Electromagnetic couplings of higher-mass N^* resonances†

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The previous partial-wave analysis of $\gamma N \to N\pi$ by Moorhouse, Oberlack, and Rosenfeld has been extended to 2000 MeV.

We have previously reported[1,2] partial-wave analyses of pion photoproduction data up to center-of-mass energy 1780 MeV which yielded photoproduction amplitudes and electromagnetic couplings of the N^* resonances[3] in the first, second, and third N^* resonance regions. We have now extended our energy region and analyzed data up to center-of-mass energy 1995 MeV, obtaining electromagnetic couplings, in the fourth resonance region, of $P_{13}(1800)$, $P_{31}(1860)$, $P_{33}(2000)$, $F_{35}(1860)$, and $F_{37}(1930)$. Previously, in analyzing to 1780 MeV we had only been able to obtain, in the fourth resonance region, indications on the couplings of the prominent $F_{37}(1930)$ through the low-energy tail of its real part; similarly we now obtain some preliminary and tentative indications on the couplings of the relatively prominent $H_{3,11}(2400)$.

Our method is the same as before,[1,2] with some trivial modifications: We have extended the region where we fit data up to 1995 MeV center-of-mass energy, and we have increased the upper limit of the fixed-t dispersion integration to 2440 MeV center-of-mass energy; we have incorporated additional high-energy resonances in the region between 1995 and 2500 MeV, and we have refined the integration treatment of these resonances outside our data region.[4] To our data set we have added 391 data points between 1780 and 1995 MeV—essentially all data available to us in that energy region. We have also added about 100 recent data points of the reaction $\gamma p \to \pi^0 p$ with a polarized target.[5] Simply for reasons of computer memory space we have had to take out about 600 lower-energy data points, so that our total data set now comprises 3806 data points on $\gamma p \to \pi^+ n$, $\gamma p \to \pi^0 p$, and $\gamma n \to \pi^- p$ from threshold to 1995 MeV center-of-mass energy. As before, our data set contains inconsistencies which we are unable to resolve.

Our four best fits have values of χ^2/data-point equal to 4.66, 4.56, 3.88, and 3.48; in the latter case we varied 98 electromagnetic couplings[6] of resonances and background and in the other cases somewhat fewer. A more detailed breakdown of the χ^2 ratio is given in Table I.

With our increased data range we now have numbers for the electromagnetic couplings of all those resonances that have been assigned to the $\{56\}_2$ $L = 2^+$ multiplet of the quark model, albeit that the couplings of the smaller angular momentum resonances have very large errors. We show these couplings from our present solutions, labeled KMO in Table II, where we also display the electromagnetic couplings of the resonances belonging to $\{70\}_1$ $L = 1^-$ multiplet; also shown for comparison are our previous solutions.[1,2] The extension of our fitted data to higher energy and the extension of our knowledge of the amplitudes are also very relevant to our fits in the lower-energy region of the $\{70\}_1$ $L = 1^-$ multiplet. This is because the real-part tails of the high-energy features (such as resonances) are important in the low-energy region: In our method these real-part tails enter through the fixed-t dispersion relation. Consequently we would expect a better determination of $\{70\}_1$ $L = 1^-$ couplings with our present program than previously.[1,2]

The couplings of the most prominent resonance of the $\{70\}_1$ $L = 1^-$, the $D_{13}(1512)$, are nearly the same as for our previous fits[1,2]; in particular, the helicity-$\frac{3}{2}$ couplings are not pure isovector (if pure, they would be equal and opposite), but have a small though non-negligible isoscalar component. Without further discussing the detailed comparison of the couplings of the $\{70\}_1$ $L = 1^-$ with our previous results,[1,2] we may say that the over-all picture is one of agreement, with no marked disagreements.

The comparison of the electromagnetic couplings with the unmixed quark model (evaluated as previously[1,2] in the four-dimensional oscillator ver-

TABLE I. The values of χ^2/(data point). Over-all χ^2/(data point) values for our different solutions are given in the last column. In the first 12 columns we show the χ^2/(data point) values separately for each type of reaction and observable quantity: σ denotes differential cross section, P the polarization of the recoil nucleon, Σ the asymmetry produced by linearly polarized photons, and T the asymmetry produced by polarization of the nucleon target.

Reaction	$\pi^+ n$				$\pi^0 p$				$\pi^- p$				All
Quantity	σ	P	Σ	T	σ	P	Σ	T	σ	P	Σ	T	All data
Number of data points (d.p.)	1614	12	91	24	1209	139	38	98	526	1	54	0	3806
						χ^2/d.p.							
Solution 1	3.34	5.51	2.62	1.89	6.91	3.95	3.09	1.98	4.89	0.64	3.64	···	4.66
Solution 2	4.66	5.45	3.18	2.28	5.01	3.57	3.03	3.27	4.32	0.00	3.25	···	4.56
Solution 3	3.46	5.05	2.64	3.46	5.06	3.58	3.25	1.44	3.48	2.54	3.25	···	3.88
Solution 4	3.40	6.07	2.18	4.22	4.13	3.25	4.03	1.42	2.85	0.02	2.76	···	3.48

TABLE II. Resonance couplings $A(\gamma NN^*)$. The result from the present partial-wave analysis (labeled KMO) is an average over four fits, and the error is the spread over the four fits. For comparison we show the results from previous fits (labeled MOR[2] and MO[1]), and also the quark-model result for the usual assignment of the resonance to an $\{SU(6)\}_n L^P$; $[SU(3), 2S+1]J^P$ multiplet, where the subscript n denotes harmonic-oscillator excitation number. The superscripts on the resonance couplings A refer to the isospin; for $I=\frac{3}{2}$ N^* resonances, there is only one γN coupling, denoted by A^{V3}. For $I=\frac{1}{2}$ N^* resonances, we use the two independent γp and γn couplings, denoted by A^p and A^n. The subscripts $\frac{1}{2}$ and $\frac{3}{2}$ denote the total γ-nucleon helicity. Units of the couplings are 10^{-3} GeV$^{-1/2}$. An asterisk on a quark-model prediction denotes that the result does not involve a difference of two terms. The resonance masses shown are averages over the four KMO fits (they are varied within a small range of about 5 MeV from our πN elastic scattering input values).

{SU(6)}L	N^*(mass) [SU(3), $2S_q+1$]J^P	$I=\frac{3}{2}$: / $I=\frac{1}{2}$:	$A^{V3}_{1/2}$ / $A^p_{1/2}$	$A^{V3}_{3/2}$ / $A^p_{3/2}$	$A^n_{1/2}$	$A^n_{3/2}$
$\{56\}_0$ $L=0^+$	$P_{33}(1232)$	KMO	-138 ± 4	-253 ± 2		
		MOR	-142 ± 1	-261 ± 1		
		MO	-142 ± 6	-259 ± 16		
	$[10,4]\frac{3}{2}^+$	Quarks	-108*	-187*		
$\{70\}_1$ $L=1^-$	$S_{11}(1535)$	KMO	56 ± 20		-52 ± 5	
		MOR	36 ± 2		-27 ± 9	
		MO	53 ± 20		-48 ± 21	
	$[8,2]\frac{1}{2}^-$	Quarks	156		-108	
	$D_{13}(1508)$	KMO	-19 ± 8	169 ± 12	-77 ± 5	-120 ± 10
		MOR	0 ± 6	174 ± 6	-88 ± 7	-119 ± 25
		MO	-26 ± 15	194 ± 31	-85 ± 14	-124 ± 13
	$[8,2]\frac{3}{2}^-$	Quarks	-34	109*	-31	-109*
	$S_{31}(1630)$	KMO	33 ± 15			
		MOR	78 ± 6			
		MO	90 ± 76			
	$[10,2]\frac{1}{2}^-$	Quarks	47			
	$D_{33}(1685)$	KMO	78 ± 9	70 ± 9		
		MOR	41 ± 28	21 ± 20		
		MO	68 ± 42	22 ± 52		
	$[10,2]\frac{3}{2}^-$	Quarks	88	84*		
	$S_{11}(1705)$	KMO	58 ± 18		-15 ± 35	
		MOR	54 ± 5		-82 ± 19	
		MO	66 ± 42		-72 ± 66	
	$[8,4]\frac{1}{2}^-$	Quarks	0		30	

TABLE II *(continued)*

{SU(6)}L	N^*(mass)	$I=\frac{3}{2}$:	$A^{V3}_{1/2}$	$A^{V3}_{3/2}$		
	[SU(3), $2S_q+1$] J^P	$I=\frac{1}{2}$:	$A^p_{1/2}$	$A^p_{3/2}$	$A^n_{1/2}$	$A^n_{3/2}$
$\{70\}_1$ $L=1^-$	$D_{13}(1690)$	KMO	−15 ± 40	30 ± 40 [a]	−36 ± 40	24 ± 24
		MOR	23 ± ?	35 ± ?	−15 ± ?	28 ± ?
		MO	3 ± ?	20 ± ?	−28 ± ?	27 ± ?
	$[8,4]\frac{3}{2}^-$	Quarks	0*	0*	−10*	−40*
	$D_{15}(1660)$	KMO	13 ± 14	14 ± 8	−43 ± 6	−71 ± 30
		MOR	19 ± 7	16 ± 2	−17 ± 4	−49 ± 4
		MO	11 ± 12	21 ± 20	10 ± 40	−35 ± 14
	$[8,4]\frac{5}{2}^-$	Quarks	0*	0*	−38*	−53*
$\{56\}_2$ $L=2^+$	$P_{13}(1770)$	KMO	−4 ± 32	−6 ± 30	14 ± 14	−8 ± 25
		MOR				
		MO				
	$[8,2]\frac{3}{2}^+$	Quarks	+100	−30	−30	0*
	$F_{15}(1680)$	KMO	−16 ± 14	97 ± 7	23 ± 5	1 ± 18
		MOR	−14 ± 3	147 ± 6	23 ± 3	−41 ± 4
		MO	−8 ± 4	100 ± 12	17 ± 14	−5 ± 18
	$[8,2]\frac{5}{2}^+$	Quarks	−10	60*	30	0*
	$P_{31}(1860)$	KMO	10 ± 12			
		MOR				
		MO				
	$[10,4]\frac{1}{2}^+$	Quarks	+30			
	$P_{33}(2000)$	KMO	−32 ± 20	13 ± 8		
		MOR				
		MO				
	$[10,4]\frac{3}{2}^+$	Quarks	+30	−50		
	$F_{35}(1860)$	KMO	42 ± 16	−22 ± 20		
		MOR	?	?		
		MO	−60 ± ?	−100 ± ?		
	$[10,4]\frac{5}{2}^+$	Quarks	−20	−90		
	$F_{37}(1920)$	KMO	−70 ± 12	−78 ± 10		
		MOR	−80	−180		
		MO	−133 ± 46	−100 ± 41		
	$[10,4]\frac{7}{2}^+$	Quarks	−50	−70*		
$\{56\}_2$ $L=0^+$	$P_{11}(1470)$	KMO	−66 ± 13		0 ± 13	
		MOR	−87 ± 2		33 ± 13	
		MO	−55 ± 28		2 ± 25	
	$[8,2]\frac{1}{2}^+$	Quarks	27*		−18*	
	$P_{33}(1718)$	KMO	3 ± 15	−34 ± 22		
		MOR				
		MO				
	$[10,4]\frac{3}{2}^+$	Quarks	34*	20*		
$\{70\}_2$ $L=0^+$	$P_{11}(1800)$	KMO	22 ± 15		27 ± 15	
		MOR	16 ± 25		57 ± 22	
		MO	26 ± 28		27 ± 22	
	$[8,2]\frac{1}{2}^+$	Quarks	−40*		10*	

[a] Solutions 1, 3, and 4 yield 57 ± 11, while solution 2 gives −9.

sion of Feynman, Kislinger, and Ravndal[7]) is also given in Table II. For the $\{70\}_1$ $L=1^-$ the striking agreements in sign have been commented on previously.[1,2] If we were to explain some discrepancies in magnitude by mixing, we would discern large mixing between the two S_{11} states and small mixing between the two D_{13} states. The $S_{31}(1630)$ and the $D_{33}(1685)$, which have no obvious mixing partners, have become satisfactorily close to the quark-model predicted values. The $D_{15}(1660)$ also has no evident mixing partners, and the smallness of the positive-charge couplings tends to verify the selection rule[8] on the vanishing of couplings of positive-charge resonances belonging to $[8,4]$ submultiplets of $\{70\}$ multiplets.

Turning now to the $\{56\}_2$ $L=2^+$ multiplet, we see that the $F_{15}(1680)$ couplings like the $D_{13}(1512)$ are satisfactory in sign and relative magnitude but, also like the $D_{13}(1512)$ and the $P_{33}(1232)$, those couplings that are predicted to be large are even larger than the prediction. There is consistency with the selection rule[9] on the vanishing of the helicity-$\frac{3}{2}$ couplings of charge-zero resonances of $[8,2]$ submultiplets of $\{56\}$ multiplets.

For the first time[10] we publish a close determination of the couplings of the $F_{37}(1930)$ and we see a good agreement with the quark model. On the other hand, there is a disagreement for the $F_{35}(1860)$. The quark model suggests a possible mixing partner for the $F_{35}(1860)$ (though *not* for the F_{37}) in the F_{35} member of the $\{70\}_2$ $L=2^+$ multiplet. The existence of this multiplet is not only an *a priori* theoretical suggestion of the symmetric quark model, but is also additionally suggested, on the basis of that model, by the probable existence of an $F_{17}(2000)$.[11] And of course, if such an F_{35} resonance exists, say, at a mass of around 2000 MeV,[11] then its presence, unallowed for in our analysis, could markedly affect the value we obtain for the couplings of the $F_{35}(1860)$ since those couplings would be doing double duty. Such problems could arise for all members of the $\{56\}_2$ $L=2^+$ except the F_{37}.

The selection rule[12] on the vanishing of the $E2$ (E_{1+}) transition on the $P_{33}(1232)$ can be extended to the $E4$ (E_{3+}) transition of the $F_{37}(1930)$ and the $E6$ (E_{5+}) transition of the $H_{3,11}(2400)$. We display the comparison in Table III, and we see good agreement for the $P_{33}(1232)$, as is well known; the agreement is almost as good for the $F_{37}(1930)$. The comparison for the $H_{3,11}$ is highly tentative since that resonance is well outside our data region and is only observed through the contribution to the fixed-t dispersion relation (that is, essentially the low-energy tail of the real part of the resonance). In our previous work[1,2] we gave similar tentative numbers for the F_{37} which, though of the correct sign, were of larger magnitude than our present numbers (similarly our numbers for the $H_{3,11}$ have now the quark-model sign, but are much too large in magnitude); so we present these $H_{3,11}$ ratios more in the spirit of a challenge to experiment and to partial-wave analysis.

In a recent paper, Devenish, Rankin, and Lyth[13] also used a method of fixed-t dispersion relations to fit 2913 data points over an energy range through the third resonance region, comparable with our previous fits.[1,2] If we compare our present values for resonances within their range, we see a fair agreement, though the errors they quote are generally much bigger than the spread of our present four solutions.

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TABLE III. The ratio $R = A_{3/2}/A_{1/2}$, from the Becchi-Morpurgo selection rule (Ref. 12) $E_{l+}=0$, is compared with R from our four partial-wave solutions.

Resonance	R (selection rule)	R from solutions			
		I	II	III	IV
$P_{33}(1232)$	+1.73	+1.86	+1.83	+1.87	+1.78
$F_{37}(1930)$	+1.29	+1.23	+1.14	+1.10	+1.00
$H_{3,11}(2400)$	+1.18	+0.562	+0.75	+0.79	+1.52

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[3]We include both $N_{1/2}$ resonances and $N_{3/2}(\Delta)$ resonances under the general term "N^* resonances."

[4]Previously the fixed-t dispersion integrand whose energy was above the data region was approximated by a δ-function representation of the imaginary parts of the high-energy resonances. We now take an exact treatment of these resonances in the integrand.

[5]J. R. Holt *et al.*, contribution of the International Symposium on Electron and Photon Interactions at High Energies, Bonn, 1973 (unpublished).

[6]This number does not include the special parameters (Ref. 2) we used as a precaution against the nonconvergence of the partial-wave series and which seem to be relatively uninfluential and to have little effect upon χ^2.

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[10]The only other fit over the energy range of the F_{37} that we know of is from a fit by W. J. Metcalf and R. L. Walker [Caltech Report No. CALT-68-425, 1974 (unpublished)]. They give -59 and -89 for $A_{1/2}$ and $A_{3/2}$, respectively.

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Threshold connection between semi-inclusive deep electroproduction and annihilation

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A unified discussion is given of the scaling limit for one-particle inclusive electroproduction, $e+p\to e+\text{hadron}+\text{anything}$, and for its crossed reactions such as $e^++e^-\to\bar{p}+\text{hadron}+\text{anything}$, $p+\text{hadron}\to e^++e^-+\text{anything}$, etc. General threshold relations, which hold also in absence of analytic continuation across the kinematical thresholds, are derived, generalizing previous results for the scaling functions of deep-inelastic scattering and annihilation.

It was pointed out by Drell, Levy, and Yan[1] that for a class of graphs in the cutoff Yukawa theory the scaling functions for e^+-e^- annihilation are analytic continuations of those for inelastic electron scattering. Detailed investigations[2] have shown, however, that this is not true in general, because of certain "double discontinuity" graphs. Nevertheless, it was possible to show that a threshold ($\omega=1$) connection is still expected to hold between the two processes, independently of analytical continuation.[3] Experimentally, two-particle inclusive e^+e^- annihilation is more directly related to the triggering system required for colliding-beam experiments, whereas detector limitations may render more difficult the verification of the threshold relation for one-particle inclusive annihilation. For this reason, among others, we have undertaken the effort to provide the threshold relations among the inclusive processes with two observed hadrons (initial or final). Similarly, as for deep-inelastic scattering and annihilation and on the same assumptions, we have derived such threshold relations which again hold independently of analytic continuation.

We consider the processes (l stands for lepton and h for hadron)

(A) $l+\bar{l}\to h_1+h_2+\text{anything}$,

(B) $h_1+h_2\to l+\bar{l}+\text{anything}$,

(C) $l+\bar{l}+h_1\to h_2+\text{anything}$,

(D) $l+h_1\to l+h_2+\text{anything}$,

(E) $l+\bar{l}+h_2\to h_1+\text{anything}$,