q^2) + \frac{i\sigma_{\lambda\nu}q^\nu\xi F_V^2(q^2)}{2M}\right. \\ &\quad\quad \left. + \gamma_\lambda\gamma_5 F_A(q^2) + \frac{q_\lambda\gamma_5 F_P(q^2)}{M}\right]u(p_1), \end{aligned}$$

where $q = k_\nu - k_\mu$, $\xi = (\mu_p - 1) - \mu_n$, and $M = (m_p + m_n)/2$. Here, μ_p and μ_n are the proton and neutron magnetic moments. We assume that there are no second class currents, so the scalar form factor F_V^3 and the tensor form factor F_A^3 need not be included. Using the above current, the cross section is

$$\frac{d\sigma^{\nu,\,\overline{\nu}}}{dq^2} = \frac{M^2 G_F^2 cos^2\theta_c}{8\pi E_\nu^2}\times \left[A(q^2) \mp \frac{(s-u)B(q^2)}{M^2} + \frac{C(q^2)(s-u)^2}{M^4}\right],$$

where

$$A(q^2) = \frac{m^2 - q^2}{4M^2}\left[\left(4 - \frac{q^2}{M^2}\right)|F_A|^2 - \left(4 + \frac{q^2}{M^2}\right)|F_V^1|^2 - \frac{q^2}{M^2}|\xi F_V^2|^2\left(1 + \frac{q^2}{4M^2}\right) - \frac{4q^2 Re F_V^{1*}\xi F_V^2}{M^2}\right],$$

$$B(q^2) = -\frac{q^2}{M^2} Re F_A^*(F_V^1 + \xi F_V^2),$$

$$C(q^2) = \frac{1}{4}\left(|F_A|^2 + |F_V^1|^2 - \frac{q^2}{M^2}\left|\frac{\xi F_V^2}{2}\right|^2\right).$$

Although we have have not shown terms of order $(m_l/M)^2$, and terms including $F_P(Q^2)$ (which is multiplied by $(m_l/M)^2$), these terms are included in our calculations [4].) The form factors $F_V^1(Q^2)$ and $\xi F_V^2(Q^2)$ are given by:

$$F_V^1(Q^2) = \frac{G_E^V(Q^2) + \tau G_M^V(Q^2)}{1+\tau}, \tag{20}$$

$$\xi F_V^2(Q^2) = \frac{G_M^V(Q^2) - G_E^V(Q^2)}{1+\tau}. \tag{21}$$

Note that from CVC we can relate $G_E^V(Q^2)$ and $G_M^V(Q^2)$ the electron scattering form factors $G_E^p(Q^2)$, $G_E^n(Q^2)$, $G_M^p(Q^2)$, and $G_M^n(Q^2)$:

$$G_E^V(Q^2) = G_E^p(Q^2) - G_E^n(Q^2) \tag{22}$$

$$G_M^V(Q^2) = G_M^p(Q^2) - G_M^n(Q^2). \tag{23}$$

At low Q^2 the axial form factor F_A and the pseudoscalar form factor F_P (related to F_A by PCAC) are given by dipole approximation:

$$F_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}, \tag{24}$$

$$F_P(Q^2) = \frac{2M^2 F_A(Q^2)}{M_\pi^2 + Q^2}. \tag{25}$$

g_A	-1.267
G_F	1.1803×10^{-5} GeV^{-2}
$\cos\theta_c$	0.9740
μ_p	2.793 μ_N
μ_n	-1.913 μ_N
ξ	3.706 μ_N
M_V^2	0.71 GeV^2
M_A	1.014 GeV

TABLE I. The most recent values of the parameters used in the our calculations.

In the expression for the cross section, $F_P(q^2)$ is multiplied by $(m_l/M)^2$. Therefore, in muon neutrino interactions, this effect is very small except at very low energy, below 0.2 GeV.

The axial form factor is normalized to measurements from neutron decay ($F_A(0) = g_A = -1.267$). At low Q^2, M_A can be extracted from pion electroproduction data. The theoretically corrected value of M_A from pion electroproduction [7] is 1.014 $\pm$ 0.016 GeV. This value is in agreement with the updated value [8] of M_A 1.00 $\pm$ 0.020 GeV, which has been re-extracted from previous neutrino data using current values for g_A and the updated 'BBA-2003' vector form factors.

Table I summarizes the most up to date values of the coupling constants and magnetic moments that we use in our calculations. Note that G_E^p, G_M^p, and G_E^n are positive, while G_M^n and the axial form factor F_A are negative.

Experimental investigation of deviations from the dipole form of $F_A(Q^2)$ will be done in the next generation high precision neutrino quasieastic scattering experiments such as MINERvA [24]. However, as we show below, current experiments such as K2K, MiniBoone and NOMAD, are already affected by non-dipole corrections to both the vector and axial form factors.

For neutrino scattering, inelastic structure functions (in the elastic limit) are related to the quasielastic form factors (G_M^V, G_E^V) as follows:

$$2xF_1^{\nu-vec}(x,Q^2) = x^2[G_M^V]^2(Q^2)\delta(x-1) \tag{26}$$

$$F_2^{\nu-vec}(x,Q^2) = x\frac{[G_E^V(Q^2)]^2 + \tau[G_M^V(Q^2)]^2}{1+\tau}\delta(x-1) \tag{27}$$

$$2xF_1^{\nu-axi}(x,Q^2) = x^2\frac{(1+\tau)[FA(Q^2)]^2}{\tau}\delta(x-1) \tag{28}$$

$$F_2^{\nu-ax}(x,Q^2) = xF_A(Q^2)]^2\delta(x-1) \tag{29}$$

The above relationships between the elastic form factors and the inelastic structure functions (equations 26 and 27) yield (using equation 9) a very simple expression for R_ν at $x = 1$.

$$R_\nu^{vec}(x=1,Q^2) = \frac{4M^2}{Q^2}\left(\frac{[G_E^V(Q^2)]^2}{[G_M^V(Q^2)]^2}\right) \tag{30}$$

$$R_\nu^{ax}(x=1,Q^2) = 0 \tag{31}$$

THE ADLER SUM RULES

In deep inelastic charged current scattering from quarks, the vector and axial couplings are equal (V-A). Similarly, in electron scattering from quarks (vector current), there is a well defined ratio (R) between electric and magnetic scattering from point-like Dirac quarks. All of these relations break down at low momentum transfers. For example, in quasielastic and resonance production at low momentum transfers, the measured charge and anomalous magnetic moments of the neutron and proton imply a ratio of electric and magnetic scattering for the vector current on nucleons is not the same as for point like spin 1/2 quarks. Similarly, from neutron decay, we know that $|g_a(Q^2=0)| = 1.267$ instead of 1.0, so vector and axial scattering are not equal at Q^2=0.

There are theoretical calculations (e.g. lattice gauge calculations, consituent quark models, etc.), of the anomalous magnetic moments of the proton and neutron and also of the Q^2 dependence of the electric, magnetic, and axial form factors of the neutron and proton. The calculations of of the magnetic form factors at Q^2=0 can be compared to the measured charge and anomalous (vector and axial) magnetic moments. The calculated slopes at low Q^2 are related to the measured mean square charge radii of the proton and neutron. The dipole form assumes that the charge and magnetization distributions of the various types of quarks and antiquarks have an exponential form. The fact that G_E^p/G_D is found to be close to 1.0 at low Q^2 implies that the charge and magnetization distribution of the proton are the same. For Q^2 above $\approx$ 0.5-1.0 $(GeV/c)^2$ this non-relativistic picture breaks down.

Recent polarization transfer experiment at Hall A at Jefferson Lab find that at higher Q^2 the ratio G_E^p/G_D becomes much smaller than 1.0. This implies that the simple non-relativsitc picture breaks down, and more sophisticated models need to be used. Here we show that if quark-hadron duality is approximately valid above Q^2 of $\approx$ 0.5-1.0 $(GeV/c)^2$, the Adler sum rules predict that the dipole forms must break down in a way which is confirmed by current experimental data.

The Adler sum rules are integrals from $\nu = q_0 = 0$ to $\nu = \infty$ (including the elastic/quasielastic contribution). The functions α, β and γ as defined by Adler correspond to the structure functions W_1 ,W_2, and W_3 in modern notation. Here W_1^-=α^-, $W_2^-= \beta^-$ and W_3^-=γ^- refer to antineutrino structure functions, and W_1^+=α^+, W_2^+=β^+ and W_3^+=γ^+ refer to neutrino structure functions, respectively. If no subscripts are used then we refer to scattering on protons. Otherwise the subscripts p and n refer to scattering from protons and neutrons respectively. The superscripts *vec* and *ax* refer to vector and axial scattering respectively. There more than a dozen Adler sum rules for the three structure functions on neutrons and protons for strangeness-conserving and strangeness-changing processes. At high Q^2 all of the β sum rules are equivalent to the statement that the number of valence u quarks is =2.

The following are the Adler sum rules for strangeness conserving charged-current reactions for $\beta_{p\Delta S=0}$:

$$\int_0^\infty \left[\beta^{vec-}_{p\Delta S=0}(\nu,Q^2) - \beta^{vec+}_{p\Delta S=0}(\nu,Q^2)\right] d\nu = cos^2\theta_c$$

$$\int_0^\infty \left[\beta^{ax-}_{p\Delta S=0}(\nu,Q^2) - \beta^{ax+}_{p\Delta S=0}(\nu,Q^2)\right] d\nu = cos^2\theta_c$$

At high Q^2 the above vector/axial pairs of equations are equal to a difference between pairs of equations below (i.e. equations 1 - 2, or equations 3-4) which are equivalent to the statement that $(u_v - d_v = 1)$. (Here + denotes neutrinos and $-$ denotes antineutrinos)

1. $\beta^-_{p\Delta S=0}= cos^2\theta_c[u+\overline{d}]+(N = udd\ discrete)$
2. $\beta^+_{p\Delta S=0}= cos^2\theta_c[d+\overline{u}]+(\Delta^{++} = uuu\ broad)$
3. $\beta^+_{n\Delta S=0}= cos^2\theta_c[u+\overline{d}]\ +(P = uud\ discrete)$
4. $\beta^-_{n\Delta S=0}= cos^2\theta_c[d+\overline{u}]+(\Delta^- = ddd\ broad)$

At Q^2=0 some of the above equations have discrete contributions at $W = M$ (for strangeness conserving $\beta^-_{p\Delta S=0}$ and $\beta^+_{n\Delta S=0}$) .

To calculated the discrete contribution for the $W = M$ we express the following functions used by Adler [1] in the notation of C.H. Llewellyn Smith [4] (which we use here).

$$|F_V(Q^2)|^2 = |F_V^1(Q^2)|^2 - \frac{q^2}{M^2}\left|\frac{\xi F_V^2(Q^2)}{2}\right|^2$$

which reduces to:

$$|F_V(Q^2)|^2 = \frac{(G_E^V)^2(Q^2) + \tau(G_M^V(^2))^2}{(1+\tau)} \qquad (32)$$

and

$$G_M^V(Q^2) = F_V^1(Q^2) + \xi F_V^2(Q^2)$$

which reduces to:

$$G_M^V(Q^2) = G_M^p(Q^2) - G_M^n(Q^2). \qquad (33)$$

Note that $F_A(Q^2)$ in our notation is the same as $g_A(q^2)$ as defined by Adler, and $F_P(Q^2)$ in our notation is the same as $h_A(q^2)/M$ as defined by Adler, and $G_M^V(Q^2)$ in our notation is the same as $g_V(q^2)$ as defined by Adler. Adler defines q^2 as positive, while we define q^2 as negative and Q^2 as positive.

If we separate out the discrete quasielastic delta function contributions, the two equations for the strangeness-conserving ($\Delta S = 0$) Adler sum rules for β_p become:

$$|F_V(Q^2)|^2 + [1/(cos^2\theta_c)] \quad x$$

$$\int_{\nu_0}^{\infty} \left[W_{2p\Delta S=0}^{vec-}(\nu, Q^2) - W_{2p\Delta S=0}^{vec+}(\nu, Q^2)\right] d\nu = 1$$

$$|F_A(Q^2)|^2 + [1/(cos^2\theta_c)] \quad x$$

$$\int_{\nu_0}^{\infty} \left[W_{2p\Delta S=0}^{ax-}(\nu, Q^2) - W_{2p\Delta S=0}^{ax+}(\nu, Q^2)\right] d\nu = 1$$

Where the limits of the integrals are from pion threshold ν_0 where $W = M_\pi + M_P$ to $\nu_=\infty$.

The two equations for the strangeness-conserving ($\Delta S = 0$) Adler sum rules for α_p become:

$$[\tau][G_M^V(Q^2)]^2 + [1/(cos^2\theta_c)] \quad x$$

$$\int_{\nu_0}^{\infty} \left[W_{1p\Delta S=0}^{vec-}(\nu, Q^2) - W_{1p\Delta S=0}^{vec+}(\nu, Q^2)\right] d\nu = C_{I,1}$$

$$[1+\tau][F_A(Q^2)]^2 + [1/(cos^2\theta_c)] \quad x$$

$$\int_{\nu_0}^{\infty} \left[W_{1p\Delta S=0}^{ax-}(\nu, Q^2) - W_{1p\Delta S=0}^{ax+}(\nu, Q^2)\right] d\nu = C_{I,2}$$

Where the limits of the integrals are from pion threshold ν_0 where $W = M_\pi + M_P$ to $\nu_=\infty$.

PREDICTIONS FROM ADLER SUM RULE AND GLOBAL QUARK-HADRON DUALITY FOR $F_A(Q^2)$

If quark hadron duality holds for W_1 then the QCD PDFs describe average of the resonances (above pion production threshold) for Q^2 above ~ 0.5 $(GeV/c)^2$. Therefore the integral for the vector and axial parts of the inelastic contributions to the W_1 integral are equal. If we assume that this equality *also holds* for the contribution of the broad $\Delta^{++} = uuu$ 1.232 GeV resonance, then the Adler sum rule for vector and axial parts of W_1 implies also that the vector and axial components of $W_1^{elastic}$ are equal at large Q^2. If we use the current best prametrizations of the form factors [8] (BBA2003), we find that the axial and vector contributions become equal at $Q^2 \approx 0.5$ $(GeV/c)^2$. The requirement that this ratio of vector and axial contribution is also equal to 1.0

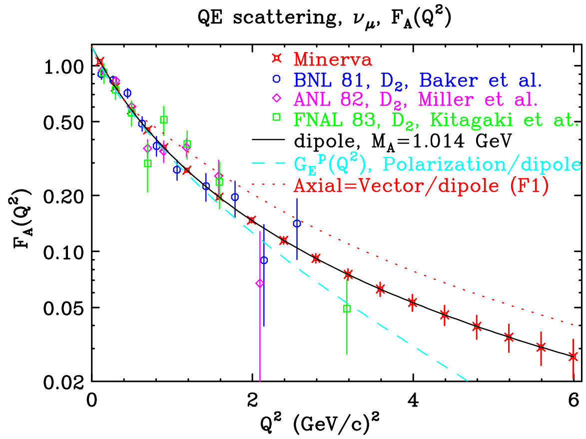

FIG. 1. Axial Form Factor: shown are $|F_A(Q^2)|$ values extracted from deuterium bubble chamber experiments using the $d\sigma/dQ^2$ from the papers of FNAL 1983 [12] BNL 1981 [9], and ANL 1982 [11]. Also shown is the expetations from a pure dipole form for $|F_A|$ with $M_A = 1.014$ GeV$/c^2$, (solid line) and our predictions (dotted line) assuming $V = A$ for Q^2 greater than ≈ 0.5 $(GeV/c)^2$ (equation 27). Also shown are the prediction from the consituent quark model of Wagenbrunn et. al. [19].

for higher Q^2 yields a prediction that the axial form factor is a factor of 1.4 times the dipole dipole parametrization at higher Q^2. Specifically, that for Q^2 greater than ≈ 0.5 $(GeV/c)^2$

$$[\tau][G_M^V Q^2)]^2 = [1+\tau][F_A(Q2)]^2$$

or

$$[F_A(Q^2)]^2 = [\tau/(1+\tau)][G_M^V(Q^2)]^2 \tag{34}$$

Where

$$[G_M^V(Q^2)]^2 = [G_M^p(Q^2) - G_M^n(Q^2)]^2$$

We compare this prediction the Dipole Parametrization for $F_A(Q^2)$ (equation 20), with $M_A = 1.014$ GeV$/c^2$, Note that we only use the dipole approximation as a scale (e.g. to show ratio to dipole in our plots). In all of our calculations, we use BBA2003 updated form factors. The dipole approximation for $G_M^p(Q^2)$ and $G_M^n(Q^2)$ yields the following simple expression:

$$[G_M^V(Q^2)]^2 \approx [4.706\ G_D(Q^2)]^2$$

.

Figure 1 and 2 shows the re-extraction of $F_A(Q^2)$ by Bodek, Budd and Arrington using published neutrino data. Here neutrino data for $d\sigma/dQ^2$ was used in combination with BBA2003 Vector Form Factors to extract values of the axial form factor versus momentum transfer. Figure 2 shows the same data as a ratio to the

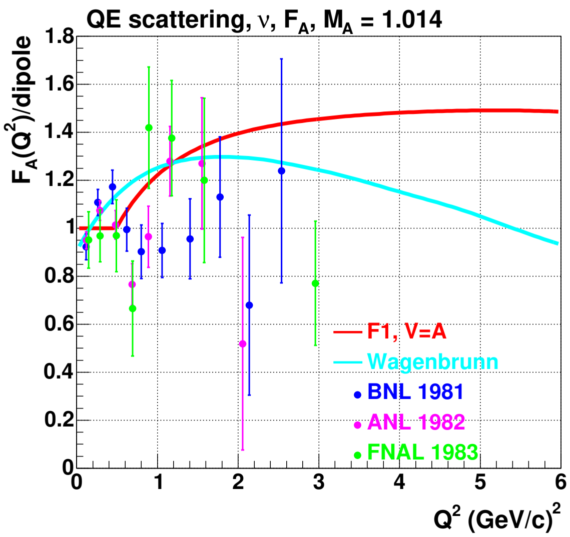

FIG. 2. Extracted values of $F_A(Q^2)/dipole$ (with $M_A = 1.014$ GeV$/c^2$) for deuterium bubble chamber experiments Baker *et al.* [9], Miller *et al.* [11], and Kitagaki *et al.* [12]. Also shown are our predictions (dotted line) assuming $V = A$ for Q^2 greater than ≈ 0.5 $(GeV/c)^2$ (equation 27). Also shown are the prediction from the consituent quark model of Wagenbrunn et. al. [19].

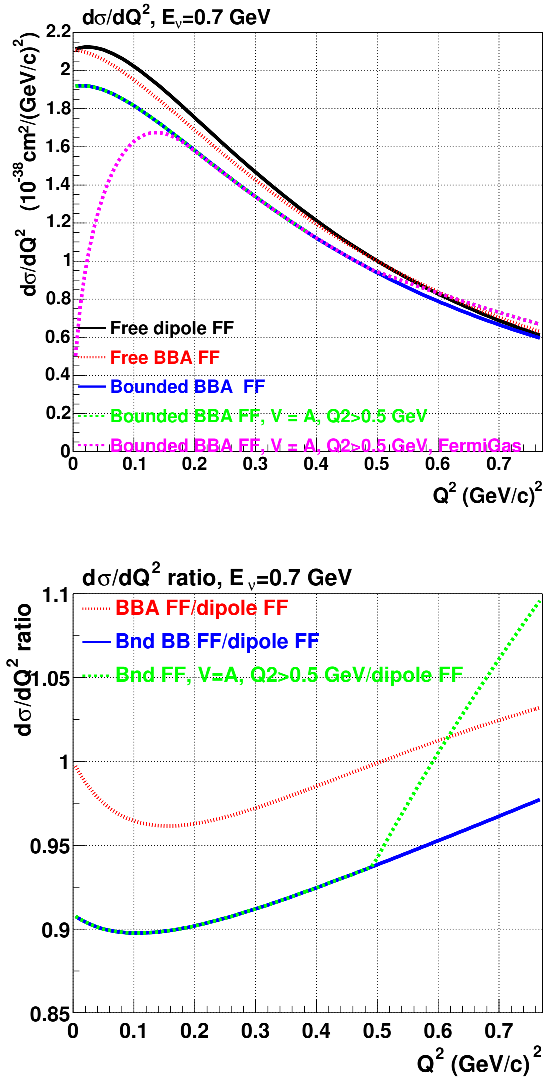

FIG. 3. $d\sigma/dQ^2$ for a neutrino energy of 0.7 GeV, assuming various models of form factors. We use $M_A = 1.014$ GeV/c^2. The curves labeled "Free dipole FF", Free BBA FF" "Bounded FF, V=A", and "Bounded FF, V=A, Pauli" are described in the text. The top set of plots shows $d\sigma/dQ^2$, while the bottom set of plots shows the ratio of $d\sigma/dq^2$ to $d\sigma/dQ^2$ with dipole free form factors. The Q^2 upper limit on the plot is the kinematic limit for an incident neutrino energy of 0.7 GeV.

dipole prediction F_A with $M_A = 1.014$ GeV$/c^2$. The dipole form factor for F_A can only be assumed to be valid close $Q^2 = 0$, which the region for which the value of M_A can also be extracted from data on the electroproduction of pions. We conclude that the Adler sum rule and quark hadron duality imply that for Q^2 greater than ≈ 0.5 $(GeV/c)^2$ the axial form factor is larger than the dipole form currently assumed in most neutrino experiments. Our prediction are also in qualitative agreement the predictions of the consituent quark model of Wagenbrunn et. al. [19]. Note however, that constituent quark models are not expected to be valid at large Q^2, and that this specific model also yields a value of $g_a = 1.15$ which deviates from the experimental value of 1.267.

The maximum Q^2 values that can be achieved with incident neutrino energies of 0.5, 1.0, 1.5 and 2 GeV are 0.5, 1.2, 2.1 and 2.9 (GeV/c)^2, respectively. Since K2K, MiniBooNE, and J-PARCnu energies are in the 0.7–2.0 GeV range, our results indicate that the there will be a relative enhancement in the number of events with Q^2 greater than ~ 0.5 $(GeV/c)^2$ with respect to the number of events at lower Q^2. This is discussed in more detail at the end of this paper.

NUCLEAR BINDING EFFECTS.

Current neutrino experiments such as MiniBoone, and K2K probe the low $Q^2 < 1$ (GeV/c)^2 region where nuclear effects are large. The low Q^2 MiniBooNE and K2K experiments have begun to investigate the various nuclear effects in carbon and oxygen. Higher Q^2 data from the NOMAD experiment will also come out soon. Although the neutrino flux is not as well know for these current experiments, a variety of effects such non-dipole form factors, and nuclear correction must be included in

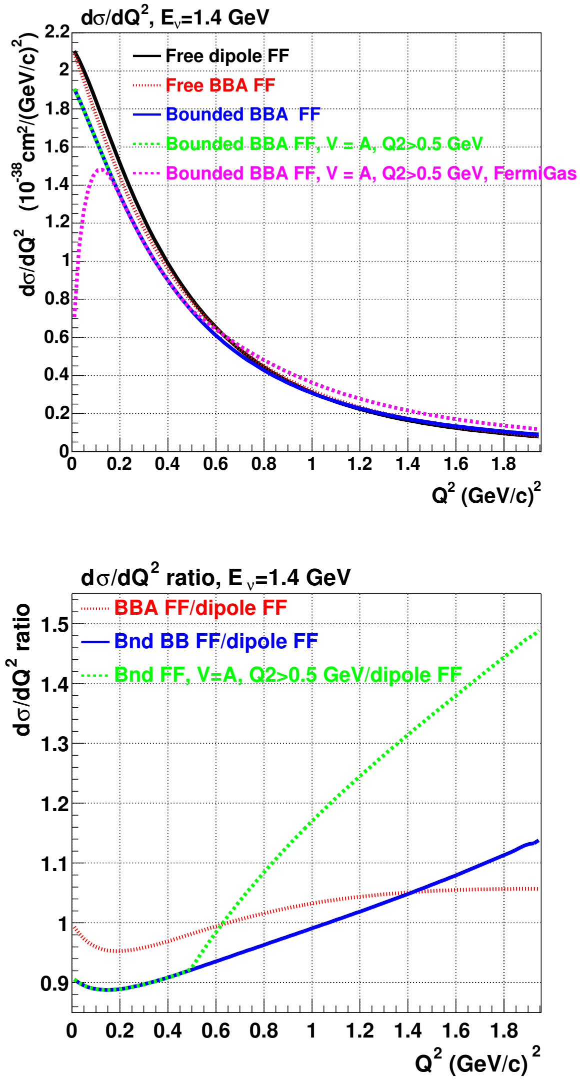

FIG. 4. $d\sigma/dQ^2$ for a neutrino energy of 1.4 GeV, assuming various models of form factors. We use $M_A = 1.014\ GeV/c^2$. The curves labeled "Free dipole FF", Free BBA FF" "Bounded FF, V=A", and "Bounded FF, V=A, Pauli" are described in the text. The top set of plots shows $d\sigma/dQ^2$, while the bottom set of plots shows the ratio of $d\sigma/dq^2$ to $d\sigma/dQ^2$ with dipole free form factors. The Q^2 upper limit on the plot is the kinematic limit for an incident neutrino energy of 1.4 GeV.

the comparison of the Q^2 distributions between data and theory.

Figures 3 and 4 show the expected $d\sigma/dQ^2$ assuming various models of form factors for neutrino energies of 0.7 and 1.4 GeV, respectively. Here we start with a dipole form factors and $M_A = 1.014\ GeV/c^2$. The curve labeled "Free dipole FF" assumes free nucleon and dipole fomr factors. The curve "Free BBA FF" assumes BBA2003 vector form factors and a dipole form for the axial form factor (with $M_A = 1.014\ GeV/c^2$). The cureve "Bounded FF" includes the binding effects on the form factors from ref. [22] (which should be applied if the nuclear model is a simple Fermi motion model. The cuve "Bounded FF, V=A" shows what happens when the a non-dipole form factor (equation 27) is also used for the axial form factor, as predicted by our duality mode in addition to nuclear binding corrections. The last curve "Bounded FF, V=A, Pauli" includes final state Pauli exclusion effects at low Q^2 for a simple Fermi gas model on Carbon [23]. The top set of plots shows $d\sigma/dQ^2$, while the bottom set of plots shows the ratio of $d\sigma/dQ^2$ to $d\sigma/dQ^2$ with dipole free form factors. The Q^2 upper limit on the plot is the kinematic limit.

Currently, neutrino experiments such as MIniBoone, K2K and NOMAD typically use Dipole form factors and include Fermi-Gas and Pauli blocking (which results in a depletion at low Q^2 the ther Monte Carlo predictions. Our results indicate each of the following additional corrections: (a) Updated non-dipole vector form factors ; (b) A non-dipole axial form factor, and also (c) modifications to the free form factors for nuclear binding corrections result in an additional relative depletion at the low Q^2 with respect to high Q^2 events.

A similar argument can be made for the axial form factors if one assumes that $V = A$ at high Q^2. If we assume quark-hadron duality, we also expect that the axial and vector contribution to W_1 for the Δ should become equal with increasing Q^2. This provides a possible explanation for the experimental observation that the axial mass for the Δ is higher than the axial mass for the nucleon (equality of V=A within the framework of the dipole approximation requires a higher axial mass).

More precise higher Q^2 data will become accessible in higher-energy experiments such as MINERvA which can span the 2–8 GeV neutrino energy range. MINERvA's measurement of the axial form-factor at high Q^2 will greatly contribute to a better understanding of the vector and axial structure of the neutron and proton.

GLOBAL VERSUS LOCAL QUARK-HADRON DUALITY

In the previous section we used global quark-hadron duality, which assumes that the integral of the structure function W_1 for inelastic scattering scattering (including the resonance region) is equal to the integral of the QCD predictions from Parton Distribution Function (including target mass effects). Global quark-hadron duality implies that the lowest order moments of the QCD predictions for scattering on quarks are the same as those for the measured structure functions. This is equivalent to the assumption that the lowest moment integrals of the structure functions as predicted from QCD are not modified by the final state interactions which result in final state resonances.

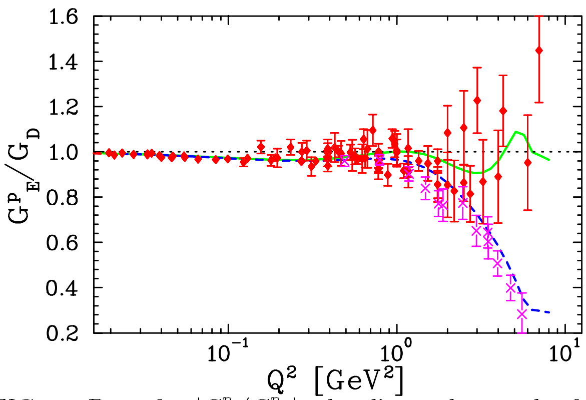

FIG. 5. Data for $|G_E^p/G_M^p|$, the diamonds are the from Rosenbluth extractions and the crosses are the polarization transfer data. Also shown are of values $|G_E^n/G_M^n|$ using polarization trasfer data. The duality prediction that $|G_E^p/G_M^p|$ $\approx |G_E^n/G_M^n|$ with increasing Q^2 is in agreement with the polarization transfer data. Also shown are the prediction from the consituent quark model of Wagenbrunn et. al.

Local quark-hadron duality implies that the same holds in the region of specific resonances. Local quark-hadron duality taken to an extreme implies that the integral of the QCD predictions above pion threshold are equal to the measured elastic form factors.

However, local quark hadron duality, even if found to be approximately valid in electron nucleon scattering, must be modified in the case of charged current neutrino and antineutrino reactions on nucleons. Another example is the 5/18 rule for the ratio of the structure functions of neutrino and electron scattering on an isoscalar (e.g. D) target, for which local duality cannot hold unless the contribution of the Δ is also included. Near x=1, the inelastic struture functions fpr electron scattering on D=n+p is proportional to

$$F_2^{e-[n+p]}(x,Q^2) \sim (5/9)[xu(x) + xd(x)] \tag{35}$$

while neutrino scattering from D=n+l is proporational to

$$F_2^{\nu-[n+p]}(x,Q^2) \sim 2[xu(x) + xd(x)] \tag{36}$$

In the case of elastic scattering in the electromagnetic case the final state in elastic scattering is always a neutron or a proton. In contract, in neutrino and antineutrino charged-current scattering, the hadronic final states include (in addition to the discrete ($neutron = udd$) and ($proton = uud$) M= 0.938 GeV/c^2) the broad $\Delta^{++} = uuu$ and Δ^- resonances a mass of 1.232 GeV/c^2. Therefore, any comparisons between electron and neutrino scattering within the framework of local quark-hadron duality must account for these additional transitions. A recent article by Steffens and Tsuchima [20] has concluded that there are violations of local quark-hadron in the comparison between electron and neutrino scattering. These violations were incorrectly taken to be evidence for Charge Symmetry Violations of nucleon Parton Distribution Functions. However, since all the vector form factors are extracted under the assumption of charge symmetry, charge symmetry (at least for the vector form factors) should be valid by definition. Therefore, comparisons of neutrino and electron-nucleon interactions must include transitions to both the nucleon and Δ resonances final states.

Here we investigate additional predictions that rely on local duality. Steffens and Tsuchima [20], state that at high Q^2 local duality for electron scattering from neutrons and protons implies the following:

$$\int_{\xi_{th}}^1 F_1^{eP}(\xi,Q^2)d\xi = \eta\left[G_M^p(Q^2)\right]^2 \tag{37}$$

$$\int_{\xi_{th}}^1 F_2^{eP}(\xi,Q^2)d\xi = \eta\left[\frac{[G_E^p(Q^2)]^2 + \tau[G_M^p(Q^2)]^2}{(1+\tau})\right] \tag{38}$$

$$\int_{\xi_{th}}^1 F_1^{eN}(\xi,Q^2)d\xi = \eta\left[G_M^n(Q^2)\right]^2 \tag{39}$$

$$\int_{\xi_{th}}^1 F_2^{eN}(\xi,Q^2)d\xi = \eta\frac{[G_E^n(Q^2)]^2 + \tau[G_M^n(Q^2)]^2}{(1+\tau)} \tag{40}$$

where, $\xi = xQ^2/[0.5Q^2(1+[1+(2Mx)^2/Q^2]^{1/2})]$ is the Nachtmann variable and ξ_0 is the Nachtman variable at the nucleon pole with x=1, ξ_{th} is Nachtman variable at the pion threshold with $x_{th} = Q^2/(Q^2 + m_\pi(2M + m_\pi))$, and $\eta = \xi_0^2/(4-2\xi_0)$.

The following are a few predictions of local duality for electron/muon scattering.

$$\frac{F_1^{inel-eN}(\xi_{th},Q^2)}{F_1^{inel-eP}(\xi,Q^2)} = \frac{d[G_M^n(Q^2)]^2/dQ^2}{d[G_M^n(Q^2)]^2/dQ^2} \tag{41}$$

which experimentally is in reasonable agreement with the data.

QCD calculations, including target mass effects agree with experimental measurements that in electron scattering, the ratio of transverse to longitudinal cross sections, R, is the same for the neutron and the proton. Therefore, local duality implies that:

$$\frac{|G_E^n(Q^2)|^2}{|G_M^n(Q^2)|^2} = \frac{|G_E^p(Q^2)|^2}{|G_M^p(Q^2)|^2} \tag{42}$$

Electron scattering experiments at SLAC and Jefferson Lab (JLab) have measured the proton and neutron electromagnetic (vector) form-factors with high precision. The vector form-factors can be determined from electron scattering cross-sections using the standard Rosenbluth separation technique [14], which is sensitive to (two-photon) radiative corrections, or from polarization measurements using the newer polarization transfer technique [15]. Polarization measurements do not directly measure form-factors, but rather the ratio G_E/G_M. Recently, discrepancies in electron scattering measurements of some vector form-factors have appeared. Figure 5 shows the current two sets of results for $|G_E^p/G_M^p|$. There appears to be a difference between the two different methods of measuring this ratio. The newer polarization transfer technique yields a much lower value at high Q^2 and indicates a difference between the electric charge and magnetization distributions. The polarization transfer technique is believed to be more reliable and less sensitive to (two-photon) radiative effects from two-photon corrections. Figure 5 shows that dipole amplitudes provide only a first-order description of form-factor behavior at high Q^2. Also shown are of values for the neutron $|G_E^n/G_M^n|$ using polarization trasfer data [18]. Our prediction that the absolute value of the two ratios become eaual with increasing Q^2 is in better agreement with the polarization transfer results for $|G_E^p/G_M^p|$ and do not agree with the the results of the Rosebluth separation method which yield $G_E^p/G_M^p = (1/mu_p$. These data shows that for $Q^2 \approx 4\ (GeV/c)^2$, both the electric form factor of the proton and electric form factor of the neutron become small, as expected from local duality. Also shown are the prediction from the consituent quark model of Wagenbrunn et. al. [19], which give similar qualitative results.

MORE ON THE ADLER SUM RULE AND GLOBAL QUARK-HADRON DUALITY ELECTRIC FORM FACTORS OF THE PROTON AND NEUTRON

In previous sections we assumed that the inelastic part of the Adler sum rule for the vector and axial part of W_1 were equal to each other as expected for scattering from point like quarks. We assumed that the axial and vector contributions of the Δ resonance to W_1 were also equal, and obtained (for Q^2 above $\approx 0.5\ (GeV/c)^2$).

$$[F_A(Q^2)]^2 = [\tau/(1+\tau)][G_M^V(Q^2)]^2 \tag{43}$$

At low Q^2, the structure function W_2 includes both transverse and longitudinal contributions, and the contribution of the Δ resonance resonance cannot be neglected. If we assume that the vector and axial inealstic parts of W_2 are also equal we get the following relation at high Q^2:

$$F_A(Q^2)]^2 + F_A^\Delta(Q^2)]^2 = |F_V(Q^2)|^2 + [F_V^\Delta(Q^2)]^2$$

with

$$|F_V(Q^2)|^2 = \frac{[G_E^V(Q^2)]^2 + \tau[G_M^V(Q^2)^2}{(1+\tau)}$$

If we neglect the contribution of the Δ, then the only way this equation can hold is if the first term in the above equation is much smaller than the second term, or:

$$\frac{|G_E^V(Q^2)|^2}{|G_M^V(Q^2)|^2} \approx 0$$

The above could be satisfied if the electric form factors of the neutron and proton are equal: $G_E^n(Q^2) \approx G_E^p(Q^2)$ which is in disagreement with the predictions from local duality $|G_E^p/G_M^p| \approx |G_E^n/G_M^n|$ (except when both form factors are zero at very high Q^2). This is just one of the examples that illustrates that the contribution of the Δ needs to be included for neutrino scattering at lower Q^2.

More precise higher Q^2 data will become accessible in higher-energy experiments such as MINERvA which can span the 2–8 GeV neutrino energy range. MINERvA's measurement of the axial form-factor at high Q^2 will greatly contribute to a better understanding of the vector and axial structure of the neutron and proton.

CONCLUSION

In summary, we have shown that global and local duality imply that vector and axial form factors of the nucleon significantly deviate from the dipole approximation with increasing Q^2, in agreement with indications from present experimental data. Additional studies of the implication of global and local duality on nucleon form factors will be presented in a future communiction,

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