Chapter 2 workshop module

1. Spend a few minutes looking over the following problems, paying particular attention to the problems that you think you might have trouble with. All of the problems are taken from an introductory physics course on mechanics, so this should seem like review material. After you have had some time to look over the problems, you will take turns stepping up to the board to solve one. When it is your turn, you may pick ANY of the problems that have not already been solved. Depending on the number of students in the recitation, you may be asked to solve more than one problem. Good luck!

   (a) Justin fires a 12-gram bullet into a block of wood. The bullet travels at 190 m/s, penetrates the 2.0-kg block of wood, and emerges going 150 m/s. If the block is stationary on a frictionless surface when hit, how fast does it move after the bullet emerges?

   (b) A mass m at the end of a spring vibrates with a frequency of 0.88 Hz; when an additional 1.25 kg mass is added to m, the frequency is 0.48 Hz. What is the value of m?

   (c) Dan has a new chandelier in his living room. The chandelier is 27-kg and it hangs from the ceiling on a vertical 4.0-m-long wire. What horizontal force would Dan need to use to displace its position 0.10 m to one side? What will be the tension in the wire?

   (d) Dianne has a new spring with a spring constant of 900 N/m that she bought at Springs-R-Us. She places it vertically on a table and compresses it by 0.150 m. What upward speed can it give to a 0.300-kg ball when released?

   (e) A tiger leaps horizontally from a 6.5-m-high rock with a speed of 4.0 m/s. How far from the base of the rock will she land?

   (f) How much work must SuperRyan do to stop a 1300-kg car traveling at 100 km/hr?

   (g) Jason catches a baseball 3.1 s after throwing it vertically upward. With what speed did he throw it and what height did it reach?

   (h) Laura is practicing her figure skating and during her finale she can increase her rotation rate from an initial rate of 1.0 rev every 2.0 s to a final rate of 3.0 rev/s. If her initial moment of inertia was 4.6 kg·m^2, what is her final moment of inertia?

   (i) On an icy day in Rochester (imagine that!), you worry about parking your car in your driveway, which has an incline of 12°. Your neighbor Emily's driveway has an incline of 9°, and Brian's driveway across the street has one of 6°. The coefficient of static friction between tire rubber and ice is 0.15. Which driveway(s) will be safe to park in?

2. Two particles are projected from the same point with velocities v_1 and v_2, at elevations α_1 and α_2, respectively ($\alpha_1 > \alpha_2$). Show that if they are to collide in mid-air the interval between the firings must be

$$\frac{2v_1 v_2 \sin(\alpha_1 - \alpha_2)}{g(v_1 \cos\alpha_1 + v_2 \cos\alpha_2)}.$$

   (If you don't have time to solve this problem completely, then at least give an outline of how you would go about solving the problem.)

3. Read each of the following statements and, without consulting anyone else, mark them true or false. If you are unsure of any of them, make a guess. Once everyone has answered each of the statements individually, break into small groups and compare your answers. Try to come to an agreement as a group. The TA will then make sure everyone has the correct answer. Good luck!

   (a) The conservation of linear momentum is a consequence of translational symmetry, or the homogeneity of space.

   (b) For an isolated system with no external forces acting on it, the angular momentum will remain constant in both magnitude and direction.

   (c) A reference frame is called an inertial frame if Newton's laws are valid in that frame.

(d) Newtonian mechanics and the laws of electromagnetism are invariant under Galilean transformations.

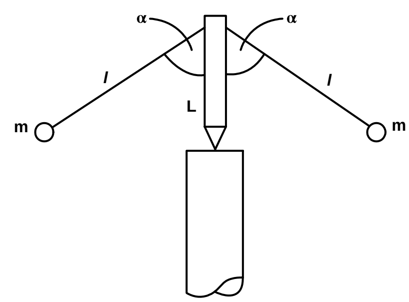
(e) The law of conservation of angular momentum is a consequence of rotational symmetry, or the isotropy of space.

(f) The center of mass of a system of particles moves like a single particle of mass M (total mass of the system) acted on by a single force F that is equal to the sum of all the external forces acting on the system.

(g) If Newton's laws are valid in one reference frame, then they are also valid in any reference frame accelerated with respect to the first system.

(h) The law of conservation of energy is a consequence of inversion symmetry, or the invertibility of space.

4. The teeter totter comprises two identical weights.which hang on drooping arms attached to a peg as shown. The arrangement is unexpectedly stable and can be spun and rocked with little danger of toppling over.



(a) Find an expression for the potential energy of the teeter toy as a function of θ when the teeter toy is cocked at an angle θ about the pivot point. For simplicity, consider only rocking motion in the vertical plane.

(b) Determine the equilibrium values(s) of θ.

(c) Determine whether the equilibrium is stable, unstable, or neutral for the value(s) of θ found in part (b).

(d) How could you determine the answers to parts (b) and (c) from a graph of the potential energy versus θ?

(e) Expand the expression for the potential energy about $\theta = 0$ and determine the frequency of small oscillations.

5. For each of the situations described below, determine which of the four functional forms of the force is most appropriate. Consider motion only along one dimension.

- Constant force: $F = constant$
- Time-dependent force: $F = F(t)$
- Velocity-dependent force: $F = F(v)$
- Distance-dependent force: $F = F(x)$

Go around the room and take turns answering a question. When it is your turn, pick a functional form and explain why you chose the one you did. If you are unsure, make a guess or ask a question to get help from the rest of the workshop. There may be more than one answer depending on your interpretation of the situation, so be sure to explore all of the possibilities.

(a) A mass resting on a frictionless table is attached to a spring, which in turn is attached to a wall. The mass is pulled to the side and executes simple harmonic motion in the horizontal direction.
(b) A freely-falling body subject to a constant gravitational field with no air resistance.
(c) An electron, initially at rest (treat it classically!), encounters an incoming electromagnetic wave of electric field intensity E given by $E = E_0 \sin(\omega t + \phi)$.
(d) A large mass is affected by the gravitational field of another mass a distance d away.
(e) A freely-falling body subject to a constant gravitational field with air resistance.
(f) A charged point particle is affected by the presence of another charged point particle a distance d away.

6. A particle of mass m is constrained to move on the frictionless inner surface of a cone of half-angle α.

   (a) Find the restrictions on the initial conditions such that the particle moves in a circular orbit about the vertical axis.
   (b) Determine whether this kind of orbit is stable.A particle of mass m is constrained to move on the frictionless inner surface of a cone of half-angle α, as shown in the figure.

7. Consider a thin rod of length L and mass M.

   (a) Draw gravitational field lines and equipotential lines for the rod. What can you say about the equipotential surfaces of the rod?
   (b) Calculate the gravitational potential at a point P that is a distance r from one end of the rod and in a direction perpendicular to the rod.
   (c) Calculate the gravitational field at P by direct integration.
   (d) Could you have used Gauss's law to find the gravitational field at P? Why or why not?

8. *Consider a single particle of mass m.

   (a) Determine the position r and velocity v of a particle in spherical coordinates.
   (b) Determine the total mechanical energy of the particle in potential V.
   (c) Assume the force is conservative. Show that $F = -\nabla V$. Show that it agrees with Stoke's theorem.
   (d) Show that the angular momentum $L = r \times p$ of the particle is conserved. Hint: $\frac{\mathrm{d}}{\mathrm{d}t}(A \times B) = A \times \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} + \frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} \times B$.

9. *Consider a fluid with density ρ and velocity v in some volume V. The mass current $J = \rho v$ determines the amount of mass exiting the surface per unit time by the integral $\int_S J \cdot dA$.

   (a) Using the divergence theorem, prove the continuity equation, $\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$

10. *A rocket of initial mass M burns fuel at constant rate k (kilograms per second), producing a constant force f. The total mass of available fuel is m_o. Assume the rocket starts from rest and moves in a fixed direction with no external forces acting on it.

   (a) Determine the equation of motion of the rocket.
   (b) Determine the final velocity of the rocket.
   (c) Determine the displacement of the rocket in time.

* Diagnostic questions