Useful equations

1. **Radioisotope age measurement for igneous rocks** (see lecture notes for 31 August 2021):

$$t = \frac{t_{1/2}}{\ln 2} \ln\left(\frac{D_A - D_B}{N_A - N_B} + 1 \right)$$

where $t_{1/2}$ is the halflife of the radioactive species.

N_A, N_B are the present-day concentrations of the radioisotope in minerals A and B, respectively. Concentration means the numbers of atoms of the radioisotope divided by the numbers of atoms of the non-radiogenic reference isotope.

D_A, D_B are the present-day concentrations of the daughter isotope in minerals A and B, respectively.

ln means "the natural logarithm of;" e.g. ln 2 = 0.693147…

t is the time since the igneous rock solidified, that provided the mineral samples A and B.

2. **Surface temperature of a *rapidly*-rotating, spherical, starlit body in outer space** (see lecture notes for 16 September 2021):

$$T = \left(\frac{(1-A)L}{16\pi\sigma r^2} \right)^{1/4} = 279\ \mathrm{K} \times \left(\frac{(1-A)L\left[L_\odot\right]}{\left(r[\mathrm{AU}]\right)^2} \right)^{1/4} \quad \text{or} \quad r[\mathrm{AU}] = \left(\frac{279\ \mathrm{K}}{T} \right)^2 \sqrt{(1-A)L\left[L_\odot\right]}$$

where A is the albedo, or reflectivity, of the spherical body's surface.

L is the luminosity, or total power output in light, of the star which illuminates the spherical body – in whatever system of units – and $L\left[L_\odot\right]$ is the same luminosity in units of the Sun's luminosity.

r is the distance between star and sphere – in whatever system of units – and $r[\mathrm{AU}]$ is that distance expressed in astronomical units (AU), the distance between Earth and Sun.

σ is the Stefan-Boltzmann constant, 5.67×10^{-5} erg sec^{-1}cm^{-2}K^{-4}.

T is the temperature of the sphere's surface, which comes out in units of degrees Kelvin (K) if all has gone well.

3. **Radius of exoplanet from transit light curve** (see lecture notes for 21 September 2021):

$$R_P = R_* \sqrt{1 - \frac{F_T}{F}} \quad ,$$

where	R_*	is the radius of the exoplanet's host star.
	F	is the flux from the star outside the exoplanet's transit.
	F_T	is the flux from the star during the exoplanet's transit.
	R_P	is the radius of the exoplanet.

4. **Radius of exoplanet orbit from orbital period** (see lecture notes for 21 September 2021):

$$r[\mathrm{AU}] = \left(M_*[M_\odot]P[\mathrm{year}]^2\right)^{1/3}$$

where	M_*	is the mass of the exoplanet's host star, and $M_*[M_\odot]$ is that mass in units of solar masses.
	P	is the orbital period of the exoplanet, and $P[\mathrm{year}]$ is that period in units of one Earth year.
	r	is the radius of the exoplanet's orbit around its host star, and $r[\mathrm{AU}]$ is that same radius expressed in AU, the distance between Earth and Sun.

5. **Surface temperature of a *slowly*-rotating, spherical, starlit body in outer space** (see lecture notes for 23 September 2021):

$$T = \left(\frac{(1-A)L}{4\pi\sigma r^2}\right)^{1/4} = 394\ \mathrm{K} \times \left(\frac{(1-A)L[L_\odot]}{(r[\mathrm{AU}])^2}\right)^{1/4} \quad \text{or} \quad r[\mathrm{AU}] = \left(\frac{394\ \mathrm{K}}{T}\right)^2 \sqrt{(1-A)L[L_\odot]}$$

where the symbols mean the same things they did for the fast rotator:

A	is the albedo, or reflectivity, of the slowly-rotating spherical body's surface.
L	is the luminosity, or total power output in light, of the star which illuminates the spherical body – in whatever system of units – and $L[L_\odot]$ is the same luminosity in units of the Sun's luminosity.
r	is the distance between star and sphere – in whatever system of units – and $r[\mathrm{AU}]$ is that distance expressed in astronomical units (AU), the distance between Earth and Sun.
σ	is the Stefan-Boltzmann constant, 5.67×10^{-5} erg $\mathrm{sec^{-1}cm^{-2}K^{-4}}$.
T	is the temperature of the sphere's surface, which comes out in units of degrees Kelvin (K) if all has gone well.

6. **The habitable zone** (see lecture notes for 23 September 2015). The inner and outer radii of the habitable zone are given by the equations above (3-4) as solved for *r*:

$$r[\mathrm{AU}] = \left(\frac{T_0}{T}\right)^2 \sqrt{(1-A)L[L_\odot]}$$

where the symbols mean the same things they did for the fast rotator:

A	is the albedo, or reflectivity, of the hypothetical habitable planets, usually taken to be about 0.3.
L	again is the luminosity, or total power output in light, of the star which illuminates the spherical body – in whatever system of units – and $L[L_\odot]$ is the same luminosity in units of the Sun's luminosity.
r	is the distance between star and the edge of the habitable zone which corresponds with temperature T – in whatever system of units – and $r[\mathrm{AU}]$ is that distance expressed in astronomical units (AU), the distance between Earth and Sun.
T_0	is 279 K for rapidly-rotating planets and 394 K for slowly-rotating ones.
T	is the temperature at the top of the atmosphere (e.g. the cloud tops) for a habitable planet placed at radius r. For an atmosphere with *no* greenhouse effect, T = 273 K and 373 K – the freezing and boiling points of pure water at standard atmospheric pressure – give the outer and inner radii of the habitable zone. For a realistic atmosphere, the greenhouse effect can be accounted for to pretty good approximation by taking T = 182 K and 249 K at the cloud tops, which will produce *surface* temperatures of 273 K and 373 K. For bodies too cold for water it is safe to assume no greenhouse effect, and to use the freezing and boiling points of the solvents, e.g. T = 90 and 109 K for methane.

7. **Radiocarbon (^{14}C) dating** (see lecture notes for 26 October 2021). The age t of organic remains, since the death of the animal or plant of which they were part, is given by

$$t = \frac{t_{1/2}}{\ln 2}\ln\left(\frac{n_0}{n(t)}\right) = \frac{t_{1/2}}{\ln 2}\ln\left(\frac{r_0}{r(t)}\right) \quad ,$$

where

$t_{1/2}$	is the radioactive halflife of ^{14}C, 5730 years.
ln	means to take the natural (base-*e* logarithm), e.g. ln 2 = 0.693.
n_0, r_0	are the original ^{14}C concentration and decay rate for contemporary living organisms. Those which die today have $n_0 = {}^{14}\mathrm{C}/{}^{12}\mathrm{C} = 1.18\times10^{-12}$ and $r_0 = 13.56$ decays per minute per gram of C.
$n(t), r(t)$	the ^{14}C concentration and decay rate measured for the remains, today, a time t since the organism died. Concentration is usually measured with a mass spectrometer, like isotope concentrations in the usual age-dating

experiments. Decay rate can be measured more easily in large samples with a Geiger counter.

8. **Typical distance between civilizations** (see lecture notes for 23 November 2021). The typical distance r between civilizations is, very roughly, given by

$$r = \left(\frac{2\pi R^2 H}{N}\right)^{1/3} \quad \text{for } N > 40000,$$
$$= R\sqrt{\frac{\pi}{N}} \quad \text{for } N < 40000,$$

where

R	is the radius of the Solar system's orbit in the Galaxy: R = 27 kly.
H	is the scale height of stars above (and below) the disk of the Galaxy: H = 300 ly.
N	is the number of communicable civilizations in the Galaxy: the result given by the Drake equation.
ly	is one light year, and kly is 1000 ly. A light year is the distance light travels in one year: 9.46×10^{17} cm $=9.46\times10^{12}$ km $=6.32\times10^{4}$ AU.

