Classical Mechanics - Module 8

1. Consider a fixed reference frame S and a rotating frame S'. The origins of the two coordinate systems always coincide. By carefully drawing a diagram, derive an expression relating the coordinates of a point P in the two systems. (This was covered in Appendix D, but it is worth reviewing now.

2. The effective force observed in a rotating coordinate system is given by

$$\mathbf{F}_{rot}^{eff} = m\mathbf{a}''_{rot} = \mathbf{F}_{fix} - m\left(\mathbf{A}_{fix} + 2\boldsymbol{\omega} \times \mathbf{v}''_{rot} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}'_{mov}) + \dot{\boldsymbol{\omega}} \times \mathbf{r}'_{mov}\right) \tag{1}$$

   (a) What is the significance of each term in this expression?

   (b) Suppose you wanted to measure the gravitational force, both magnitude and direction, on a body of mass m at rest on the surface of the Earth. What terms in the effective force can be neglected?

   (c) Suppose you wanted to calculate the deflection of a projectile fired horizontally along the Earth's surface. What terms in the effective force can be neglected?

   (d) Suppose you wanted to calculate the effective force on a small block of mass m placed on a frictionless turntable rotating with a time-dependent angular velocity $\omega(t)$. What terms in the effective force can be neglected?

3. A plumb line is carried along in a moving train, with m the mass of the plumb bob. Neglect any effects due to the rotation of the Earth and work in the noninertial frame of reference of the train.

   (a) Find the tension in the cord and the deflection from the local vertical if the train is moving with constant acceleration a_0.

   (b) Find the tension in the cord and the deflection from the local vertical if the train is rounding a curve of radius ρ with constant speed v_0.

4. A bead on a rotating rod is free to slide without friction. The rod has a length L and rotates about its end with angular velocity ω. The bead is initially released from rest (relative to the rod) at the midpoint of the rod.

   (a) Find the displacement of the bead along the wire as a function of time.

   (b) Find the time when the bead leaves the end of the rod.

   (c) Find the velocity (relative to the rod) of the bead when it leaves the end of the rod.

5. Here is a "thought experiment" for you to consider. Suppose you are in a small sailboat of mass M at the Earth's equator. At the equator there is very little wind (this is known as the "equatorial doldrums"), so your sailboat is, more or less, sitting still. You have a small anchor of mass m on deck and a single mast of height h in the middle of the boat. How can you use the anchor to put the boat into motion? In which direction will the boat move?

6. Does water really flow in the other direction when you flush a toilet in the southern hemisphere? What (if anything) does the Coriolis force have to do with this?