P235 - PROBLEM SET 4

To be handed in by **1700 hr on Friday, 2 October 2015**.

1. A sphere of radius ρ is constrained to roll without slipping on the lower half of the inner surface of a hollow cylinder of radius R. Detemine the Lagrangian function, the equation of constraint, and the Lagrange equations of motion, Find the frequency of small oscillations.

2. A particle moves in a plane under the influence of a force $f = -Ar^{\alpha-1}$ directed toward the origin; A and $\alpha\,(>0)$ are constants. Choose appropriate generalized coordinates, and let the potential energy be zero at the origin.

   a) Find the Lagrangian equations of motion.

   b) Is the angular momentum about the origin conserved?

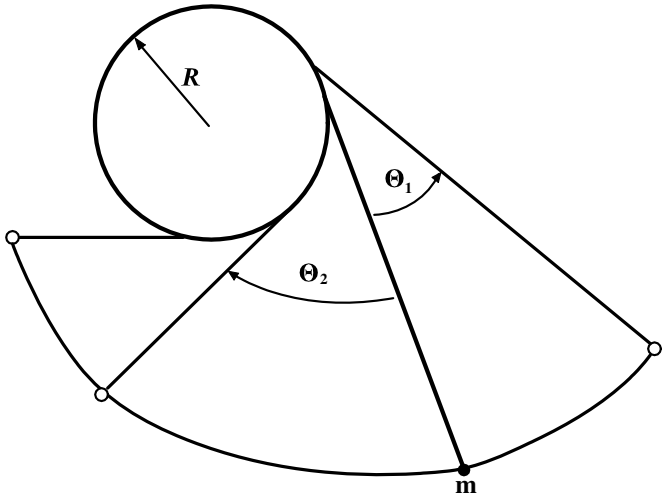
   c) Is the total energy conserved?

3. A pendulum is constructed by attaching a mass m to an extensionless string of length l. The upper end of the string is connected to the uppermost point on a vertical disk of radius R, $(R < l/\pi)$ as shown in the figure.

   a) Obtain the pendulum's equation of motion,

   b) Find the frequency of small oscillations.

   c) Find the line about which the angular motion extends equally in either direction (i.e. $\theta_1 = \theta_2$).



4. Two blocks, each of mass M, are connected by an extensionless, uniform string of length l. One block is placed on a frictionless horizontal surface, and the other block hangs over the side, the string passing over a frictionless pulley. Describe the motion of the system:

   a) when the mass of the string is negligble

   b) when the string has mass m.

5. Two masses m_1 and m_2 $(m_1 \neq m_2)$ are connected by a rigid rod of length d and of negligible mass. An extensionless string of length l_1 is attached to m_1 and connected to a fixed point of the support P. Similarly a string of length l_2 $(l_1 \neq l_2)$ connects m_2 and P. Obtain the equation of motion describing the motion in the plane of m_1, m_2, and P, and find the frequency of small oscillation around the equilibrium position.