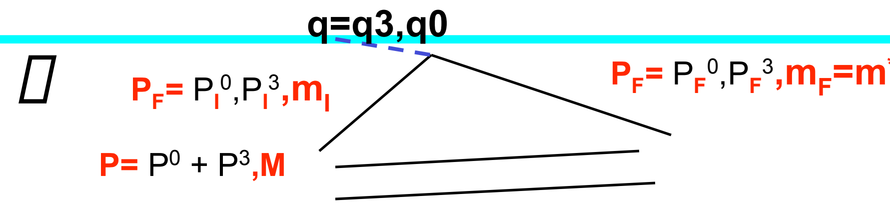
- A. Bodek – Feb 9. 2004   updated , 2002
- This same WWW area has PDF file copies of most of the references used.

Initial quark mass m_I and final mass $m_F = m^*$ bound in a proton of mass M -- Summary: INCLUDE quark initial Pt) Get □ scaling (not $x = Q^2/2M$□)

□ Is the correct variable which is Invariant in any frame : q3 and P in opposite directions.

□



$PI, P0$ quark $\qquad q3, q0$ photon

$$\square = \frac{P_I^0 + P_I^3}{P_P^0 + P_P^3} \quad \square\square\square\square\square\square \; \square\square\square\square\square\square\square$$

$$(q + P_I)^2 = P_F^2 \quad \square \quad q^2 + 2P_I \cdot q + P_I^2 = m_F^2$$

$$\square = \frac{Q^2 + m_F^2}{M\square[1 + \sqrt{(1 + Q^2/\square^2)}]} \qquad for\ m_I^2, Pt = 0$$

Special cases:

Numerator m_F^2: Slow Rescaling □ as in charm production

Denominator: Target mass effect, e.g. Nachtman Variable □, Light Cone Variable □, Georgi Politzer Target Mass □

Most General Case:

$$\square'_w = [Q'^2 + B] / [M\square(1 + (1 + Q^2/\square^2))^{1/2} + A]$$

where $2Q'^2 = [Q^2 + m_F^2 - m_I^2] + [(Q^2 + m_F^2 - m_I^2)^2 + 4Q^2(m_I^2 + P^2t)]^{1/2}$

For the case of Pt2=0 see R. Barbieri et al Phys. Lett. 64B, 1717 (1976) and Nucl. Phys. B117, 50 (1976)

Add B and A to account for effects of additional □ m^2 from NLO and NNLO (up to infinite order) QCD effects.

Initial quark mass m_I and final mass, $m_F = m^*$ bound in a proton of mass M -- Page 1 INCLUDE quark initial Pt) Get □ scaling (not $x=Q^2/2M$□) DETAILS

□ Is the correct variable which is Invariant in any frame : q3 and P in opposite directions.

$$PI, P0 \qquad q3, q0$$

$$\square = \frac{P_I^0 + P_I^3}{P_P^0 + P_P^3} \quad \text{□□□□□ quark □□□□□□ photon}$$

$$In \square LAB \square Frame : \square \qquad P_P^0 = M, P_P^3 = 0$$

$$\square = \frac{P^0_{I\square LAB} + P^3_{I\square LAB}}{M} \square \; P^0_{I\square LAB} + P^3_{I\square LAB} = \square M$$

$$\square = \frac{(P_I^0 + P_I^3)(P_I^0 \square P_I^3)}{M(P_I^0 \square P_I^3)} = \frac{(P_I^0)^2 \square (P_I^3)^2}{M(P_I^0 \square P_I^3)}$$

$$\square M(P_I^0 \square P_I^3) = (m_I^2 + Pt^2)$$

$$\square \; P_I^0 \square P_I^3 = (m_I^2 + Pt^2)/(\square M)$$

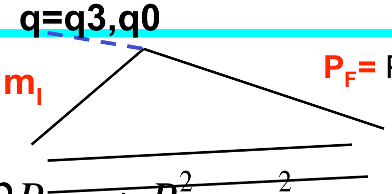
$$(1): \quad P_I^0 \square P_I^3 = (m_I^2 + Pt^2)/(\square M)$$

$$(2): \quad P_I^0 + P_I^3 = \square M$$

$$2P_I^0 = \square M + (m_I^2 + Pt^2)/(\square M) \square \square\square^{m_I, Pt \square 0}\square\square \; \square M$$

$$2P_I^3 = \square M \square (m_I^2 + Pt^2)/(\square M) \square \square\square^{m_I, Pt \square 0}\square\square \; \square M$$

□



$$(q + P_I)^2 = P_F^2 \quad \square \; q^2 + 2P_I \cdot q + P_I^2 = m_F^2$$

$$2(P_I^0 q^0 + P_I^3 q^3) = Q^2 + m_F^2 \square m_I^2 \qquad Q^2 = \square q^2 = (q^3)^2 \square (q^0)^2$$

$$In \square LAB \square Frame : \square \qquad Q^2 = \square q^2 = (q^3)^2 \square \square^2$$

$$[\square M + (m_I^2 + Pt^2)/(\square M)]\square + [\square M \square (m_I^2 + Pt^2)/(\square M)]q^3$$

$$= Q^2 + m_F^2 \square m_I^2 \quad : General$$

$$Set: m_I^2, Pt = 0 \qquad (for\ now)$$

$$\square M \square + \square M q^3 = Q^2 + m_F^2$$

$$\square = \frac{Q^2 + m_F^2}{M(\square + q^3)} = \frac{Q^2 + m_F^2}{M\square(1 + q^3/\square)} \qquad for\ m_I^2, Pt = 0$$

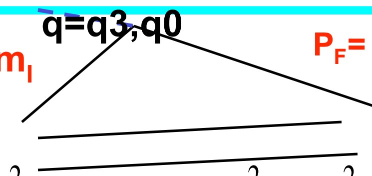
$$\square = \frac{Q^2 + m_F^2}{M\square[1 + \sqrt{(1 + Q^2/\square^2)}]} \qquad for\ m_I^2, Pt = 0$$

Special cases : Denom □ TM term, Num □ Slow rescaling

initial quark mass m_I and final mass $m_F=m^*$ bound in a proton of mass M -- Page 2 INCLUDE quark initial Pt) DETAILS

□ For the case of non zero m_I, P_t

(note P and q3 are opposite)



□

$PI, P0$ quark $\quad q3, q0$ photon

$$□ = \frac{P_I^0 + P_I^3}{P_P^0 + P_P^3}$$

In □ LAB □ $Frame$: □ $P_P^0 = M, P_P^3 = 0$

$$(1): \ 2P_I^0 = □M + (m_I^2 + Pt^2)/(□M)$$

$$(1): \ 2P_I^3 = □M □ (m_I^2 + Pt^2)/(□M)$$

$$(q + P_I)^2 = P_F^2 \quad □ \quad q^2 + 2P_I \cdot q + P_I^2 = m_F^2$$

$$Q^2 = □q^2 = (q^3)^2 □□^2$$

$$[□M + (m_I^2 + Pt^2)/(□M)]□ + [□M □ (m_I^2 + Pt^2)/(□M)]q^3$$

$$= Q^2 + m_F^2 □ m_I^2$$

Keep all terms here and : multiply by □M and group terms in □ qnd □^{2}

$□^2 \ M^2 (□+ q3) \ - □ \ M \ [Q^2 + m_F^2 - m_I^2] \ + [m_I^2 + Pt^2 \ (□□ q3)^2] = 0$ General Equation

a b c

=> solution of quadratic equation □ = [-b +(b ^{2} - 4ac) $^{1/2}$] / 2a

use (□^{2}□ q3 ^{2}) = q ^{2} = -Q ^{2} and (□+ q3) = □ + □[1 + Q ^{2}/□^{2}] $^{1/2}$ = □ + □[1 + 4M^2 x^2/ Q ^{2}] $^{1/2}$

□'$_w$ = [Q'^{2} +B] / [M□ (1+(1+Q^2/□^{2})) $^{1/2}$ +A]

where 2Q'^{2} = [Q^2+ m $_F$ ^{2} - m $_I$ ^{2}] + [(Q^2+m $_F$ ^{2} - m $_I$ ^{2}) ^{2} + 4Q^2 (m $_I$ ^{2} +P^{2}t)] $^{1/2}$

Add B and A to account for effects of additional □m^2 from NLO and NNLO effects.

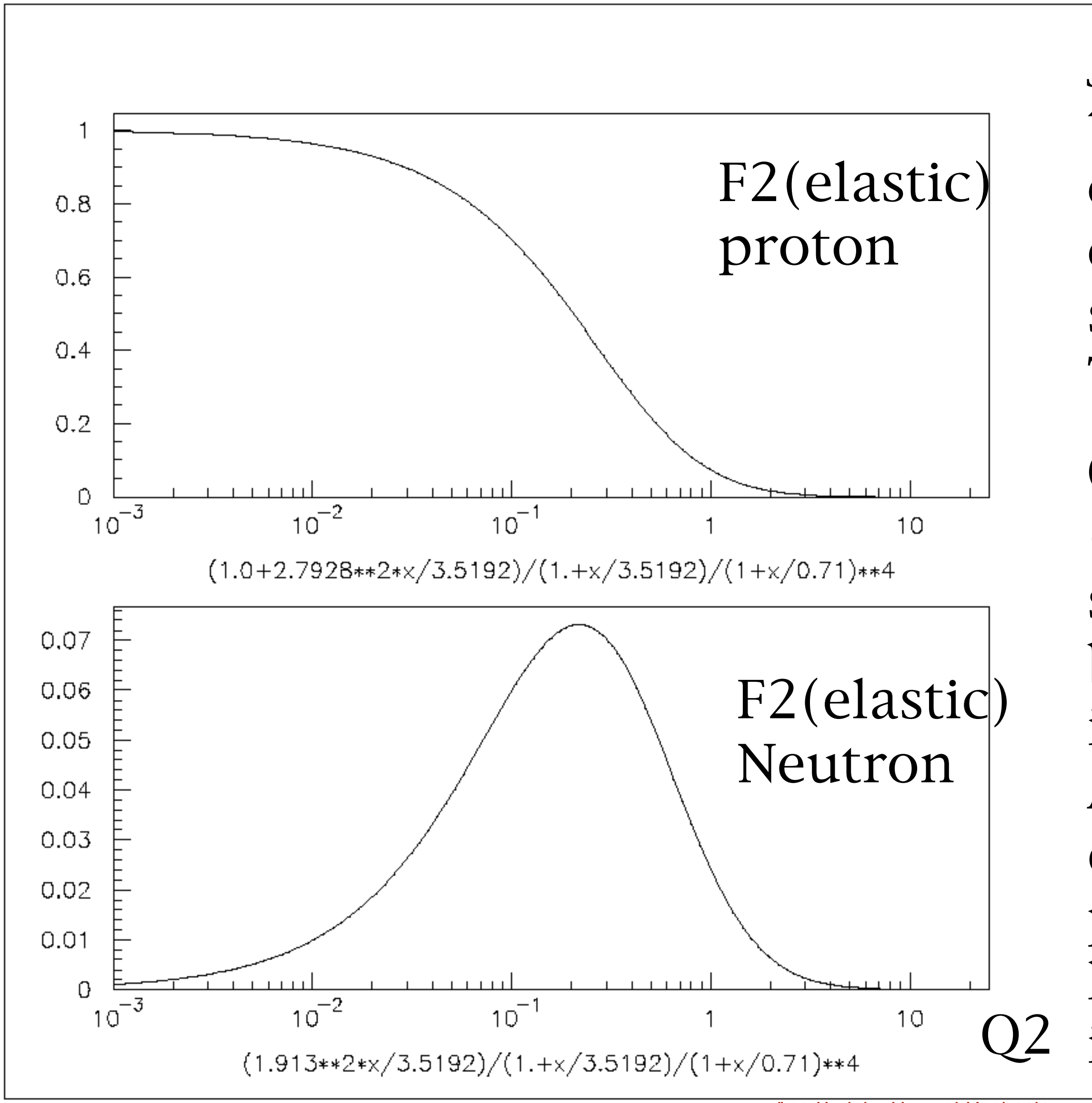
or 2Q'^{2} = [Q^2+ m $_F$^{2} - m $_I$^{2}] + [Q^4 +2 Q^2(m $_F$^{2} + m $_I$^{2} +2P^{2}t) + (m $_F$^{2} - m $_I$^{2}) ^{2}] $^{1/2}$

□$_w$ = [Q'^{2} +B] / [M □(1 + [1 + 4M^2 x^2/ Q ^{2}] $^{1/2}$) +A] (equivalent form)

□$_w$ = x [2Q'^{2} + 2B] / [Q^2 + (Q^4 +4x^2 M^2 Q^2) $^{1/2}$ +2Ax] (equivalent form)

Very low Q2: Revenge of the Spectator Quarks

F2(elastic) versus Q2 (GeV2)



Just like in p-p scattering there is a strong connection between elastic and inelastic scattering (Optical Theorem).

Quantum Mechanics (Closure) requires a strong connection between elastic and inelastic scattering. Although spectator quarks were ignored in pQCD - they rebel at lowQ2 and will not be ignored.

Revenge of the Spectator Quarks

Stein et al PRD 12, 1884 (1975)-1

$$\nu W_{2p}(q^2,\nu) = [1 - W_2^{el}(q^2)] F_{2p}(\omega') , \tag{13}$$

where $F_{2p}(\omega')$ is the scaling limit structure function and

$$W_2^{el}(q^2) = \frac{G_E^2(q^2) + \tau G_M^2(q^2)}{1+\tau} , \quad \tau = \frac{q^2}{4M^2} \tag{14}$$

is the counterpart of W_2 for elastic scattering (see Appendix B), where G_E and G_M are, respectively, the elastic electric and magnetic form factors for the proton. This form satisfies the constraint that W_2 vanish at $q^2 = 0$. Integrating W_{2p} over all values of ν yields

$$\int_{\text{inelastic}} d\nu\, W_{2p}(q^2,\nu) = [1 - W_2^{el}(q^2)] \int_{\text{inelastic}} \frac{d\nu}{\nu} F_{2p}(\omega') . \tag{15}$$

But this is the Gottfried sum rule[27] for the proton,

where

$$\int_{\text{inelastic}} \frac{d\nu}{\nu} F_{2p}(\omega') = \sum_i q_i^2 \tag{16}$$

is the sum of the parton charges squared.

2. Application

We can now apply these results to the proton and neutron if we consider them as being made of constituents. These yield immediately

$$\int_{\text{inel}} d\nu\, W_{2p}(q^2,\nu) = \left(\sum_{i=1}^{N} e_i^2\right)_p [1 - |F_{el}^p(q^2)|^2] + C_p(q^2) \left(\sum_{i\neq j}^{N}\sum e_i e_j\right)_p , \tag{B15}$$

$$\int_{\text{inel}} d\nu\, W_{2n}(q^2,\nu) = \left(\sum_{i=1}^{N} e_i^2\right)_n [1 - |F_{el}^N(q^2)|^2] + C_n(q^2) \left(\sum_{i\neq j}^{N}\sum e_i e_j\right)_n . \tag{B16}$$

F_{el}^p and F_{el}^n would be equal if the momentum distributions of the constituents were the same in the proton and neutron, so if the correlation terms were negligible, one might expect W_{2n}/W_{2p} to scale to lower values of q^2 than either W_{2p} or W_{2n} alone. Gottfried noted that in the simple quark model the charge sum in the correlation contribution vanishes for the proton, but not for the neutron.[27]

For the case of particles with spin, magnetic moments, and more realistic ground states, the results get much more complicated. There are several more detailed accounts in the case of nuclear scattering in the literature.[41] However, the simple approach stated here agrees with the spirit of the more complex analyses.

Revenge of the Spectator Quarks

Stein etal PRD 12, 1884 (1975)-2

[41]For more detailed treatment of closure, see, for example O. Kofoed-Hanson and C. Wilkin, Ann. Phys. (N.Y.) 63, 309 (1971); K. W. McVoy and L. Van Hove, Phys. Rev. 125, 1034 (1962).

[27]K. Gottfried, Phys. Rev. Lett. 18, 1174 (1967).

$$G_{el}(q^2) = \left|\sum_{i=1}^{N} e_i\right|^2 |F_{el}(q^2)|^2 ,$$

$$G_{inel}(q^2) = \sum_{i=1}^{N} e_i^2[1-|F_{el}(q^2)|^2] + C(q^2)\sum_{i \neq j}^{N}\sum e_i e_j . \quad \text{(B14)}$$

$$\nu W_{2p}(q^2,\nu) = [1 - W_2^{el}(q^2)]\, F_{2p}(\omega') , \quad (13)$$

where $F_{2p}(\omega')$ is the scaling limit structure function and

$$W_2^{el}(q^2) = \frac{G_E^2(q^2) + \tau G_M^2(q^2)}{1+\tau} , \quad \tau = \frac{q^2}{4M^2} \quad (14)$$

$$G_{Ep} = P(q^2)/(1+q^2/0.71)^2 ,$$

P is close to 1 and gives deviations

From Dipole form factor (5%)

Note: at low Q2

$[1 - W_2^{el}] = 1 - 1/(1+Q^2/0.71)^4$

$= 1-(1-4Q^2/0.71) =$

$= 1- (1-Q^2/0.178) =$

$\rightarrow Q^2/0.178$ as $Q^2 \rightarrow 0$

Versus Our GRV98 fit with

$Q^2/(Q^2+C) \rightarrow Q^2/C$

c = 0.1797 +- 0.0036

Revenge of the Spectator Quarks -3 - History of Inelastic Sum rules C. H. Llewellyn Smith hep-ph/981230

Talk given at the Sid Drell Symposium
SLAC, Stanford, California, July 31st, 1998

Gottfried noted that in the *'breathtakingly crude'* naïve three-quark model the second term in the following equation vanishes for the proton (it also vanishes for the neutron, but neutrons are not mentioned):

$$\sum_{i,j} Q_i Q_j \equiv \sum_i Q_i^2 + \sum_{i \neq j} Q_i Q_j \ . \tag{5}$$

Thus for any charge-weighted, flavour-independent, one-body operator all correlations vanish, and therefore using the closure approximation the following sum rule can be derived:

$$\int_{\nu_0} W_2^{ep}(\nu, q^2) d\nu = 1 - \frac{G_E^2 - q^2 G_M^2/4m^2}{1 - q^2/4m^2} \ , \tag{6}$$

where ν_0 is the inelastic threshold (the methods used to derive this sum rule are those that have long been used to derive sum rules in atomic and nuclear physics, for example the sum rule [13] derived in 1955 by Drell and Schwarz). After observing that this sum

Revenge of the Spectator Quarks -4 - History of Inelastic Sum rules C. H. Llewellyn Smith hep-ph/981230

rule appears to be oversaturated in photoproduction (we now know that the integral is actually infinite in the deep inelastic region), Gottfried asked whether it was '*idiotic*', and stated that if, on the contrary there is some truth in it, one would want a '*derivation that a well-educated person could believe*'.

In his talk at the 1967 SLAC conference Bj quoted Gottfried's paper and stated that diffractive contributions should presumably be excluded from the integral, which could be done by taking the difference between protons and neutrons, leading to the following result, in modern notation:

$$\int \left(F_2^{ep}(x,q^2) - F_2^{en}(x,q^2)\right)\frac{dx}{x} = \frac{1}{3}\,. \qquad (7)$$

This result, which is generally known as the Gottfried sum rule, is not respected by the data which give the value [14] 0.235 ± 0.026. In parton notation, the left-hand side can be written

$$\frac{1}{3}(n_u + n_{\bar{u}} - n_d - n_{\bar{d}}) = \frac{1}{3} + \frac{2}{3}(n_{\bar{u}} - n_{\bar{d}})\,, \qquad (8)$$

S. Adler, Phys. Rev. 143, 1144 (1966) Exact Sum rules from Current Algebra. Valid at all Q2 from zero to infinity. - 5A

$\alpha = W_1$

$\beta = W_2$

Strangeness-Conserving Case

The kinematic analysis of Sec. 3 shows that we may write the reaction differential cross section in the form

$$d^2\sigma\left(\binom{\nu}{\bar{\nu}}+p\rightarrow\binom{l}{\bar{l}}+\beta(S=0)\right)\Big/d\Omega_l dE_l=\frac{G^2\cos^2\theta_C}{(2\pi)^2}\frac{E_l}{E_\nu}$$

$$\times[q^2\alpha^{(\pm)}(q^2,W)+2E_\nu E_l\cos^2(\tfrac{1}{2}\phi)\beta^{(\pm)}(q^2,W)\mp(E_\nu+E_l)q^2\gamma^{(\pm)}(q^2,W)]. \quad (13)$$

By measuring $d^2\sigma/d\Omega_l dE_l$ for various values of the neutrino energy E_ν, the lepton energy E_l, and the lepton-neutrino angle ϕ, we can determine the form factors $\alpha^{(\pm)}$, $\beta^{(\pm)}$, and $\gamma^{(\pm)}$ for all $q^2>0$ and for all W above threshold.

In Sec. 4 we prove that:

(i) the local commutation relations of Eq. (1a) and Eq. (1c) imply

$$2=g_A(q^2)^2+F_1^V(q^2)^2+q^2F_2^V(q^2)^2+\int_{M_N+M_\pi}^{\infty}\frac{W}{M_N}dW[\beta^{(-)}(q^2,W)-\beta^{(+)}(q^2,W)]; \quad (14)$$

Strangeness-Changing Case

$$(4,2)=\int\frac{W}{M_N}dW[\beta_{(p,n)}{}^{(-)}(q^2,W)-\beta_{(p,n)}{}^{(+)}(q^2,W)]; \quad (18)$$

The integrals of Eqs. (18)–(20) have discrete contributions at $W=M_\Lambda$ and/or M_Σ and a continuum extending from $W=M_\Lambda+M_\pi$ or from $W=M_\Sigma+M_\pi$ to $W=\infty$. We have not explicitly separated off the discrete contributions to the integrals, as was done in Eqs. (14)–(16) for the strangeness-conserving case. It would, of course, be straightforward to do this.

S. Adler, Phys. Rev. 143, 1144 (1966) Exact Sum rules from Current Algebra. Valid at all Q2 from zero to infinity. - 5B

$\alpha = W_1$

$\beta = W_2$

(B) Sum Rule for $\beta^{(\pm)}$

The sum rule on $\beta^{(\pm)}$ of Eq. (14) is obtained by adding together two separately derived sum rules on the axial-vector and the vector parts of $\beta^{(\pm)}$, $\beta_A{}^{(\pm)}$, and $\beta_V{}^{(\pm)}$:

$$1 = g_A(q^2)^2 + \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW \times [\beta_A{}^{(-)}(q^2,W) - \beta_A{}^{(+)}(q^2,W)], \quad (53a)$$

$$1 = F_1{}^V(q^2)^2 + q^2 F_2{}^V(q^2)^2 + \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW \times [\beta_V{}^{(-)}(q^2,W) - \beta_V{}^{(+)}(q^2,W)]. \quad (53b)$$

In terms of the structure functions defined in Eq. (41),

$$\beta_A{}^{(\pm)}(q^2,W) = [q^2 A_1{}^{(\pm)}(q^2,W) + (q^2)^2 A_2{}^{(\pm)}(q^2,W) + q^2 I_A{}^{(\pm)}(q^2,W) + D_A{}^{(\pm)}(q^2,W)] \times 4M_N{}^2/(W^2 - M_N{}^2 + q^2)^2, \quad (54)$$

$$\beta_V{}^{(\pm)}(q^2,W) = q^2[V_1{}^{(\pm)}(q^2,W) + q^2 V_2{}^{(\pm)}(q^2,W)] \times 4M_N{}^2/(W^2 - M_N{}^2 + q^2)^2.$$

[The structure functions $I_V{}^{(\pm)}(q^2,W)$ and $D_V{}^{(\pm)}(q^2,W)$ vanish identically in the strangeness-conserving case, because of conservation of the vector current.] Since the derivations of Eqs. (53a) and (53b) are identical, we will treat explicitly only the axial-vector case, Eq. (53a).

(C) Sum Rule for $\alpha^{(\pm)}$

The sum rule on $\alpha^{(\pm)}$ of Eq. (15) is obtained by adding together the two identities

$$C_I{}^2 = \left(1 + \frac{q^2}{4M_N{}^2}\right) g_A(q^2)^2 + \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW [\alpha_A{}^{(-)}(q^2,W) - \alpha_A{}^{(+)}(q^2,W)], \quad (73a)$$

$$C_I{}^1 = \left(\frac{q^2}{4M_N{}^2}\right) g_V(q^2)^2 + \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW [\alpha_V{}^{(-)}(q^2,W) - \alpha_V{}^{(+)}(q^2,W)]. \quad (73b)$$

Here $\alpha_A{}^{(\pm)}$ and $\alpha_V{}^{(\pm)}$ are, respectively, the axial-vector and the vector parts of $\alpha^{(\pm)}$,

$$\alpha_A{}^{(\pm)} = A_1{}^{(\pm)}(q^2,W), \quad \alpha_V{}^{(\pm)} = V_1{}^{(\pm)}(q^2,W). \quad (74)$$

$$g_V(q^2) = F_1{}^V(q^2) + 2M_N F_2{}^V(q^2),$$

$$C_I{}^2 = \int dq_0 (A_1{}^{(-)} - A_1{}^{(+)}) = \int \frac{W}{M_N} dW [A_1{}^{(-)}(q^2,W) - A_1{}^{(+)}(q^2,W)], \quad (82)$$

NEUTRINO REACTIONS AT ACCELERATOR ENERGIES.
By C.H. Llewellyn Smith (SLAC). SLAC-PUB-0958, May 1971. 243pp.
Published in Phys.Rept.3:261,1972 - 6

http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-0958b.pdf

(5) Isotriplet current

$$F_V^1(q^2) = \left[F_1^p(q^2) - F_1^n(q^2)\right] = \text{Dirac electromagnetic isovector form factor.} \qquad (3.15)$$

$\xi = \mu_p - \mu_n = 3.71$ (μ = anomalous magnetic moment)

$$F_V^2(q^2) = \frac{\mu_p F_2^p(q^2) - \mu_n F_2^n(q^2)}{\mu_p - \mu_n} = \text{Pauli electromagnetic isovector form factor.}$$

In terms of the Sachs form factors

$$F_V^1(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1}\left[G_E^V(q^2) - \frac{q^2}{4M^2}\, G_M^V(q^2)\right]$$

$$\xi F_V^2(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1}\left[G_M^V(q^2) - G_E^V(q^2)\right] \qquad (3.16)$$

Experimentally, the G's are described to within ± 10% by:

$$G_E^V(q^2) = \frac{1}{\left(1 - \frac{q^2}{0.71\ \text{GeV}^2}\right)^2}$$

$$G_M^V(q^2) = \frac{1 + \mu_p - \mu_n}{\left(1 - \frac{q^2}{0.71\ \text{GeV}^2}\right)^2} \qquad (3.17)$$

Note that LS define q2 as negative while Gillman and Adler it is positive. So all the –q2 here should be written as +Q2, while for Alder and Gillman q2=+Q2. Also, in modern notation Fa is -1.26 and for n=2, Ma = 1.0 GeV2. We define GE (vector)= Gep-Gen We need to put in non zero Gen

$$F_A(q^2) = -1.23 \Big/ \left(1 - \frac{q^2}{M_A^2}\right)^n \qquad (3.24)$$

NEUTRINO REACTIONS AT ACCELERATOR ENERGIES.
By C.H. Llewellyn Smith (SLAC). SLAC-PUB-0958, May 1971. 243pp.
Published in Phys.Rept.3:261,1972 - 7

http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-0958b.pdf

(5) Isotriplet current

$$F_V^1(q^2) = \left[F_1^p(q^2) - F_1^n(q^2)\right] = \text{Dirac electromagnetic isovector form factor.} \quad (3.15)$$

$$\xi = \mu_p - \mu_n = 3.71 \quad (\mu = \text{anomalous magnetic moment})$$

$$F_V^2(q^2) = \frac{\mu_p F_2^p(q^2) - \mu_n F_2^n(q^2)}{\mu_p - \mu_n} = \text{Pauli electromagnetic isovector form factor.}$$

In terms of the Sachs form factors

$$F_V^1(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1} \left[G_E^V(q^2) - \frac{q^2}{4M^2} G_M^V(q^2)\right] \quad (3.16)$$

$$\xi F_V^2(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1} \left[G_M^V(q^2) - G_E^V(q^2)\right]$$

Experimentally, the G's are described to within ± 10% by:

$$G_E^V(q^2) = \frac{1}{\left(1 - \frac{q^2}{0.71\ \text{GeV}^2}\right)^2} \quad (3.17)$$

$$G_M^V(q^2) = \frac{1 + \mu_p - \mu_n}{\left(1 - \frac{q^2}{0.71\ \text{GeV}^2}\right)^2}$$

Using these equations on the left we get:

F1v=GD [1-GenF +4.71*Q2/(4M2)]/[1+Q2/(4M2)]

F2v=(1/3.71) GD [4.71–1+GenF]/ [1+Q2/4M2]

Equation 14 in Adler's paper is in a different notation so we need to use equation 19 in Gillman

(see next page)

SV= [F1v**2 + Tau* 3.71**2* F2v**2]

And (1-SV) is the vector suppression

(1-Fa**2) is the axial suppression

See next page for GenF

Need to devide by integral from Xsithreshold

To 1.0 of (Dv-Uv). Where Xsi threshold

Is the Xsi for pion threshold

$$F_A(q^2) = -1.23 / \left(1 - \frac{q^2}{M_A^2}\right)^n \quad (3.24)$$

F2 Adler includes Gen term (from equation 13 in hep-ph/0202183 Krutov
(extraction of the neutron charge form factor, Feb. 2002).

MuN = -1.913

GD = 1/(1+Q2/0.71)**2

Tau = Q2/(4*Mp**2)

a = 0.942 and b= 4.62

Gen = GenF * GD (GenF is the factor that multiplies GD to get Gen)

GenF = -MuN * a * tau/ (1+b* tau) , So Gen is positive

F. Gillman, Phys. Rev. 167, 1365 (1968)- 8

Adler like Sum rules for electron scattering get same ...ssion

$\alpha = W_1$

$\beta = W_2$

The vector current part of the original sum rule of Adler for neutrino scattering can be written

$$\int_0^\infty dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(+)}(q_0,q^2)]=1. \qquad (18)$$

The functions $\beta^{(\pm)}(q_0,q^2)$ are defined just as in Eq. (7) except that in place of the electromagnetic currents $J_\mu(0)$ and $J_\mu(0)$ we have put the isospin raising or lowering F-spin currents $\mathfrak{F}_{(1\pm i2)\mu}(0)$ [recall that $\mathfrak{F}_{3\mu}(0)$ is just the isovector part of the electromagnetic current]. If we explicitly separate out the nucleon Born term in Eq. (18), we have

$$[F_1^V(q^2)]^2+q^2\left(\frac{\mu^V}{2M_N}\right)^2[F_2^V(q^2)]^2 + \int_{M_\pi+(q^2+M_\pi^2)/2M_N}^\infty dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(8)}(q_0,q^2)]=1, \qquad (19)$$

where the superscript V denotes the fact that we are dealing with the isovector part of the current; the isovector anomalous magnetic moment $\mu^V=\mu_p'-\mu_n'=3.70$. As $q^2\to 0$, we see from Eq. (10) or (17) that only the first term, $[F_1^V(q^2)]^2$, on the left-hand side of Eq. (19) survives, and as $q^2\to 0$ it goes to 1, in agreement with the left-hand side.

In the derivation[3] of Eq. (18) only two assumptions enter: (1) the commutation relation Eq. (3a) of the F-spin densities, and (2) an unsubtracted dispersion relation for the forward Compton scattering amplitudes (which are the coefficients of $p_\mu p_\nu$ and $q_\mu q_\nu$ in the expansion of $T_{\mu\nu}$) corresponding to $\beta(q_0,q^2)$. It is of course the second assumption which is most open to question. However, we note the following:

(a) The fact that as $q^2\to 0$ the left- and right-hand sides of Eq. (19) as it now stands automatically become equal rules out a q^2-independent subtraction. This just means we have done nothing grossly wrong, e.g., introduced a kinematic singularity in q^2 in one of our amplitudes.

F. Gillman, Phys. Rev. 167, 1365 (1968)- 9

Adler like Sum rules for electron scattering get same exp

$\alpha = W_1$
$\beta = W_2$

$$\alpha(q_0,q^2) = (q^2/4M_N^2)[F_1(q^2)+\mu F_2(q^2)]^2\delta(q_0-q^2/2M_N)$$
$$= (q^2/4M^2)[G_M(q^2)]^2\delta(q_0-q^2/2M_N) \quad (8a)$$

and

$$\beta(q_0,q^2) = \{[F_1(q^2)]^2+(q^2\mu^2/4M_N^2)[F_2(q^2)]^2\} \times\delta(q_0-q^2/2M_N) \quad (8b)$$
$$= \frac{[G_E(q^2)]^2+(q^2/4M_N^2)[G_M(q^2)]^2}{1+q^2/4M_N^2} \times\delta(q_0-q^2/2M_N). \quad (8c)$$

It is easily verified that on putting these one-nucleon-state contributions to α and β in Eq. (6) and integrating over dE', one obtains the Rosenbluth formula for elastic electron-nucleon scattering.

Since $\alpha(q_0,q^2)$ and $\beta(q_0,q^2)$ are related to the imaginary part of forward Compton scattering of photons of

[8] The quantity $J_\mu(x)$ is the Heisenberg electromagnetic current operator divided by the electronic charge e. By the conserved-vector-current hypothesis, $J_\mu(x)$ is just the F-spin current, i.e., $J_\mu^{(s)}(x) = \mathfrak{F}_{\mu 3}(x)$.

[9] $F_1(q^2)$ and $F_2(q^2)$ are the usual Dirac and Pauli electromagnetic form factors of the nucleon, normalized so that $F_1(0)=F_2(0)=1$, and μ is the anomalous magnetic moment in Bohr magnetons. $G_E=F_1-(q^2\mu/4M_N^2)F_2$ and $G_M=F_1+\mu F_2$ are the Sachs electric and magnetic form factors of the nucleon.

. Note that Gillman has two extra factor of M in equation 12, 13 (which cancel) with respect to modern definitions so Alpha is what we call W1 and Beta is what we call W2 today.

$$\frac{d\sigma}{d\Omega' dE'} = \frac{4\alpha^2}{q^4}\frac{E'^2}{M_N} \times[2W_1(q^2,q\cdot p)\sin^2(\tfrac{1}{2}\theta)+W_2(q^2,q\cdot p)\cos^2(\tfrac{1}{2}\theta)], \quad (12)$$

so that their functions W_1 and W_2 are related to α and β by

$$\alpha = W_1/M_N,$$
$$\beta = W_2/M_N. \quad (13)$$

F. Gillman, Phys. Rev. 167, 1365 (1968)- 10 Adler like Sum rules for electron scattering.

$\alpha = W_1$

$\beta = W_2$

Before:

$[1 - G_e^{2\ el}] = 1 - 1/(1+Q^2/0.71)^4$

$= 1-(1-4Q^2/0.71) =$

$= 1- (1-Q^2/0.178) =$

$\to Q^2/0.178$ as $Q^2 \to 0$

Is valid for VALENCE QUARKS FROM THE ADLER SUM RULE FOR the Vector part of the interaction

Versus Our GRV98 fit with

$Q^2/(Q^2+C) \to Q^2/C$

c = 0.1797 +- 0.0036

And C is probably somewhat different for the sea quarks.

F2nu-p(vector)= d+ubar

F2nubar-p(vector) =u+dbar

1=F2nubar-p-F2nu-p= (u+dbar)-(d+ubar)

= (u-ubar)- (d-dbar) = 1

INCLUDING the

x=1 Elastic contribution

Therefore, the inelastic part is reduced by the elastic x=1 term.

$$2 = g_A(q^2)^2 + F_1^V(q^2)^2 + q^2 F_2^V(q^2)^2 + \int_{M_N+M_\pi}^{\infty} \frac{W}{M_N} dW[\beta^{(-)}(q^2,W) - \beta^{(+)}(q^2,W)];$$

$\alpha = W_1$

$\beta = W_2$

•

Above is integral of F2(□w) d□w/ □w
Since: □w=
[Q'2 +B] / [M□ (1+(1+Q2/□2)) 1/2 +A]
At low Q2 □w= [Q'2 +B] / 2M□
where Q'2 = [Q2+ m_F^2] And B and A
to account for effects of additional Dm
from NLO and NNLO effects.

W2=M2+2 M□ - Q2
2W dW = 2 M d□

At fixed Q2 (W/M)dW= d□
□w= [Q'2 +B] / 2M□
d□w= [Q'2 +B] / 2M□ [d□/□]
d□ = □ d□w/ □w

What about the photproduction limit

•Q2	•C=0.178 •Q2/(Q2+C)	•Adler •Vector	•Adler •Axial	• •0.71 1-GD2
•0	•0.000	•0.000	•-0.588	•0
•0.1	•0.360	•0.227	•0.063	•0.410
•0.25	•0.585	•0.470	•0.525	•0.701
•0.5	•0.738	•0.701	•0.812	•0.881
•0.75	•0.809	•0.819	•0.911	•0.944
•1	•0.849	•0.883	•0.953	•0.970
•1.25	•0.876	•0.921	•0.973	•0.983
•1.5	•0.894	•0.945	•0.983	•0.989
•2	•0.918	•0.970	•0.993	•0.995
•2.25	•0.927	•0.977	•0.995	•0.997
•2.5	•0.934	•0.983	•0.996	•0.998
•2.75	•0.939	•0.986	•0.997	•0.998
•3	•0.944	•0.989	•0.998	•0.999
•4	•0.958	•0.995	•0.999	•0.999
•5	•0.966	•0.997	•1.000	•1.000
•10	•0.983	•1.000	•1.000	•1.000
•20	•0.991	•1.000	•1.000	•1.000
•100	•0.998	•1.000	•1.000	•1.000

SV= F1v**2 + Q2 F2v**2 =
(1/[1+Q2/4M2]) *
GD**2 {
[1+4.71*Q2/(4M2)]**2 + Q2}
And (1-SV) is the vector suppression
GD2 = 1/[1+Q2/0.71]**4

At Q2=0
(1-SV)/Q2 =
(4/0.71 – 3.71/(2M2)+1) =
4.527 = 1/0.221

Will give better photoproduction
Cross section.

Stein etal PRD 12, 1884 (1975) -- getting photoproduction cross sections

$$\sigma_{tot}(q^2, W) = \frac{1}{\Gamma_T}\frac{d^2\sigma}{d\Omega dE'} = \sigma_T + \epsilon\sigma_L$$
$$= \sigma_R(q^2, W) + \sigma_{bkd}(q^2, W)\,, \qquad (20)$$

where σ_R and σ_{bkd} are the resonance and background contributions to the cross sections. In order to remove some of the known kinematic variations, we write the structure function νW_2 as

$$\nu W_2(q^2, W) = [1 - W_2^{el}(q^2)]\, F_2(\omega')\, B(q^2, W)$$
$$\times \left[4\pi^2 \alpha F_2(\infty) \lim_{q^2\to 0} \frac{1 - W_2^{el}(q^2)}{q^2}\right]^{-1} \qquad (21)$$

where the term in the large square brackets is included so that

$$\lim_{q^2\to 0} B(q^2, W) = \sigma_{\gamma p}(W) \qquad (22)$$

and $\sigma_{\gamma p}(W)$ is the total photoproduction cross section. This makes

$$B(q^2, W) = \left[\frac{q^2}{1 - W_2^{el}(q^2)} \lim_{q^2\to 0} \frac{1 - W_2^{el}(q^2)}{q^2}\right]$$
$$\times \left(\frac{\nu K}{q^2 + \nu^2}\right)\left(\frac{1 + R}{1 + \epsilon R}\right)\left[\frac{F_2(\infty)}{F_2(\omega')}\right]\sigma_{tot}(q^2, W)\,, \qquad (23)$$

where we have used $R = 0.23q^2$ which has the cor-

We can use the above form from Stein et al except:

1. We use Xsiw instead of omega prime
2. We use F2 (Q2min where QCD freezes instead of F2 (infinity)
3. We use our form for 1-SV derived from Adler sum rule for [1-W2(elastic,Q2)]
4. We use R1998 for R instead of 0.23 Q2.
5. Limit (1-SV)/Q2 is now 4.527 or 1/0.221

What about the fact that Adler sum rule is for Uv-Dv as measured in vector and axial scattering, on light quarks, what above Strangeness Changing –

One could gets the factors for Dv and Uv separately by using the Adler sum rules for the STRANGNESS CHANGING (DS=+-1 proportional to sin2 of the Cabbibo angle)(where he gets 4, 2) if one knew the Lambda and Sigma form factors (F1v, F2v, Fa) as follows. Each gives vector and axial parts here cosTC and SinTc are for the Cabbibo Angle.

1. F2nub-p (DS=0)/cosTc = u +dbar (has neutron final state udd quasielatic)
2. F2nu-p (DS=0)/(costTc = d + ubar (only inelastic final states)
3. F2nub-p (DS+-1)/sinTc = u + sbar (has Lambda and Sigma0 uds qausi)
4. F2nu-p (DS+-1)=/sinTc= s + ubar (making uud + sbar continuum only))
5. F2nub-n (DS+-1) = d + sbar (has Sigma- =dds quasi)
6. F2nu-n (DS+-1)=s + ubar (making udd + sbar continuum only))

A. strangeness conserving is Equations 1 minus 2 = Uv-DV = 1V+1A = 2 (and at Q2=0 has neutron quasielastic final state) (one for vector and one for axial)

B. strangeness changing on neutrons is Equation 5 minus 6 = Dv = 1V+1A = 2(and at Q2=0 has Sigma- qasielastic)

C. strangeness changing on protons is Equation 3 minus 4 = Uv = 2V+2A = 4 (and at Q2=0 has both Lambda0 and Sigma0 qausielastic. Note according to Physics reports artilce of Llwellyn Simth - DeltaI=1/2 rule has cross section for Simga0 at half the value of Sigma+).

What about charm? Need to see how these equations are modified (to be edited)

Need to add charm final states

1. F2nub-p (DS=0)/cosTc = u + dbar (has neutron final state udd quasielatic)
2. F2nu-p (DS=0)/(costTc = d + ubar (only inelastic final states)
3. F2nub-p (DS+-1)/sinTc = u + sbar (has Lambda and Sigma0 uds qausi)
4. F2nu-p (DS+-1)=/sinTc= s + ubar (making uud + sbar continuum only))
5. F2nub-n (DS+-1) = d + sbar (has Sigma- =dds quasi)
6. F2nu-n (DS+-1)=s + ubar (making udd + sbar continuum only))

A. strangeness conserving is Equations 1 minus 2 = Uv-DV = 1V+1A = 2 (and at Q2=0 has neutron quasielastic final state) (one for vector and one for axial)

B. strangeness changing on neutrons is Equation 5 minus 6 = Dv = 1V+1A = 2(and at Q2=0 has Sigma- qasielastic)

C. strangeness changing on protons is Equation 3 minus 4 = Uv = 2V+2A = 4 (and at Q2=0 has both Lambda0 and Sigma0 qausielastic. Note according to Physics reports artilce of Llwellyn Simth - DeltaI=1/2 rule has cross section for Simga0 at half the value of Sigma+).

.

Additional references to look at

Llewellyn Smith Phys. Reports C has most of the stuff

See also: S. Adler, Ann. Phys. 50 (1968) 189 where he does electroproduction and photoprduction in first resonance but has deviations at high Q2 (did not know about DIS)

For Lambda production in neutrino see: V. V. Ammosov et al JETP Letters 43, 716 (1986) and references in it to earlier Gargamelle data (comparison with LLS papers)

See K. H. Althoff et al Phys Lett. B37, 535 (1971) for Lamda S Form factors from decays.

For Charm production one needs to understand Charm Lambda C transition form factor to see what the low Q2 suppression is for the DIS. Is it the nucleon intital state form factor or the final state smaller LambdaC. Probably initital state/

For example http://jhep.sissa.it/archive/prhep/preproceeding/003/020/stanton.pdf

Form factors in charm meson semileptonic decays.

By E791 Collaboration (N. Stanton for the collaboration). 1999. 8pp.

Prepared for 8th International Symposium on Heavy Flavor Physics (Heavy Flavors 8), Southampton, England, 25-29 Jul 1999.

Says for single pole fits MV=2.1 and MA=2.5 for the D meson

Versus

Analysis of pion-helium scattering for the pion charge form factor.

By C.T. Mottershead (UC, Berkeley). 1972.

Published in Phys.Rev.D6:780-797,1972 which gives

For Gaussian and

Yukawa pion charge distributions. The results indicate 2.2<r pi <3.2 F

Modified LO PDFs for all Q^2 (including 0)

Results for Scaling variable

- $\xi_w = [Q^2+B] / [M\nu (1+(1+Q^2/\nu^2)^{1/2}) + A]$
- A=0.418 GeV^2, B=0.222 GeV^2 (from fit)
- A=initial binding/target mass effect plus NLO +NNLO terms)
- B= final state mass Δm^2 from gluons plus initial Pt.
- Very good fit with modified GRV98LO
- χ^2 = 1268 / 1200 DOF
- Next: Compare to Prediction for data not included in the fit

1. Compare with SLAC/Jlab resonance data (not used in our fit) -> A (w, Q^2)
2. Compare with photo production data (not used in our fit)-> check on K production threshold
3. Compare with medium energy neutrino data (not used in our fit)- except to the extent that GRV98LO originally included very high energy data on xF_3

FIT results for K photo-production threshold

$F_2(x, Q^2) = K * F_{2QCD}(\xi_w, Q^2) * A(w, Q^2)$

$F_2(x, Q^2 < 0.8) = K * F_2(\xi_w, Q^2=0.8)$

For sea Quarks we use

$K = Ksea = Q^2 / [Q^2+Csea]$

Csea = 0.381 GeV^2 (from fit)

For valence quarks (in order to satisfy the Adler Sum rule which is exact down to Q2=0) we use

K = Kvalence

$= [1- G_D{}^2(Q^2)] [Q^2+C2V] / [Q^2+C1V]$

$G_D{}^2(Q^2) = 1/[1+Q^2/0.71]^4$

= elastic nucleon dipole form factor squared. we get from the fit

C1V = 0.604 GeV^2 , C2V = 0.485 GeV^2

Which Near Q^2 =0 is equivalent to:

$Kvalence \sim Q^2 / [Q^2+Cvalence]$

With Cvalence=(0.71/4)*C1V/C2V= =0.221 GeV^2

[Ref:Bodek and Yang hep-ex/0203009]

Adler Sum rule EXACT all the way down to Q^2=0 includes W_2 quasi-elastic

Origin of low Q2 K factor for Valence Quarks

- □- = W_2 (Anti-neutrino -Proton)
- □+ = W_2 (Neutrino-Proton) q0=□

The vector current part of the original sum rule of Adler for neutrino scattering can be written

$$\int_0^\infty dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(+)}(q_0,q^2)]=1. \quad (18)$$

If we explicitly separate out the nucleon Born term in Eq. (18), we have

$$[F_1^V(q^2)]^2+q^2\left(\frac{\mu^V}{2M_N}\right)^2[F_2^V(q^2)]^2 + \int_{M_\pi+(q^2+M_\pi^2)/2M_N}^\infty dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(8)}(q_0,q^2)]=1,$$

Vector Part of W2

$$g_A(q^2)+\int_{M_\pi+(q^2+M_\pi^2)/2M_N}^\infty dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(8)}(q_0,q^2)]=1,$$

AXIAL Vector part of W_2

Adler is a number sum rule at high Q^2

$$\int_0^\infty dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(+)}(q_0,q^2)] = 1 \text{ is}$$

$$\int_0^1 \frac{[F_2^{□}(□) □ F_2^{+}(□)]}{□} d□ = \int_0^1 [U_v(□) □ D_v(□)] d□ = 2 □ 1$$

F_2^-= F_2 (Anti-neutrino -Proton) = □W_2

F_2^+= F_2 (Neutrino-Proton) = □W_2

we use: d (q0) = d (□) = (□)d □/ □

at fixed q^2= Q^2

[see Bodek and Yang hep-ex/0203009] and references therein

Valence Quarks

Fixed $q^2 = Q^2$

Adler Sum rule EXACT all the way down to $Q^2=0$ includes W_2 quasi-elastic

1 = Quasielastic δ-function $(F^-_2 - F^+_2)\, d\xi/\xi$ Integral Separated out + Integral of Inelastic $(F^-_2 - F^+_2)\, d\xi/\xi$ both resonances and DIS

$$g_V(q^2) = [F_1^V(q^2)]^2 + q^2\left(\frac{\mu^V}{2M_N}\right)^2 [F_2^V(q^2)]^2$$

For Vector Part of Uv-Dv the Form below F_2 will satisfy the Adler Number Sum rule

$$\frac{\int_0^{\xi\ \text{pion threshold}} [U_v^{QCD}(\xi_W) - U_v^{QCD}(\xi_W)][1 - g_V(Q^2)]\, d\xi_W/\xi_W}{N(Q^2) = \int_0^{\xi\ \text{pion threshold}} [\xi_W U_v^{QCD}(\xi_W) - \xi U_v^{QCD}(\xi_W)]\, d\xi_W/\xi_W} + g_V(Q^2) = 1$$

If we assume the same form for Uv and Dv --->

$$F_2^{VALENCE}(\xi_W, Q^2) = \frac{[V^{QCD}(\xi_W, Q^2)][1 - g_V(Q^2)]}{N(Q^2)}$$

[Ref:Bodek and Yang hep-ex/0203009]

Adler Sum rule EXACT all the way down to $Q^2=0$ includes W_2 quasi-elastic

$$F_2^{VALENCE\ Vector}(\xi_W, Q^2) = \frac{[\xi_W V^{QCD}(\xi_W, Q^2)][1 - g_V(Q^2)]}{N(Q^2)}$$

This form Satisfies Adler Number sum Rule at all fixed Q^2

$$\int_0^1 \frac{[F_2^{-}(\xi,Q^2) - F_2^{+}(\xi,Q^2)]}{\xi} d\xi = \int_0^1 [U_v(\xi) - D_v(\xi)] d\xi = 1\ exact$$

$F_2^{-} = F_2$ (Anti-neutrino -Proton)

$F_2^{+} = F_2$ (Neutrino-Proton)

$$\int_0^1 [F_2^{Valence}(\xi,Q^2) + F_2^{sea}(\xi,Q^2) + xg(\xi,Q^2)] d\xi \neq 1$$

While momentum sum Rule has QCD and Non Pertu. corrections

- Use : K = Kvalence = $[1 - G_D^2(Q^2)]\ [Q^2 + C2V] / [Q^2 + C1V]$
- Where C2V and C1V in the fit to account for both electric and magnetic terms
- And also account for $N(Q^2)$ which should go to 1 at high Q^2.
- This a form is consistent with the above expression (but is not exact since it assumes no dependence on ξ_w or W (assumes same form for resonance and DIS)
- Here: $G_D^2(Q^2) = 1/[1+Q^2/0.71]^4$ = elastic nucleon dipole form factor

[Ref:Bodek and Yang hep-ex/0203009]