Classical Mechanics - Module 6

1. Consider a diatomic molecule which has a symmetry axis along the line through the center of the two atoms comprising the molecule. Consider that this molecule is rotating about an axis perpendicular to the symmetry axis and that there are no external forces acting on the molecule. Use Noether's Theorem to answer the following questions:

   a) Is the total angular momentum conserved?

   b) Is the projection of the total angular momentum along a space-fixed z axis conserved?

   c) Is the projection of the angular momentum along the symmetry axis of the rotating molecule conserved?

   d) Is the projection of the angular momentum perpendicular to the rotating symmetry axis conserved?

2. A bead of mass m slides under gravity along a smooth wire bent in the shape of a parabola $x^2 = az$ in the vertical (x, z) plane.

   (a) What kind (holonomic, nonholonomic, scleronomic, rheonomic) of constraint acts on m?
   (b) Set up Lagrange's equation of motion for x with the constraint embedded.
   (c) Set up Lagrange's equations of motion for both x and z with the constraint adjoined and a Lagrangian multiplier λ introduced.
   (d) Show that the same equation of motion for x results from either of the methods used in part (b) or part (c).
   (e) Express λ in terms of x and $\dot{x}$.
   (f) What are the x and z components of the force of constraint in terms of x and $\dot{x}$?

3. A block of mass m rests on an inclined plane making an angle θ with the horizontal. The inclined plane (a triangular block of mass M) is free to slide horizontally without friction. The block of mass m is also free to slide on the larger block of mass M without friction.

   (a) Construct the Lagrangian function.
   (b) Derive the equations of motion for this system.
   (c) Calculate the canonical momenta.
   (d) Construct the Hamiltonian function.
   (e) Find which of the two momenta found in part (c) is a constant of motion and discuss why it is so. If the two blocks start from rest, what is the value of this constant of motion?

4. Discuss among yourselves the following four conditions that can exist for the Hamiltonian and give several examples of systems exhibiting each of the four conditions.

   a) The Hamiltonian is conserved and equals the total mechanical energy

   b) The Hamiltonian is conserved but does not equal the total mechanical energy

   c) The Hamiltonioan is not conserved but does equal the total mechanical energy

   d) The Hamiltonian is not conserved and does not equal the mechanical total energy

5. Compare the Lagrangian formalism and the Hamiltonian formalism by creating a two-column chart. Label one side "Lagrangian" and the other side "Hamiltonian" and discuss the similarities and differences. Here are some ideas to get you started:

   - What are the basic variables in each formalism?
   - What are the form and number of the equations of motion derived in each case?
   - How does the Lagrangian "state space" compare to the Hamiltonian "phase space"?