# Astronomy 203 Problem Set #2: Solutions

28 September 1999

1. Caustics and rainbows. *Consider light incident a distance y from the center on a dielectric sphere with radius r and index n, that is refracted at the surface, reflects internally at the next, and then refracts out of the sphere, as shown in the figure below.*

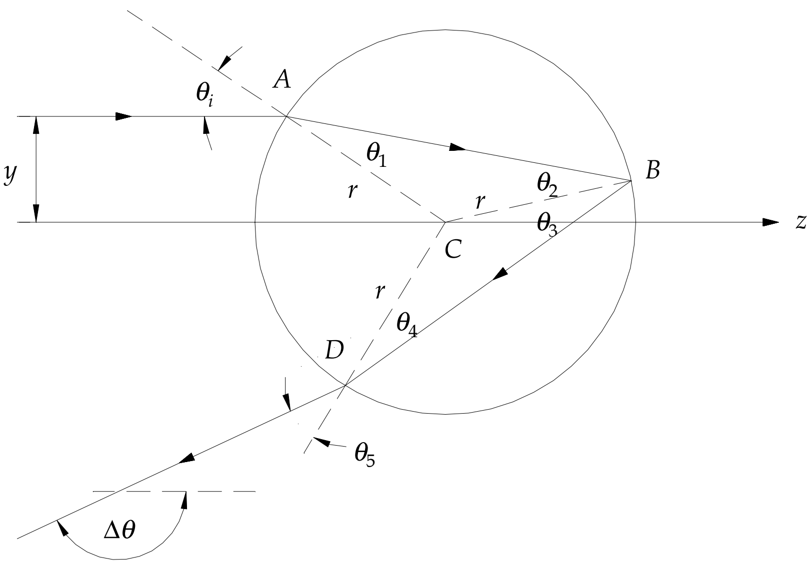


Figure 1: setup for Problem 1.

a. *Show that as one varies y from zero to r, the angle $\Delta\theta$ by which the ray is deviated by the sphere passes through a minimum. Obtain expressions for the minimum value of $\Delta\theta$ and for the value of y that corresponds to this minimum.*

*(**Hint**: prove first that all the angles of incidence, reflection and refraction within the sphere, $\theta_{1\text{-}4}$, have the same magnitude.)*

*(**Hint II**: don't try to apply the matrix formulation of ray-tracing all the way through.)*

First, the geometry. Note in Figure 1 that the two triangles *ABC* and *BCD* within the sphere that are composed of the rays and the radii drawn to the ray-sphere intersections are *isosceles,* because two sides of each triangle are radii and thus equal in length. In isosceles triangles the angles opposite the congruent sides are congruent, so $\theta_1 = \theta_2$ and $\theta_3 = \theta_4$. The ray reflects at point *B*, so $\theta_2 = \theta_3$, and thus all the labelled angles inside the sphere have the same measure. Furthermore,

$$\theta_5 = \arcsin(n \sin\theta_4) = \arcsin(n \sin\theta_1) = \theta_i \qquad (1)$$

Let's call the angles $\theta\,(=\theta_i = \theta_5)$ and $\theta'\,(=\theta_1 = \theta_2 = \theta_3 = \theta_4)$ henceforth.

Put the coordinate origin at the center of the sphere, so that the equation for the circle in Figure 1 is $y^2 + z^2 = r^2$. Then the equation that describes the left half of the circle, on which the light is

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incident, is $z = -\sqrt{r^2 - y^2}$ (note that the negative root is taken). At point *A*, the slope of the sphere is

$$\frac{dz}{dy} = \frac{y}{\sqrt{r^2 - y^2}} = \tan\theta \quad , \tag{2}$$

whence

$$\begin{aligned} \sin\theta &= \frac{y}{\sqrt{\left(r^2 - y^2\right) + y^2}} = \frac{y}{r} \quad , \\ \cos\theta &= -\frac{\sqrt{r^2 - y^2}}{\sqrt{\left(r^2 - y^2\right) + y^2}} = -\sqrt{1 - \frac{y^2}{r^2}} \quad . \end{aligned} \tag{3}$$

(Note that the signs work out so that the surface-normal unit vector, $\boldsymbol{\nu}'$, points at the center of the sphere.) By Snell's Law, we have

$$\sin\theta' = \frac{1}{n}\sin\theta = \frac{y}{nr} \quad . \tag{4}$$

This is really all we need to know to express $\Delta\theta$ in terms of *y*. At point *A*, the ray is deviated by an angle $\theta - \theta'$. At point *B,* it is deviated further by an amount $\pi - 2\theta'$. At point *D*, the last deviation, like the first, is by the amount $\theta - \theta'$. The total deviation is the sum of these:

$$\begin{aligned} \Delta\theta &= 2\theta - 4\theta' + \pi \\ &= 2\arcsin\left(\frac{y}{r}\right) - 4\arcsin\left(\frac{y}{nr}\right) + \pi \end{aligned} \tag{5}$$

As usual, we look for extrema by taking

$$\begin{aligned} \frac{d}{dy}\Delta\theta &= \frac{2}{\sqrt{r^2 - y^2}} - \frac{4}{\sqrt{n^2 r^2 - y^2}} = 0 \quad \text{at } y = y_0, \\ \text{or} \qquad r^2 - y_0^2 &= \frac{1}{4}\left(n^2 r^2 - y_0^2\right) \quad , \quad \text{and} \\ y_0 &= \frac{r}{3}\sqrt{12 - 3n^2} \quad . \end{aligned} \tag{6}$$

Thus no (real) extremum exists for $n > 2$. Take the second derivative to see what sort of extremum we have:

$$\frac{d^2}{dy^2}\Delta\theta = \frac{2y}{\left(r^2 - y^2\right)^{3/2}} - \frac{4y}{\left(n^2 r^2 - y^2\right)^{3/2}} \tag{7}$$

Substitute $y_0$ from Equation (6) into this and you get

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$$\frac{d^2}{dy^2}\Delta\theta(y_0) = \frac{\frac{2r}{3}\sqrt{12-3n^2}}{\left(r^2 - \frac{r^2}{9}\left[12-3n^2\right]\right)^{3/2}} - \frac{\frac{4r}{3}\sqrt{12-3n^2}}{\left(n^2r^2 - \frac{r^2}{9}\left[12-3n^2\right]\right)^{3/2}}$$

$$= \frac{2\sqrt{12-3n^2}}{3r^2}\left[\frac{1}{\left(\frac{n^2-1}{3}\right)^{3/2}} - \frac{2}{\left(\frac{4n^2-4}{3}\right)^{3/2}}\right] \tag{8}$$

$$= \frac{2\sqrt{12-3n^2}}{3r^2}\left[\left(\frac{3}{n^2-1}\right)^{3/2} - \frac{1}{4}\left(\frac{3}{n^2-1}\right)^{3/2}\right]$$

$$= \frac{\sqrt{12-3n^2}}{2r^2}\left(\frac{3}{n^2-1}\right)^{3/2} > 0 \quad ,$$

so if the extremum exists, it's a minimum. The minimum value of $\Delta\theta$ is given by putting the result (6) into Equation (5):

$$\Delta\theta_{\text{min}} = 2\arcsin\left(\frac{y_0}{r}\right) - 4\arcsin\left(\frac{y_0}{nr}\right) + \pi$$
$$= 2\arcsin\left(\frac{1}{3}\sqrt{12-3n^2}\right) - 4\arcsin\left(\frac{1}{3n}\sqrt{12-3n^2}\right) + \pi \quad . \tag{9}$$

b. *Make a sketch of the situation for rays incident on the sphere at several different values of y between zero and r. Use the result to argue that a uniformly illuminated sphere appears brightest in scattered light when viewed near the direction of this minimum-deviation ray, because the deviation angle passes through a minimum.*

*Such minima (or maxima) in deviation angles, and corresponding intensity peaks, produced by optical elements are called* caustics.

If the sphere is uniformly illuminated, the scattered light will be most intense in directions in which a particularly large number of rays are directed. This is the case for scattering angles near $\Delta\theta_{\text{min}}$; for light incident both at slightly larger and slightly smaller $y$, the rays will reflect and refract near the "minimum" ray: one can see the pile-up (caustic) clearly in Figure 2. Still brighter caustics are seen within the sphere.

c. *Water is a dielectric with refractive index $n \approx 4/3$; raindrops are spherical blobs of water. Use these facts with the results from parts a and b to explain how rainbows work. Calculate the angle from the direction of incident sunlight at which one should look to see a rainbow.*

Sunlight falls on raindrops with an angular spread of only about half a degree – that is, the sun's rays are nearly parallel. Thus a peak in scattered sunlight will appear at an angle

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$$\Delta\theta_{\min} = 2\arcsin\left(\frac{1}{3}\sqrt{12-3\left(\frac{4}{3}\right)^2}\right) - 4\arcsin\left(\frac{1}{4}\sqrt{12-3\left(\frac{4}{3}\right)^2}\right) + \pi = 2.4080 \text{ radians} = 137.97^\circ \quad . \tag{10}$$

from the direction of sunlight, or $42.03^\circ$ from the spot on the sky opposite the Sun. (That is, one sees light from all the drops for which the sun-drop-observer angle is $137.97^\circ$.) The refractive index of water varies with wavelength – it's about 1.330 at λ = 0.7 μm (red) and 1.342 at 0.4 μm (violet) – so the peaks in the intensity of scattered light of shorter wavelength occur at slightly larger $\Delta\theta$. We would, and do, see this scattered light as a circular arc on the sky, blue on the inside and red on the outside: a rainbow.

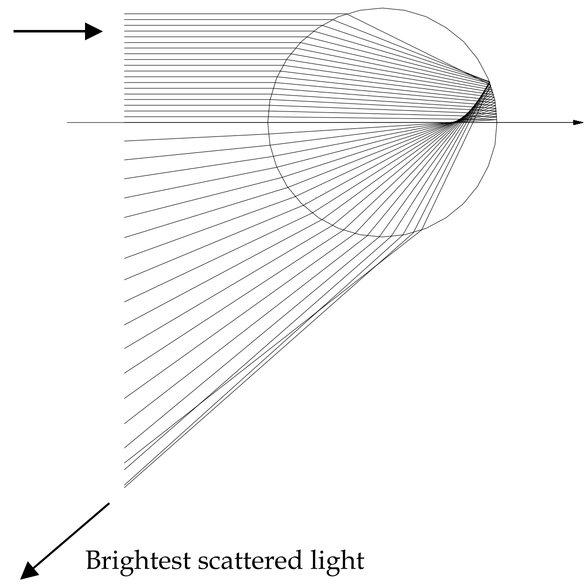


Figure 2: Uniform-intensity light incident on the upper half of a dielectric sphere, and the intensity peak in scattered light from the minimum in the deviation angle (a caustic).

The rainbow we have just considered is called the *primary* rainbow. Light that takes another internal reflection also leads to a caustic, and gives rise to the *secondary* rainbow. For this rainbow, $\Delta\theta$ turns out to be $130^\circ$ (as you can easily show!), so its circular arc on the sky is about eight degrees larger in radius than the primary, and the colors are reversed. One interesting aspect of the two rainbows is that the primary caustic is a minimum in $\Delta\theta$, but the secondary is a maximum: no light in the first two rainbows is scattered into the range $\Delta\theta = 130 - 138^\circ$. Verify this next time you see a double rainbow: the sky indeed appears to be darker between the rainbows than it does on either side of them. (This effect is known as "Alexander's dark band," after Alexander of Aphrodisias, who first described it in writing, around the turn of the third century A.D.)

2. *Consider a concave paraboloidal mirror of on-axis focal length 4 m, with incident light rays intersecting its surface at its center and at a point 0.5 m from the optical axis. If the two incident rays are inclined at an angle of 1° to the paraboloid's optical axis, but still share the same incidence plane, where do the reflected rays intersect each other? Where do the reflected rays intersect the paraxial focal plane?*

   This problem is only slightly different from Example 4.1. We shall set the problem up such that the plane of incidence (and the plane of reflection) is the *x-z* plane, as shown in Figure 1. In this coordinate system, φ and δ are both zero.

   The incident rays have the same direction cosines:

$$v = \begin{bmatrix}\gamma\\ \delta\\ \varepsilon\end{bmatrix} = \begin{bmatrix}\sin\psi\\ 0\\ \cos\psi\end{bmatrix} = \begin{bmatrix}0.01745\\ 0\\ 0.99985\end{bmatrix} \quad . \tag{11}$$


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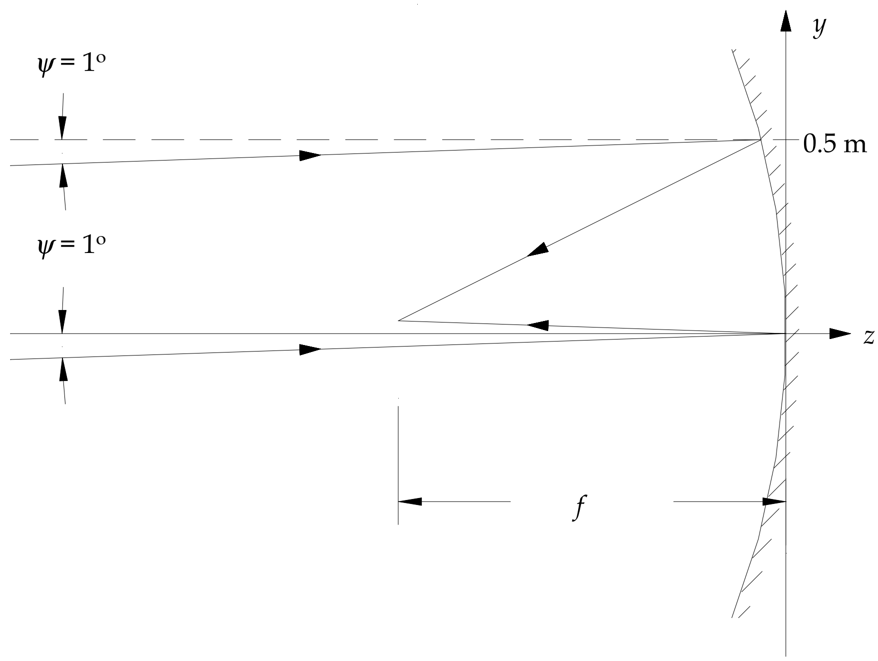


Figure 3: setup for problem 2.

As drawn, the equation describing the parabola is

$$z(x) = -\frac{x^2}{4f} \quad , \tag{12}$$

so the slope of the parabola is

$$\frac{dz}{dx} = -\frac{x}{2f} \quad , \tag{13}$$

and the slope of the *normal* to the parabola is

$$-\left[\frac{dz}{dx}\right]^{-1} = \frac{2f}{x} \quad . \tag{14}$$

Since we have calculated these in such a way that a slope is equal to tan $\theta$, where $\theta$ is the angle the line makes to the $x$-axis, the direction cosines of the normal unit vector $\mathbf{v}'$ are obtained from

$$\begin{aligned} &\tan\theta' = \frac{2f}{x} \\ &\gamma'(x) = \cos\theta' = -\frac{x}{\sqrt{4f^2 + x^2}} \quad , \\ &\varepsilon'(x) = \sin\theta' = -\frac{2f}{\sqrt{4f^2 + x^2}} \end{aligned} \tag{15}$$

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where the minus signs are chosen, as in Example 4.1, to give $\cos\theta_i = \mathbf{v}\cdot\mathbf{v}'$ the same sign as the ratio of refractive indices: negative, since reflection is considered. For the two rays incident at $x$ = 0 and $x$ = 500 mm at ψ = 1°, this comes to

$$\mathbf{v}'(x) = \begin{bmatrix} \gamma'(x) \\ \delta'(x) \\ \varepsilon'(x) \end{bmatrix} \quad , \qquad \mathbf{v}'(0) = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad , \qquad \mathbf{v}'(500\text{ mm}) = \begin{bmatrix} -0.06238 \\ 0 \\ -0.99805 \end{bmatrix} \quad . \tag{16}$$

Now we can calculate the incidence angle from $\cos\theta_i = \mathbf{v}\cdot\mathbf{v}'$:

$$\cos\theta_i = \mathbf{v}\cdot\mathbf{v}' = \gamma\gamma'(x) + \varepsilon\varepsilon'(x) \quad . \tag{17}$$

The direction cosines of the outgoing ray are given by the usual matrix equation. Since all of the action is in the $x$-$z$ plane, this equation becomes

$$\begin{aligned} \begin{bmatrix} \gamma & 0 & \varepsilon \\ \gamma'(x) & 0 & \varepsilon'(x) \\ 0 & \varepsilon'(x)\gamma - \gamma'(x)\varepsilon & 0 \end{bmatrix} \begin{bmatrix} \gamma'' \\ \delta'' \\ \varepsilon'' \end{bmatrix} &= \begin{bmatrix} -\cos 2\theta_r \\ \cos\theta_r \\ 0 \end{bmatrix} = \begin{bmatrix} 1 - 2\cos^2\theta_i \\ -\cos\theta_i \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 - 2\left(\gamma\gamma'(x) + \varepsilon\varepsilon'(x)\right)^2 \\ -\left(\gamma\gamma'(x) + \varepsilon\varepsilon'(x)\right) \\ 0 \end{bmatrix} \end{aligned} \quad . \tag{18}$$

The determinant of the coefficient matrix is

$$D = -\left(\varepsilon'(x)\gamma - \gamma'(x)\varepsilon\right) \begin{vmatrix} \gamma & \varepsilon \\ \gamma'(x) & \varepsilon'(x) \end{vmatrix} = -\left(\varepsilon'(x)\gamma - \gamma'(x)\varepsilon\right)^2 \quad . \tag{19}$$

Cramer's rule can be used here to obtain the direction cosines for the reflected ray:

$$\begin{aligned} \gamma''(x) &= \frac{1}{D} \begin{vmatrix} 1 - 2\left(\gamma\gamma'(x) + \varepsilon\varepsilon'(x)\right)^2 & 0 & \varepsilon \\ -\left(\gamma\gamma'(x) + \varepsilon\varepsilon'(x)\right) & 0 & \varepsilon'(x) \\ 0 & \varepsilon'(x)\gamma - \gamma'(x)\varepsilon & 0 \end{vmatrix} \\ &= -\frac{\varepsilon'(x)\gamma - \gamma'(x)\varepsilon}{D} \begin{vmatrix} 1 - 2\left(\gamma\gamma'(x) + \varepsilon\varepsilon'(x)\right)^2 & \varepsilon \\ -\left(\gamma\gamma'(x) + \varepsilon\varepsilon'(x)\right) & \varepsilon'(x) \end{vmatrix} \\ &= \frac{\varepsilon'(x)\left(1 - 2\left(\gamma\gamma'(x) + \varepsilon\varepsilon'(x)\right)^2\right) + \varepsilon\left(\gamma\gamma'(x) + \varepsilon\varepsilon'(x)\right)}{\varepsilon'(x)\gamma - \gamma'(x)\varepsilon} \quad ; \end{aligned} \tag{20}$$

$$\delta''(x) = 0 \quad ; \tag{21}$$

$$\begin{aligned}\varepsilon''(x) &= \frac{1}{D}\begin{vmatrix}\gamma & 0 & 1-2(\gamma\gamma'(x)+\varepsilon\varepsilon'(x))^2 \\ \gamma'(x) & 0 & -(\gamma\gamma'(x)+\varepsilon\varepsilon'(x)) \\ 0 & \varepsilon'(x)\gamma-\gamma'(x)\varepsilon & 0\end{vmatrix} \\ &= -\frac{\varepsilon'(x)\gamma-\gamma'(x)\varepsilon}{D}\begin{vmatrix}\gamma & 1-2(\gamma\gamma'(x)+\varepsilon\varepsilon'(x))^2 \\ \gamma'(x) & -(\gamma\gamma'(x)+\varepsilon\varepsilon'(x))\end{vmatrix} \\ &= -\frac{\gamma'(x)\left(1-2(\gamma\gamma'(x)+\varepsilon\varepsilon'(x))^2\right)+\gamma(\gamma\gamma'(x)+\varepsilon\varepsilon'(x))}{\varepsilon'(x)\gamma-\gamma'(x)\varepsilon} \quad .\end{aligned} \tag{22}$$

Putting the numbers in for the components of $\mathbf{v}$ and $\mathbf{v}'$ (see Equations (11) and (16)), the resulting reflected rays look like this:

$$\mathbf{v}''(x) = \begin{bmatrix}\gamma''(x) \\ \delta''(x) \\ \varepsilon''(x)\end{bmatrix} \quad , \qquad \mathbf{v}''(0) = \begin{bmatrix}0.01745 \\ 0 \\ -0.99985\end{bmatrix} \quad , \qquad \mathbf{v}''(500\text{ mm}) = \begin{bmatrix}-0.10718 \\ 0 \\ -0.99424\end{bmatrix} \quad . \tag{23}$$

Now we can figure out where they wind up after reflection. The equation of a line with the ray's slope, passing through the reflection point $(x,z)$, is

$$x''(x) = \frac{\gamma''(x)}{\varepsilon''(x)}\left(z''(x)-z(x)\right)+x \quad , \tag{24}$$

and for $z''$ = -4000 mm, we get the location of our two rays in the paraxial focal plane:

$$x''(0) = 69.820\text{ mm} \quad , \qquad x''(500\text{ mm}) = 70.488\text{ mm} \quad . \tag{25}$$

— that is, they miss each other in the focal plane by $\Delta x$ = 0.668 mm, in contrast to the behavior of the on-axis rays that reflect from the same two points on the mirror. This is an example of *coma*. Where do the rays intersect? We have two equations in two unknowns:

$$x'' = \frac{\gamma''(0)}{\varepsilon''(0)}z'' \quad , \qquad x'' = \frac{\gamma''(500\text{ mm})}{\varepsilon''(500\text{ mm})}\left(z''-15.625\text{ mm}\right)+500\text{ mm} \tag{26}$$

that can be solved easily: $x''$ = 69.91 mm, $z''$ = -4005.33 mm.

One can use RayTrace 5.0 to confirm these answers. (Try using **`DX = 1, NP = NR = 1, CR = List`**, and **`RX = 0`** and **`500`** to trace single rays that hit the mirror at $x$ = 0 and 500 mm.) In fact, RayTrace 5.0 does the calculation exactly like we did. Every dot in your spot diagram represents several calculations like that. Aren't you glad computers don't get bored?

3. *For the paraboloidal mirror of problem 2, use RayTrace 5.0, with a bullseye ray pattern, to plot spot diagrams for bundles of parallel rays incident*

   a *parallel to the paraboloid's axis, and*

   b. *inclined at an angle of 1° to the paraboloid's axis.*

   *Compare these results to those obtained in class (Example 4.1, for the on-axis rays) and in problem 2.*

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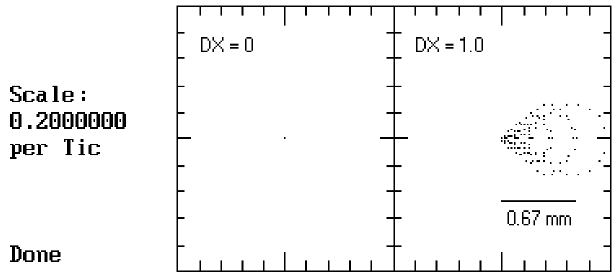


Figure 4: results of the ray traces for problems 3a (left) and 3b (right), plotted at the same scale, 0.2 mm per tic.

The result is shown in Figure 4. Important prescription parameters: **`RA = -8000, EC = 1, FT = -4000, FI = -1, FI = Far field, RP = Bull's eye, DX = 0, 1`**. I used 100 rays in the trace. Note in the context of Problem 2 that for the off-axis case a point was indeed produced about 0.67 mm from the central ray. All of the on-axis rays focus to one point, as expected from Example 4.1..

***Choose and work out only one of the following three problems. Share your results with your classmates as indicated.***

I have to work out all three, of course. Your sympathy is hereby accepted.

4. A telescope with coma. *Consider a Cassegrain telescope with a paraboloidal primary mirror with diameter 2 m and focal length 4 m. The secondary mirror is a hyperboloid with eccentricity 2.0, 0.5 m in diameter.*

   a. *Compute the radius of curvature of this mirror at its apex, and its position relative to the primary, if it produces a Cassegrain focus at the apex of the primary, with a plate scale 1/3 that of the prime focus.*

   The hyperboloid has two focal lengths, $f_1$ and $f_2$. It needs to place the Cassegrain image 4 m away from the prime focus, so

$$f_1 + f_2 = 4 \text{ m} \quad . \tag{27}$$

   Since $f = c \pm a$ for hyperboloids, this mirror has

$$c - a + c + a = 4 \text{ m} \qquad \Rightarrow \qquad c = 2 \text{ m} \quad , \tag{28}$$

   or, since the eccentricity is $\varepsilon = c/a = 2$,

$$\begin{aligned} a &= 1 \text{ m} \quad , \\ f_1 &= c - a = 1 \text{ m} \quad , \\ f_2 &= c + a = 3 \text{ m} \quad . \end{aligned} \tag{29}$$

   The radius of curvature at the apex is

$$r = \frac{b^2}{a} = \frac{c^2 - a^2}{a} = 3 \text{ m} \quad . \tag{30}$$

   Since

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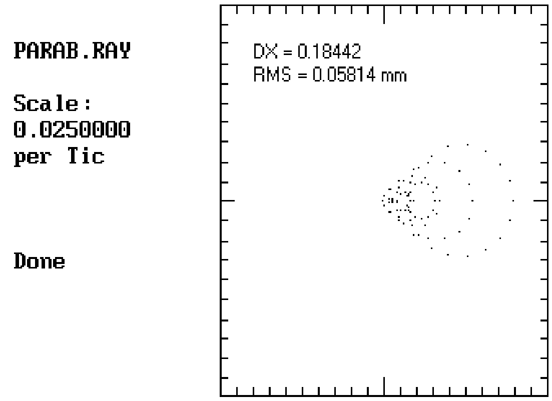


Figure 5: spot diagram for the Cassegrain telescope of problem 4, for rays incident at $\theta_{UD}$. The plot scale is 0.025 mm per tic.

$$\frac{f_2}{f_1} = 3 \quad , \tag{31}$$

a magnification of 3 results from the choice of a convex hyperboloid with its closer focus coincident with the paraboloid's focus. Thus, you should make the mirror with a radius of curvature of 3 m and put its apex 3 m from the apex of the primary.

b. *What is the Cassegrain plate scale, in arcsec mm$^{-1}$?*

The paraboloid's plate scale is

$$\frac{1}{f} = 0.25 \text{ rad m}^{-1} = 51.6 \text{ arcsec mm}^{-1} \quad ,$$

and the Cassegrain plate scale is 1/3 of that, because of the magnification, or 17.2 arcsec mm$^{-1}$.

c. *Using RayTrace 5.0, generate spot diagrams in the Cassegrain focal plane for ray bundles incident on the primary mirror, and use the RMS diameter of the spot pattern (computed by the program as* **`Sm_RMS`***) to identify the off-axis angle $\theta_{UD}$ that produces a distorted image equal in size to the atmospheric-turbulence-broadened image (the "seeing disk"), taken to be 1 arcsec in diameter. One usually calls $2\theta_{UD}$ the* unblurred field of view *of the telescope. Plot a spot diagram for the offset angle $\theta_{UD}$.*

*An example optical prescription called CASS.RAY is provided for your convenience in setting up this problem; from inside RayTrace 5.0, one can gain access to this prescription by use of the* **`Fetch`** *command.*

In RayTrace's terms, here are the dimensions of the telescope, in millimeters: **`AP = 2000, RA = -8000, EC = 1, FI = -1, FT = -3000`** for the primary mirror, and **`AP = 500, RA = -3000, EC = 2, FI = 1, FT = 3000`** for the secondary mirror. If you want to be fancy, you can include the part of the primary obscured by the secondary mirror as an extra surface in between the mirrors, as is done in the example prescription; in this case you would use **`FT = 0`** for the primary, and **`AP = 2000, FT = -3000, FI = -1, SS = CO`**, and **`S1 = 500`** for the secondary's shadow. This has a minor effect on the appearance of the spot diagram, notable only for the absence of the central ray in the result. By varying **`DX`** and/or **`DY`** in the trace menu, one

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can find the offset angle that produces an arcsecond's worth of blur, 0.058 mm (1/17.2). I get **`RMS = 0.05814`** at **`DX = 0.184°`**, or

$$2\theta_{UD} = 21' \quad .$$

Coma is the dominant aberration, as you can see in spot diagram for this offset angle, shown in Figure 5. On-axis rays (**`DX = DY = 0`**) all converge on a single point, as expected.

d. *Ask some of your classmates who did the other two problems to show you their results for $2\theta_{UD}$. Compare the performance of this telescope with the other two. Which one offers the widest unblurred field of view?*

I'll make the comparisons as I go along during the next two solutions.

5. A telescope without coma. *Repeat problem 4, using the Ritchey-Chretien telescope from problem 1: same dimensions as the Cassegrain but with primary eccentricity 1.0368 and secondary eccentricity 2.2389. Again, plot a spot diagram for the offset angle $\theta_{UD}$. Compare and comment on the appearances of the aberrations in the Cassegrain and Ritchey -- Chretien telescopes.*

a,b Since this R-C telescope has the same dimensions and focal point as the Cassegrain, it must have the same effective focal length and plate scale as well. See, therefore, the solution for parts a and b of problem 4.

c,d Input for the prescription: **`AP = 2000, RA = -8000, EC = 1, FI = -1, FT = -3000`** for the primary mirror, and **`AP = 500, RA = -3000, EC = 2, FI = 1, FT = 3000`** for the secondary mirror. The shadow of the secondary would be included in exactly the same way as the classical Cassegrain telescope. The unblurred field of view for the R-C comes out a bit larger than for the Cassegrain: I get **`Sm_RMS = 0.05814`** at **`DX = 0.21610`**, so $2\theta_{UD} = 26'$. There's no coma in the R-C; note in the spot diagram shown in Figure 4 that all the spots are centered about the ones from the central rays. This feature, and the elongation of the spot, are your clue that the dominant aberration is astigmatism. For **`DX = DY = 0`** the spot winds up with **`Sm_RMS = 0.000179 mm`**, still very much smaller than seeing or diffraction, although it's not a point, and the best focus is slightly more than 3 m from the secondary. There turns out to be a little bit of uncorrected spherical aberration present.

Because the R-C can give a bigger picture for the same blur, or less blur for the same size picture, the design is adopted more frequently as time goes on; for instance, in the Hubble Space Telescope.

6. A telescope with spherical aberrations. *Same as problem 4, but with spherical mirrors (eccentricity = 0) having the same size and apex curvature, and replacing parts c and d with the questions below.*

a,b Once again, this telescope has the same dimensions and close to the same focal point, and therefore practically the same plate scale, as the other two. See the solutions to parts a and b, problem 4.

c. *What is the position of the best focus? (That is, where does the "circle of least confusion" lie along the optical axis, judging from the RMS diameter of the focal-plane spot diagram?) Why is it different from the Cassegrain and Ritchey-Chretien telescopes?*

Judging by the minimum RMS spot size for 100 rays in a bullseye pattern, the focal plane is 2.8945 m past the secondary (0.11 m above the vertex of the primary). Thin-mirror geometrical optics

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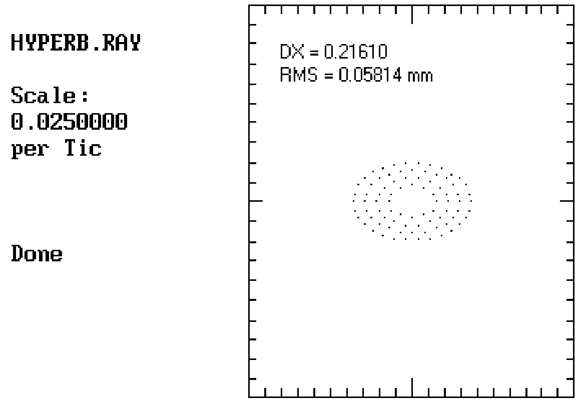


Figure 6: spot diagram for the Ritchey-Chretien telescope of problem 5, for rays incident at $\theta_{UD}$. The plot scale is 0.025 mm per tic. Compare this plot to Figure 5, which is done to the same scale.

tells you that it should be 3.00 m, same as for the Cassegrain and Ritchey-Chretien telescopes — here you see the effect of spherical aberration.

d. *Plot a spot diagram for the central focus (i.e. on-axis rays, DX = DY = 0). What is the angle on the sky that corresponds to the RMS diameter of this distorted image? Compare the performance of this telescope with the other two.*

The RMS diameter of the blurry on-axis image comes out to 1.9159 mm (34”), as is shown in Figure 7. Most of the blur is spherical aberration from the primary mirror. (How can you tell that it's not the secondary’s fault?) Note that there was very little geometrical blurring for the central focus in the Cassegrain and Ritchey-Chretien telescopes. By the standards of optical and infrared astronomy, this spherical-mirror telescope is useless.

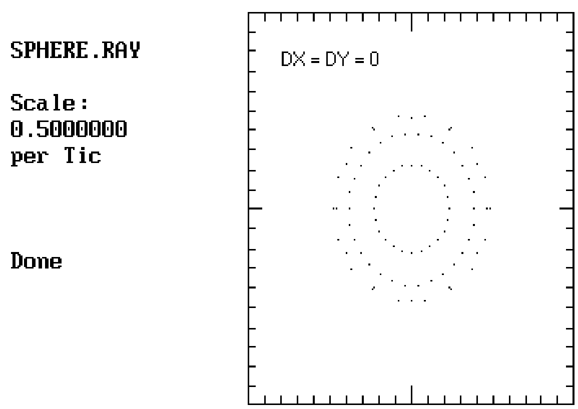


Figure 7: spot diagram for the spherical-mirror telescope of problem 6, for on-axis rays and the best focus. The plot scale is 0.5 mm per tic.