# P235 - PROBLEM SET 10

To be handed in by **1800 hr on Friday, 20 November 2015**.

1. Four identical masses $m$ are connected by four identical springs, spring constant $\kappa$, and constrained to move on a frictionless circle of radius $b$ as shown in the figure.

   a) How many normal modes of small oscillation are there?

   b) What are the eigen frequencies of the small oscillations?

   c) Describe the motion of the four masses for each eigen frequency.

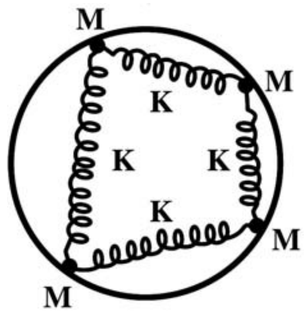
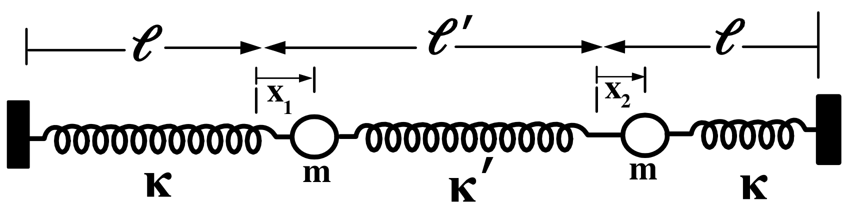



Figure 1:

2. Consider the two identical coupled oscillators given on the right in the figure assuming $\kappa_1 = \kappa_2 = \kappa$. Let both oscillators be linearly damped with a damping constant $\beta$. A force $F = F_0 \cos(\omega t)$ is applied to mass $m_1$. Write down the pair of coupled differential equations that describe the motion. Obtain a solution by expressing the differential equations in terms of the normal coordinates. Show that the normal ccordinates $\eta_1$ and $\eta_2$ exhibit resonance peaks at the characteristic frequencies $\omega_1$ and $\omega_2$ respectively.




3. Three oscillators of equal mass $m$ are coupled such that the potential energy of the system is given by

$$U = \frac{1}{2}\left[\kappa_1\left(x_1^2 + x_3^2\right) + \kappa_2 x_2^2 + \kappa_3\left(x_1 x_2 + x_2 x_3\right)\right]$$

   where $\kappa_3 = \sqrt{2\kappa_1\kappa_2}$.

   a) Find the eigen frequencies by solving the secular equation.

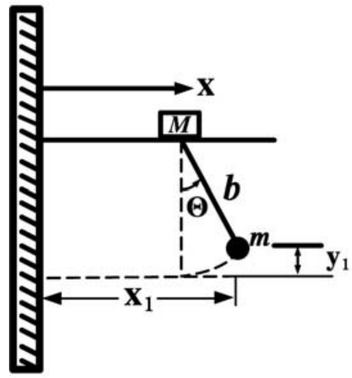
   b) What is the meaning of the zero-frequency mode?

4. As shown the mass $M$ moves horizontally along a frictionless rail. A pendulum is hung from $M$ with a weightless rod of length $b$ with a mass $m$ at its end.

a) Prove that the eigenfrequencies are

$$\begin{aligned}
\omega_1 &= 0 \\
\omega_2 &= \sqrt{\frac{g}{Mb}(M+m)}
\end{aligned}$$

b) Describe the normal modes.




5. A thin hoop of radius $R$ and mass $M$ oscillates in its own plane hanging from a single fixed point of the hoop as shown below. Attached to the hoop is a small mass $M$ at the point $Q$ that is constrained to move (in a frictionless manner) along the hoop. Consider only small amplitude oscillations, and show that the eigenfrequencies are

$$\begin{aligned}
\omega_1 &= \sqrt{2}\sqrt{\frac{g}{R}} \\
\omega_2 &= \frac{\sqrt{2}}{2}\sqrt{\frac{g}{R}}
\end{aligned}$$

Find the two sets of initial conditions that allow the system to oscillate in its normal modes. Describe the physical situation for each mode.