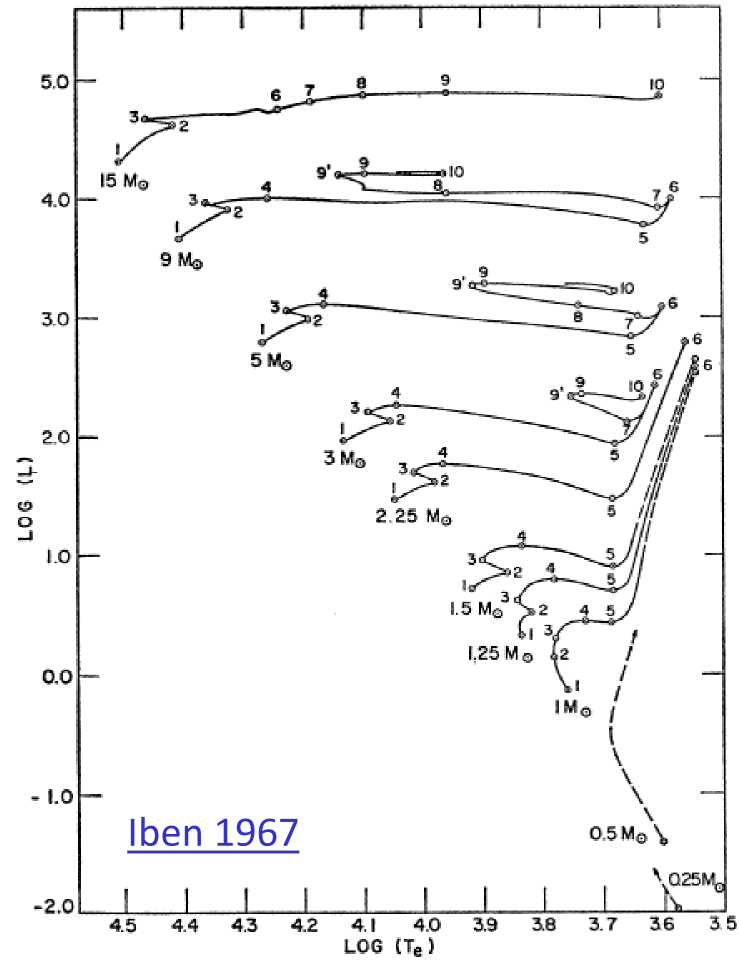
Today in Astronomy 142: stellar evolution

- The stellar scaling relationships, and the main sequence
- Changes on the main sequence
- Subgiants, giants, the horizontal branch, and supergiants
- Stellar death: formation of white dwarfs and neutron stars, and of planetary nebulae and supernovae.



Back to Live Stars

We will now learn how to scale our results on stellar structure and luminosity to normal stars of all masses.

- We will speak in terms of scaling relations, e.g.

$$P_c \propto \frac{M^2}{R^4} \quad \text{instead of} \quad P_c = 19\frac{GM^2}{R^4} \quad ,$$

  because in the end we will express everything in terms of ratios to the results on the Sun, and all the constants will cancel out, e.g.

$$\frac{P_c(M)}{P_{c\odot}} = \frac{19GM^2/R^4}{19GM_\odot^2/R_\odot^4} = \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R_\odot}{R}\right)^4$$

$$\Rightarrow P_c = P_{c\odot}\left(\frac{M}{M_\odot}\right)^2 \left(\frac{R_\odot}{R}\right)^4$$

The luminosity-mass relation

- Escape of photon produced at star's center: as we have seen (25 January), in terms of the mean free path ℓ, the number of steps N and time t are given by

$$N = \frac{3R^2}{\ell^2} \qquad t = \frac{N\ell}{c} = \frac{3R^2}{\ell c}$$

- The average photon mean free path is ℓ = 0.5 cm in the Sun. How does this scale with average temperature and density?
  - Very complicated (~AST 453) to show; skip to answer:

$$\begin{aligned} \ell &\propto T^{3.5} \big/ \bar{\rho}^2 \qquad && \text{if } M < 1M_\odot \text{ or so;} \\ &\propto 1 \big/ \bar{\rho} && \text{if } M > 1M_\odot \text{ or so.} \end{aligned}$$

$$(\bar{\rho} = 3M \big/ 4\pi R^3)$$

The luminosity-mass relation (cont'd)

- This tells us how luminosity scales (lecture on 25 January):

$$L = \frac{u_r V}{t} \qquad (u_r = \text{radiation energy density})$$

$$\approx \left(\frac{\ell c}{3R^2}\right)\left(\frac{4}{c}\sigma T^4\right)\left(\frac{4\pi R^3}{3}\right) \propto \ell R T^4$$

- Hydrostatic equilibrium and ideal-gas pressure support against the star's weight (also on 25 January) also imply

$$P \propto \frac{GM^2}{R^4} \propto \bar{\rho} T \quad ,$$

$$\text{so} \quad T \propto \frac{P}{\bar{\rho}} \propto \frac{GM^2}{R^4}\frac{R^3}{M} \propto \frac{M}{R}$$

The luminosity-mass relation (cont'd)

- Thus, for low-mass stars $\left(M \leq 1M_\odot\right)$,

$$L \propto \ell R T^4 \propto \frac{T^{3.5}}{\bar{\rho}^2} R T^4 \propto \left(\frac{M}{R}\right)^{3.5} \left(\frac{R^3}{M}\right)^2 R \left(\frac{M}{R}\right)^4$$

$$\propto \frac{M^{5.5}}{R^{0.5}}$$

and for higher-mass stars $\left(M \geq 1M_\odot\right)$,

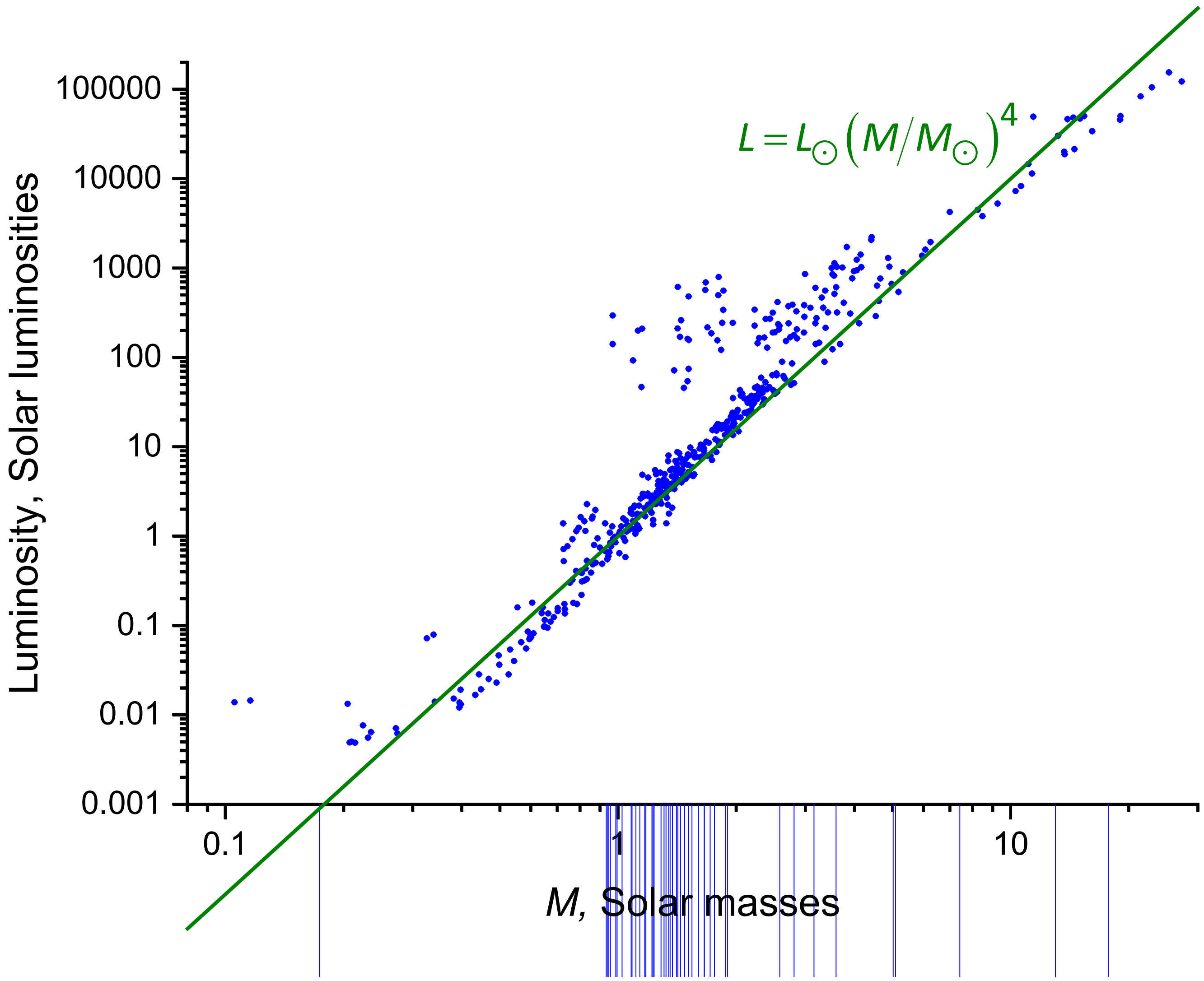
$$L \propto \ell R T^4 \propto \frac{1}{\bar{\rho}} R T^4 \propto \left(\frac{R^3}{M}\right) R \left(\frac{M}{R}\right)^4$$

$$\propto M^3$$

- Compromise:

$$L \propto M^4 \quad \Rightarrow \quad \frac{L}{L_\odot} = \frac{M^4}{M_\odot^4} \quad \Rightarrow \quad \boxed{L = L_\odot \left(\frac{M}{M_\odot}\right)^4}$$

Comparison to detached eclipsing binaries: *L vs. M*



The lifetimes of stars

Larger mass means a larger *pp*-chain fusion energy supply *E*, but leads to an even larger luminosity increase: **the more massive a star, the shorter its life**.

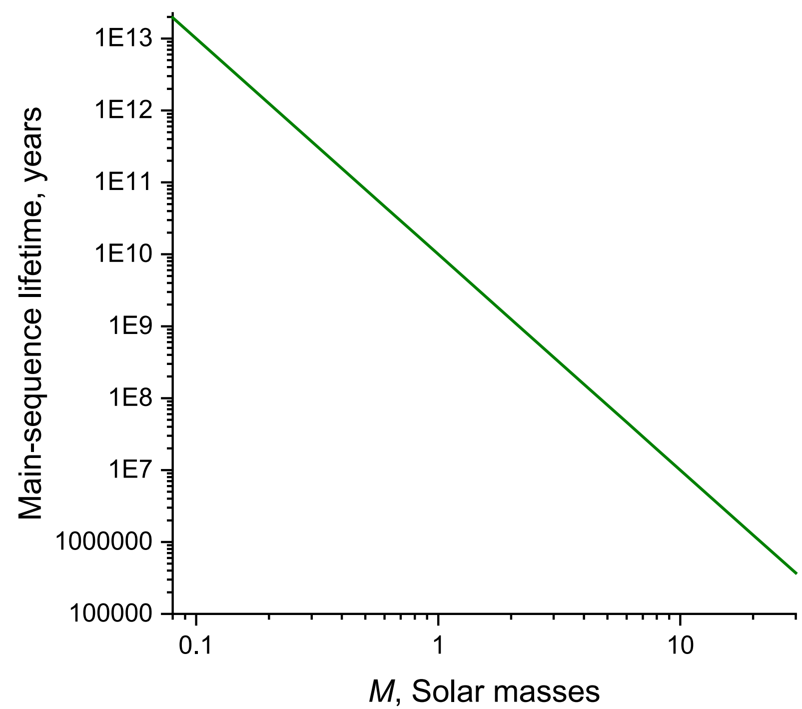
$$E \cong 0.03Mc^2 \propto M$$

$$-\frac{dE}{dt} = L \propto M^4$$

Thus $\tau = \dfrac{\Delta E}{L} \propto \dfrac{1}{M^3}$, or

$$\frac{\tau}{\tau_\odot} = \frac{M_\odot^3}{M^3}$$

$$\boxed{\tau \cong 1.0\times10^{10} \text{ years} \times \left(\frac{M_\odot}{M}\right)^3}$$



The radius-mass and temperature-mass relations

- Fusion reactions comprise a sort of thermostat: temperature in the interior of a main sequence star is only slowly dependent upon *M* and *R,* as we saw before (again in class on 29 January). Take *T* therefore to be approximately constant within a given star; then since $T \propto M/R$,

$$R \propto M \quad \Rightarrow \boxed{R = R_\odot \left(M/M_\odot\right)}$$

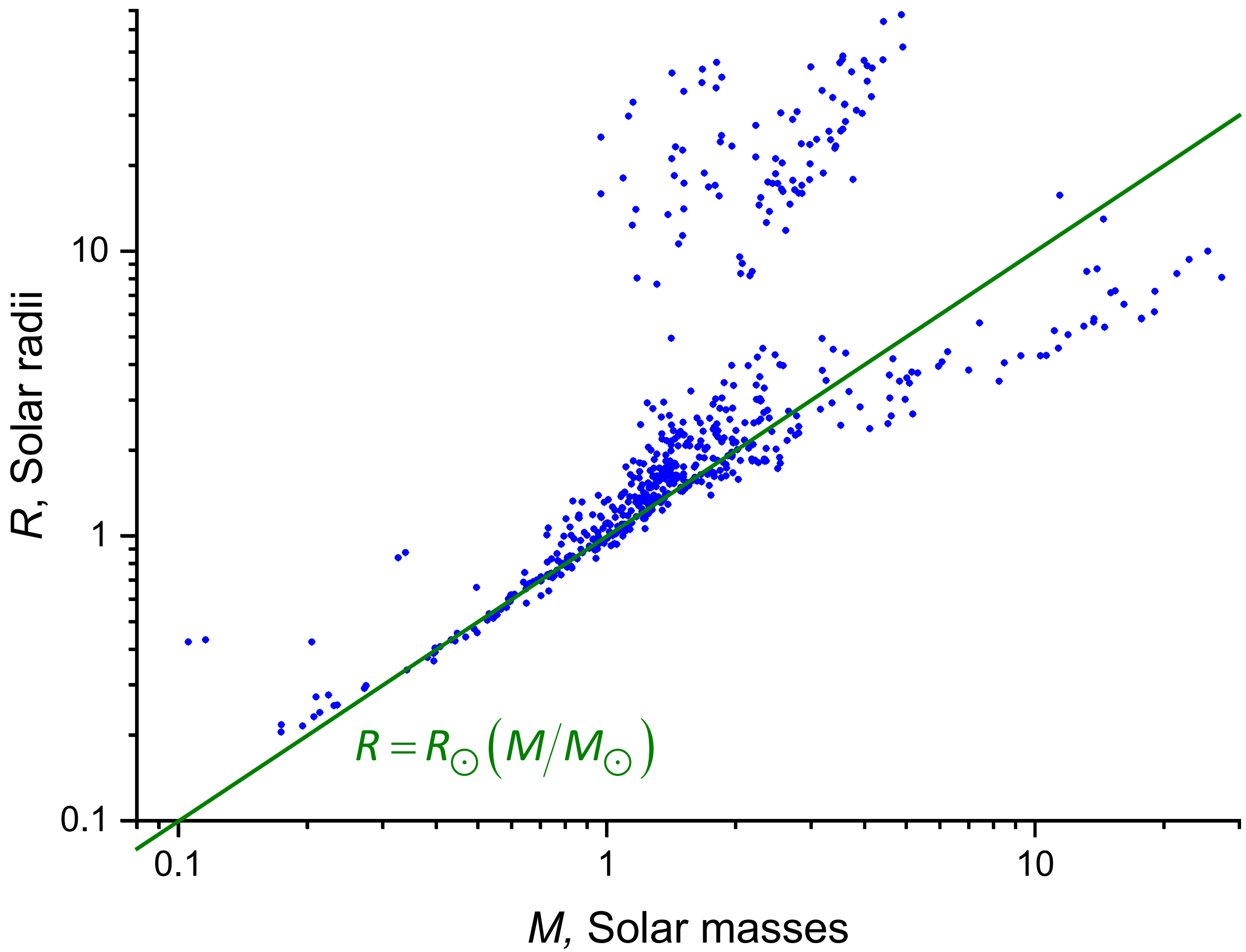
- On one hand, $L = 4\pi R^2 \sigma T_e^4$, and on the other (due to our "compromise"), $L \propto M^4$, so

$$4\pi R^2 \sigma T_e^4 \propto M^4$$

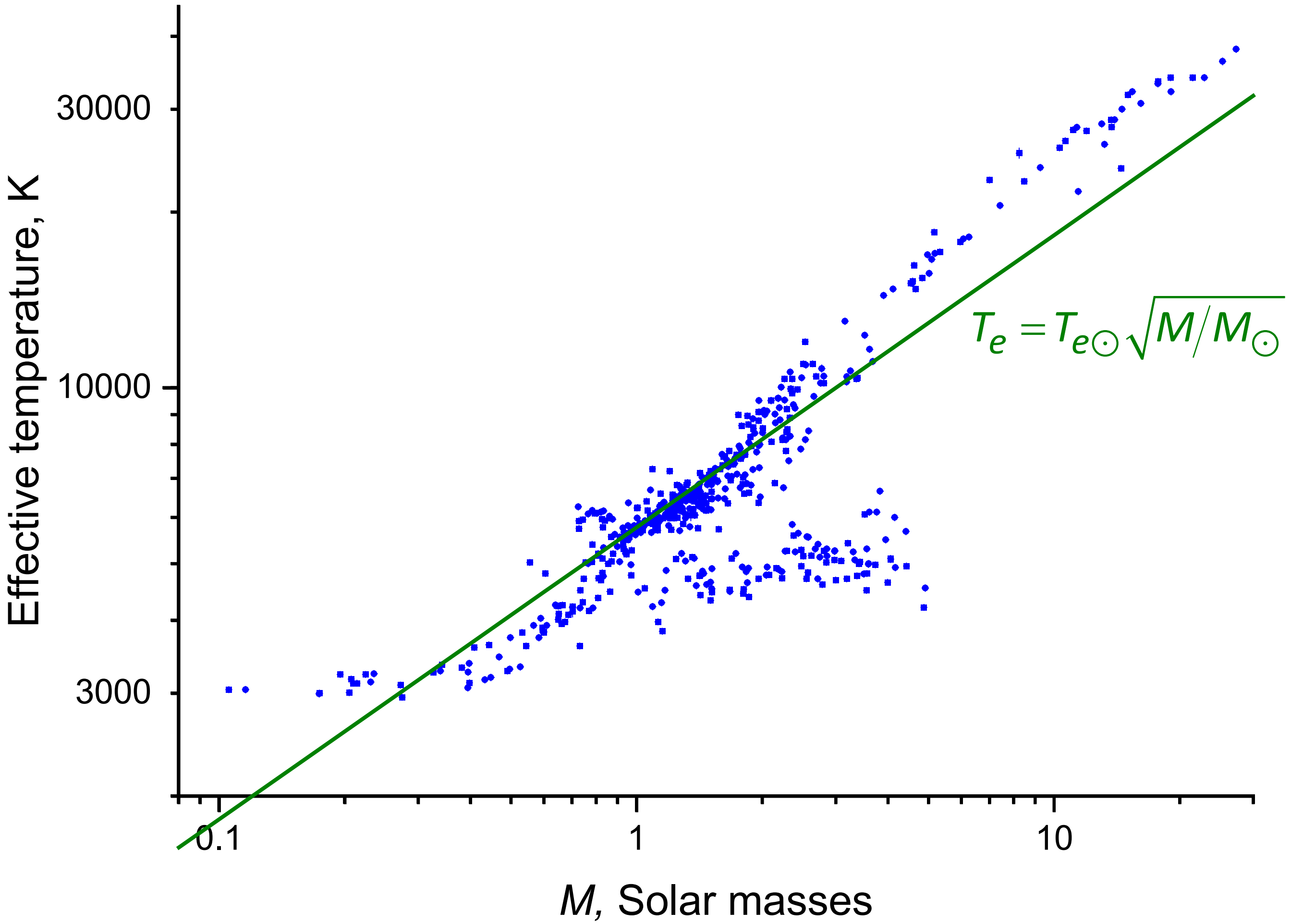
$$M^2 T_e^4 \propto M^4$$

$$T_e \propto M^{1/2} \quad \Rightarrow \boxed{T_e = T_{e\odot}\sqrt{M/M_\odot}}$$

Comparison to detached eclipsing binaries: *R vs. M*



Comparison to detached eclipsing binaries: T_e vs. M



The main sequence in the luminosity-effective temperature relation (H-R diagram)

- Combine this last result again with $L \propto M^4$:

$$L \propto M^4 \propto T_e^8$$

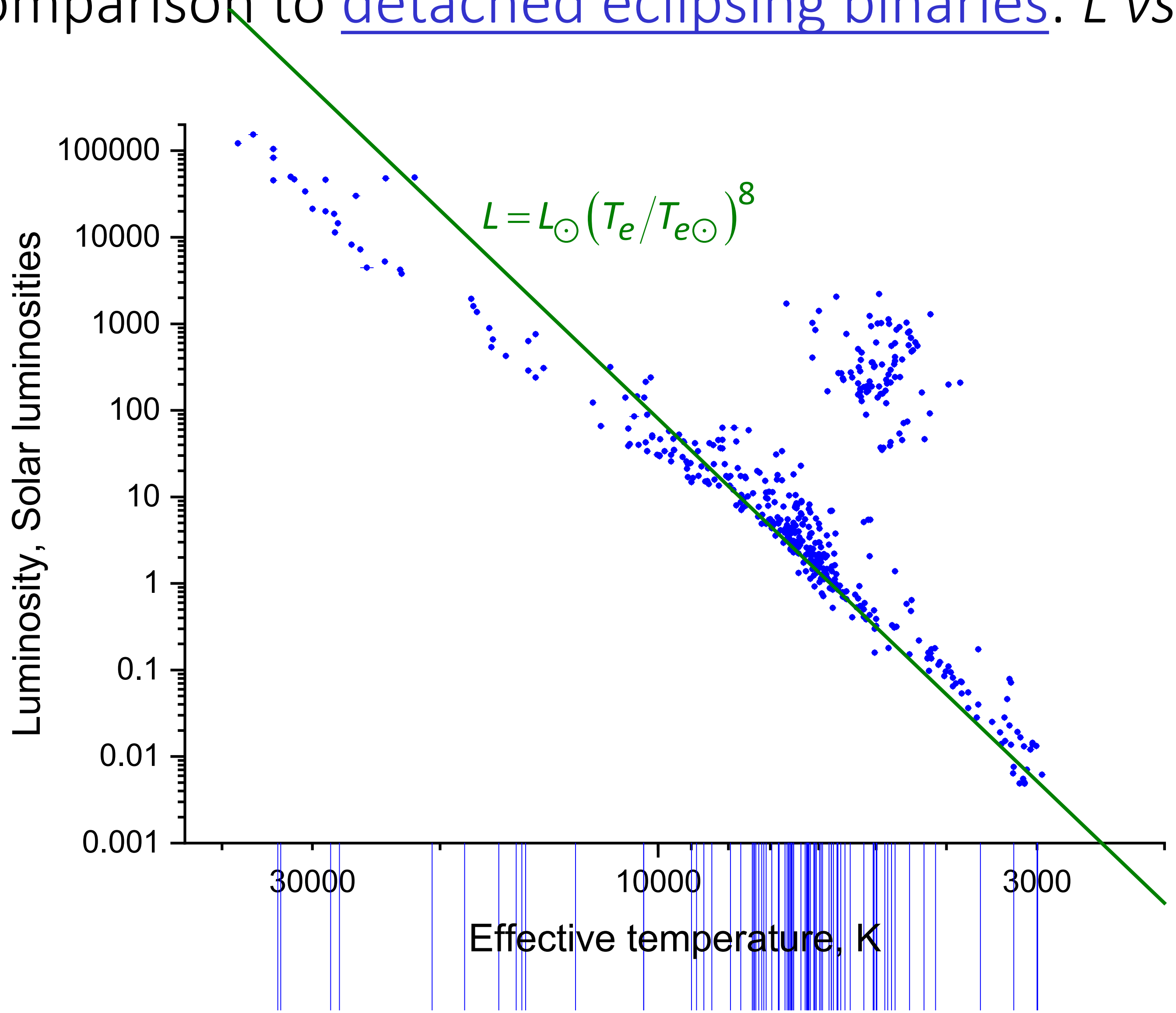
$$\boxed{L = L_\odot \left(\frac{T_e}{T_{e\odot}} \right)^8}$$

The main sequence

- All these results are in reasonable agreement with the data, which indicates that we have included most of the important physics in our discussions.

- In fact, one needs to build quite detailed models to do better – AST 453 style, not even AST 241 style.

Comparison to detached eclipsing binaries: *L vs. T_e*



Mean molecular mass

Changes in a star's mean molecular mass μ change its appearance over time.

- For pure ionized hydrogen: $\mu = \dfrac{m_p + m_e}{2} \cong 0.5 m_p$

- For pure ionized helium: $\mu = \dfrac{3.97 m_p + 2 m_e}{3} \cong 1.32 m_p$

- In general, and in terms of mass fractions: *X, Y, Z* = fraction of total mass in hydrogen, helium, total of all others, respectively, for fully ionized gas:

$$\frac{m_p}{\mu} \cong 2X + \frac{3}{4}Y + \frac{1}{2}Z$$

- Mean molecular weight for ionized gas with X = 0.70, Y = 0.28, Z = 0.02 (abundances found on the Solar surface): $\mu = 0.62 m_p$.

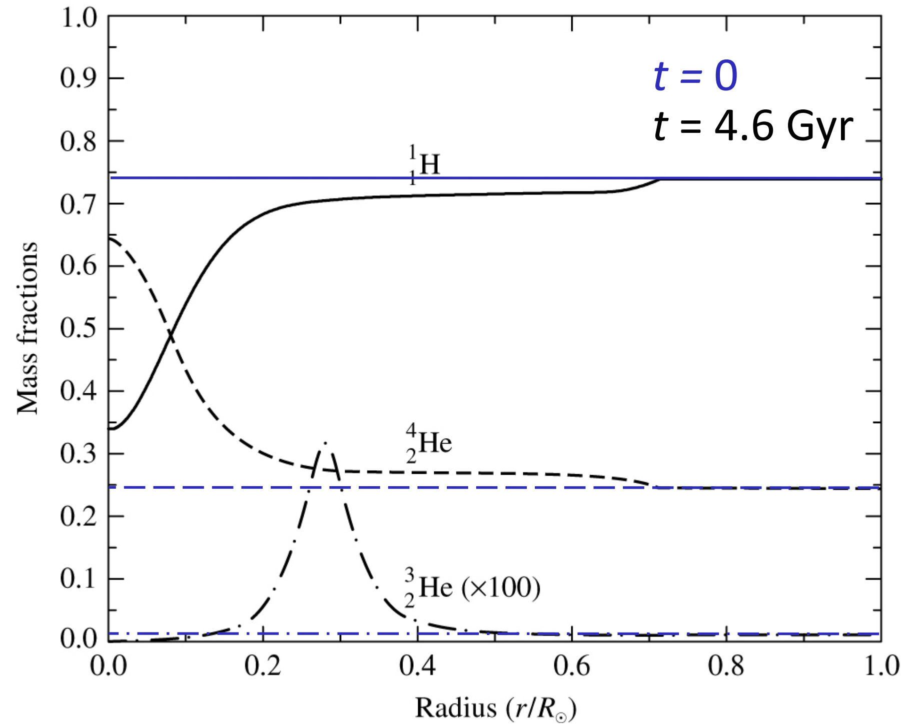
Stellar evolution on the main sequence

- As hydrogen burns in the core, fusing into heavier elements, the mean molecular mass slowly increases.
- At a given temperature the ideal gas law says this would result in a lower gas pressure, and less support for the star's weight:

$$P = \frac{\rho k T}{\mu} \quad .$$

- In the center of the Sun today,

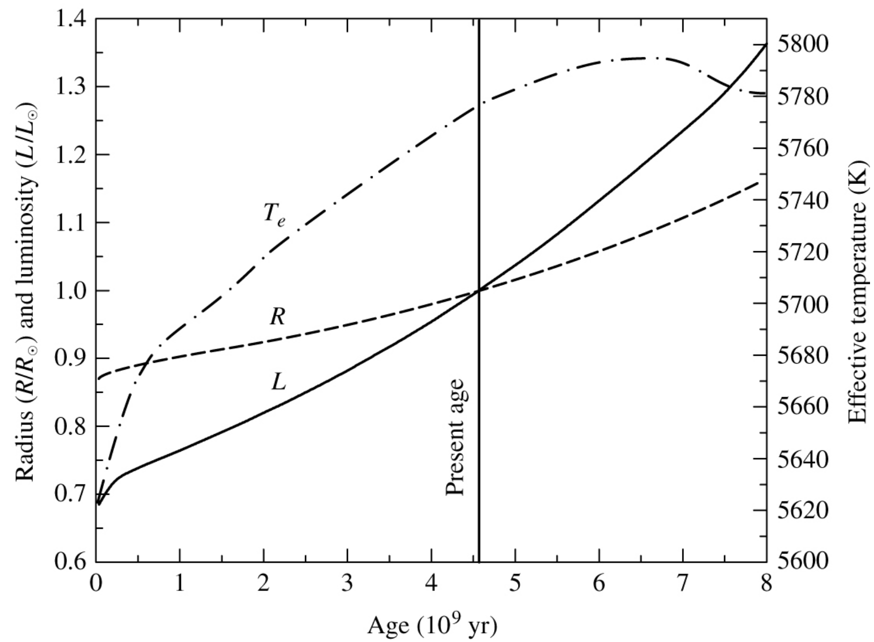
$$\mu = 1.17 m_p \quad .$$



Base figure from Carroll and Ostlie, Modern Astrophysics, 2e.

Stellar evolution on the main sequence (continued)

- Therefore, as time goes on,
  - the core of the star slowly contracts and heats up.
  - the radius and effective temperature of the star slowly increase, in response to the new internal temperature and density distribution.
  - the luminosity slowly and slightly increases, in response to the increase in radius and effective temperature.

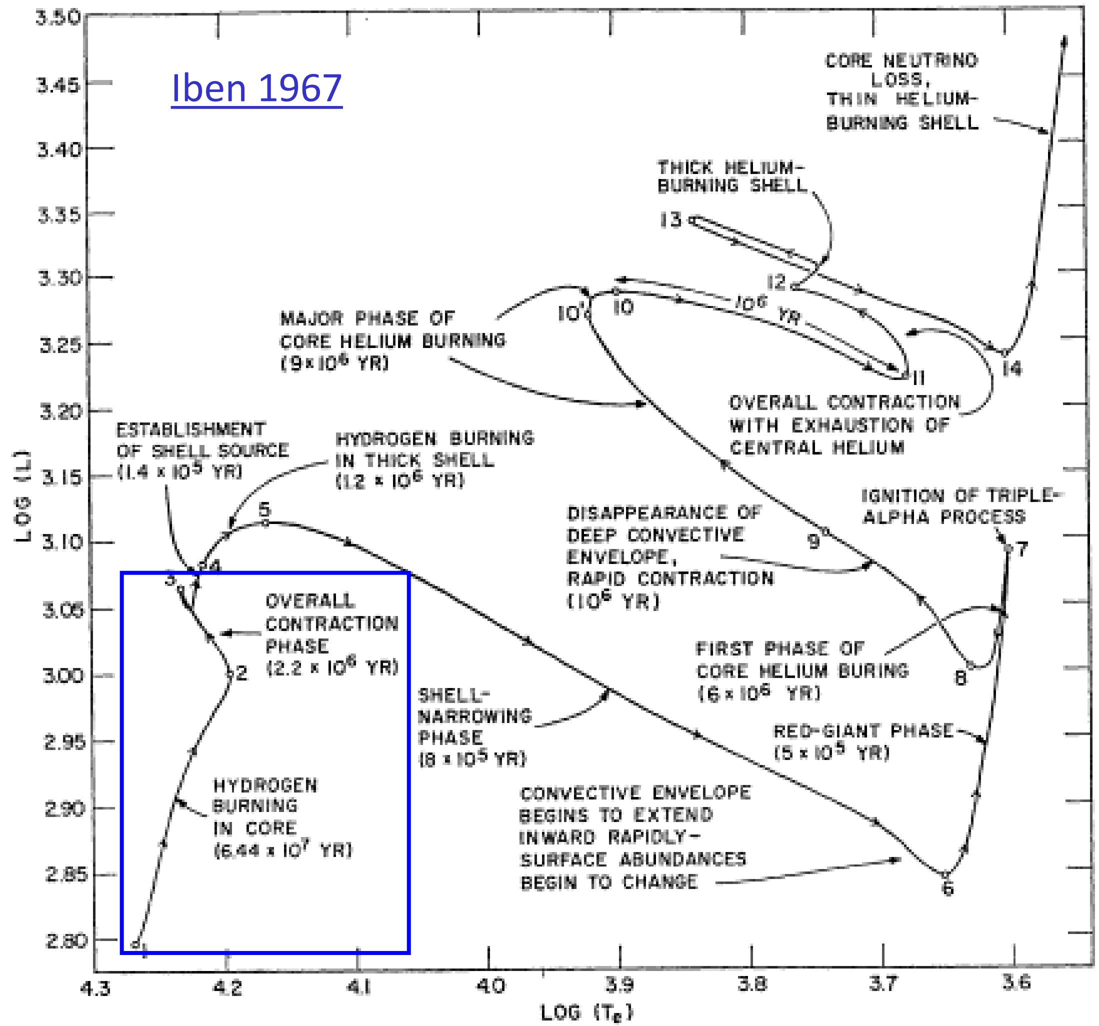


From Carroll and Ostlie, Modern Astrophysics, 2e.

Stellar evolutionary track, main sequence and thenceforth

Here's a famous, and famously busy, stellar-evolution track.

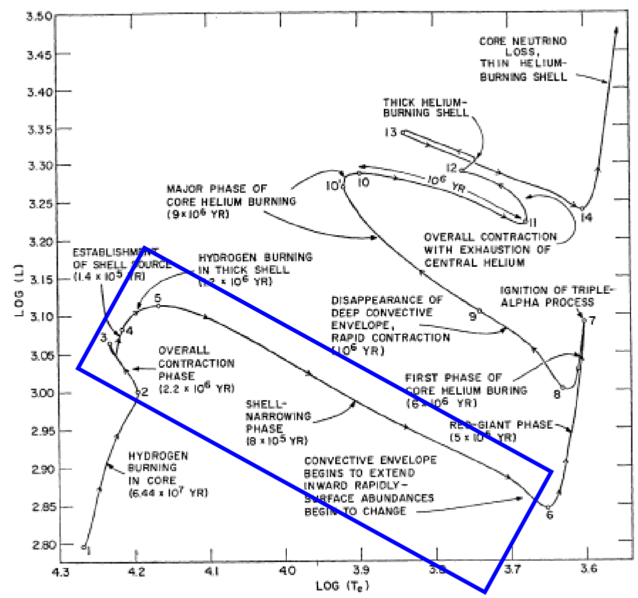
- This is for a $5M_\odot$ star, so the time scales are all about a factor of $5^3 = 125$ faster than for the Sun.
- And no low-mass star has made it to post-main-sequence yet.



Shell hydrogen burning and the subgiant phase

Eventually hydrogen is exhausted in the very center, and the temperature is insufficient to ignite helium fusion, but is high enough just outside the center for a shell of hydrogen fusion to provide support for the star.

- T is nearly constant in the core (**isothermal helium core**), which keeps increasing in mass owing to hydrogen depletion.
- L increases and R increases further.
- T_e decreases.
- This is called the **subgiant phase**: the star moves off the main sequence, to the right on the H-R diagram.



Subgiant phase

[Iben 1967]

Degeneracy in the isothermal core

- The subgiant phase ends when the mass of the isothermal core becomes too great for support of the star.
  - Reason for a maximum in the weight that can be supported by pressure in the core: electron degeneracy pressure, again. The isothermal core is a like a white dwarf, except with additional, external pressure.
  - Maximum fraction of mass in core (Schoenberg & Chandrasekhar, 1942):

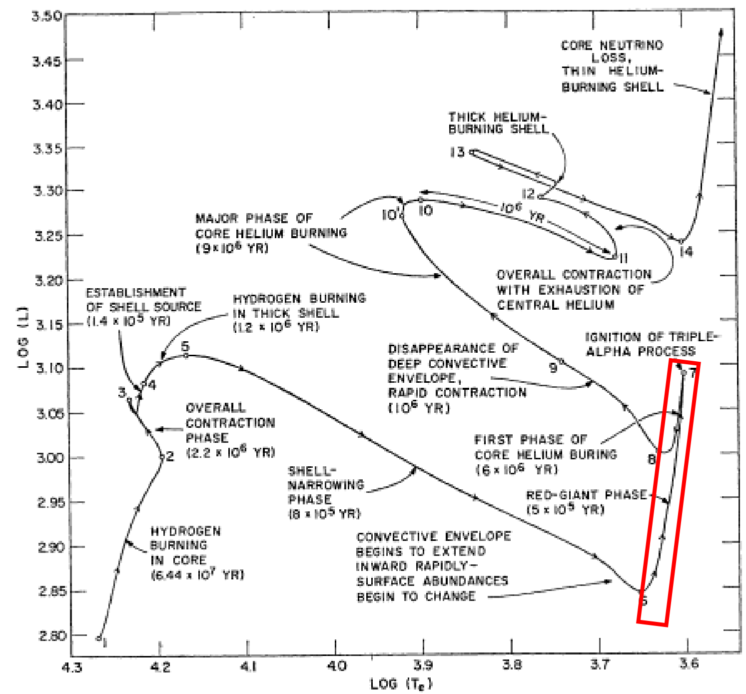
$$\frac{M_{\text{isoth. core}}}{M_{\text{total}}} \cong 0.37\left(\frac{\mu_{\text{envelope}}}{\mu_{\text{isoth. core}}}\right)^2$$

$$\cong 0.37\left(\frac{0.62}{1.32}\right)^2 = 0.08 \ \text{ for the Sun.}$$

Late stages of stellar evolution

After the maximum mass fraction of the isothermal helium core is exceeded, the star enters the…

- **Red giant phase**: moving up in H-R diagram, for lower-mass stars.
  - Core contracts, central density and temperature rise.
  - Convection zone extends inward (**dredge-up**).
  - Stellar radius increases dramatically, from sharp increase in radiation pressure from interior. Radiation pressure now dominates support against the star's weight.



Red giant phase

Iben 1967

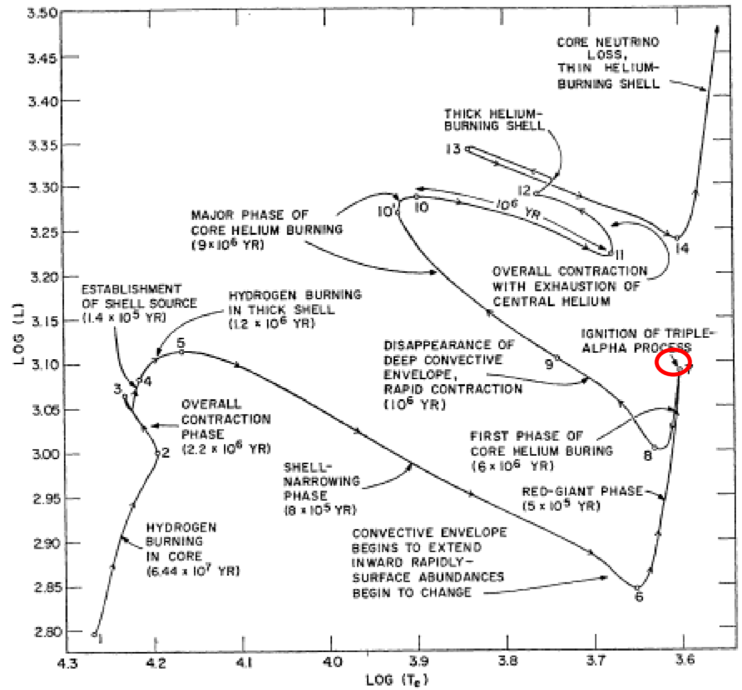
Late stages of stellar evolution (continued)

When the core T reaches 10^8 K, the **triple-α process**,

$$3\,{}^4_2\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C} + 2\,{}^0_0\gamma \quad ,$$

begins helium fusion.

- The onset of this process is very rapid in stars with $M \geq 2M_\odot$, leading to the term **helium flash**.



Helium flash

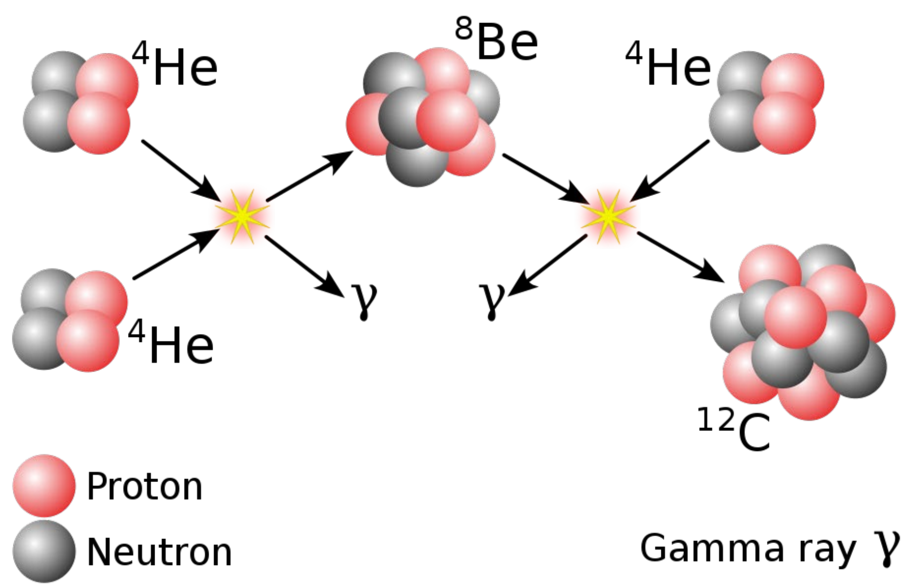
[Iben 1967]

Triple alpha

Triple alpha requires very large density because of the short lifetime (10^{-16} sec) of ^{8}Be, so it only happens in the extremes of core compression.

- We will meet this again in cosmology, under the name **^{8}Be bottleneck**.

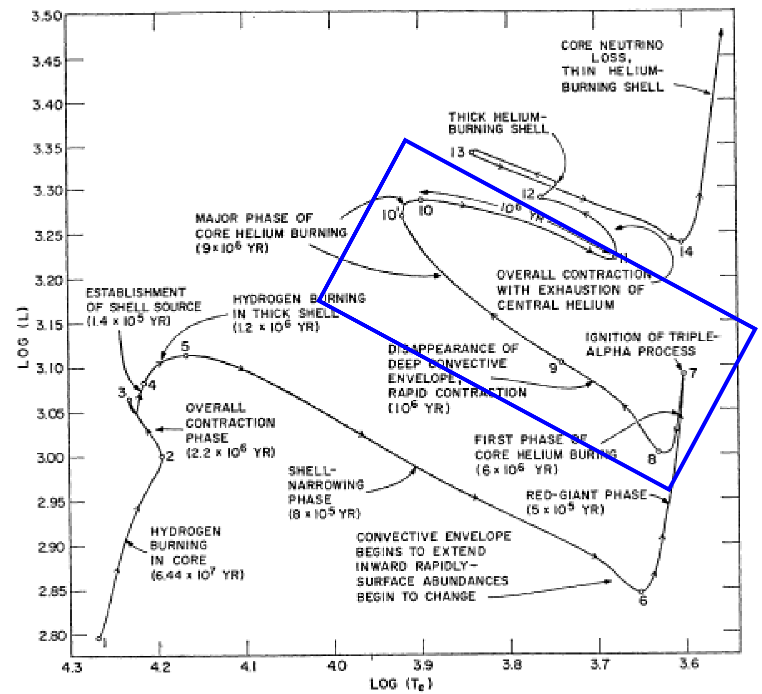
$$2\,{}^{4}_{2}\mathrm{He}$$
$$\rightarrow {}^{8}_{4}\mathrm{Be}^{*} + {}^{0}_{0}\gamma + {}^{4}_{2}\mathrm{He}$$
$$\rightarrow {}^{12}_{6}\mathrm{C} + {}^{0}_{0}\gamma$$



Late stages of stellar evolution (continued)

The **horizontal branch** is the phase after triple-α onset.

- So called because of its appearance in the *V* vs. *B-V* color-magnitude diagram of globular clusters.
- Core helium burning, shell hydrogen burning. The core is on the **helium main sequence**.
- You will explore the properties of the helium main sequence in Homework #4.



Horizontal branch
= helium main sequence

Iben 1967

After the horizontal branch

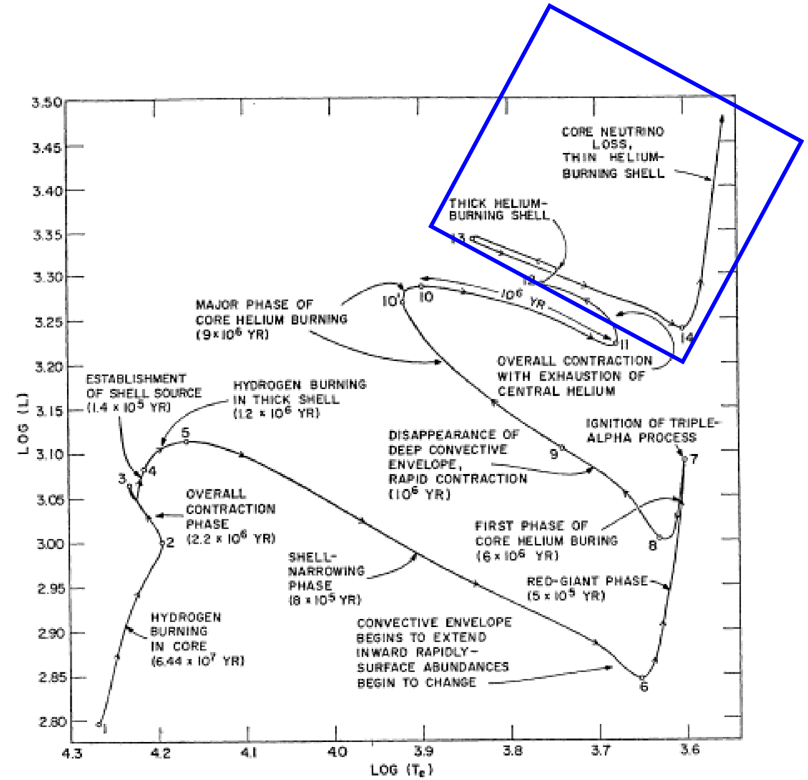
Low mass stars – those with $M < 2M_{\odot}$:

- Slowly an "isothermal carbon-oxygen core" forms in the center as the helium fuel is exhausted.
- In these stars, however, there is not enough weight to overpower degeneracy pressure, so the core can't contract and reheat to ignite carbon-oxygen fusion.
- Results:
  - H/He burning of outer layers of core, ejection of most of the rest of the star, and formation of a **planetary nebula**. This phase lasts a few thousand years, during which the ejected gas drifts away.
  - The core becomes a carbon-oxygen white dwarf with mass $M \approx 0.6M_{\odot}$ and initial temperature ~10^8 K. Lasts forever unless it has a close stellar companion.

After the horizontal branch (continued)

Massive stars – those with $M > 2M_{\odot}$ – go further, to

- **Asymptotic giant branch** (AGB, or supergiant) evolution.
- Repeated core collapse - fusion reignition - nuclear fuel exhaustion occurs, including silicon fusion to produce iron-peak elements.
- Radius and luminosity steadily increase, T_e decreases.

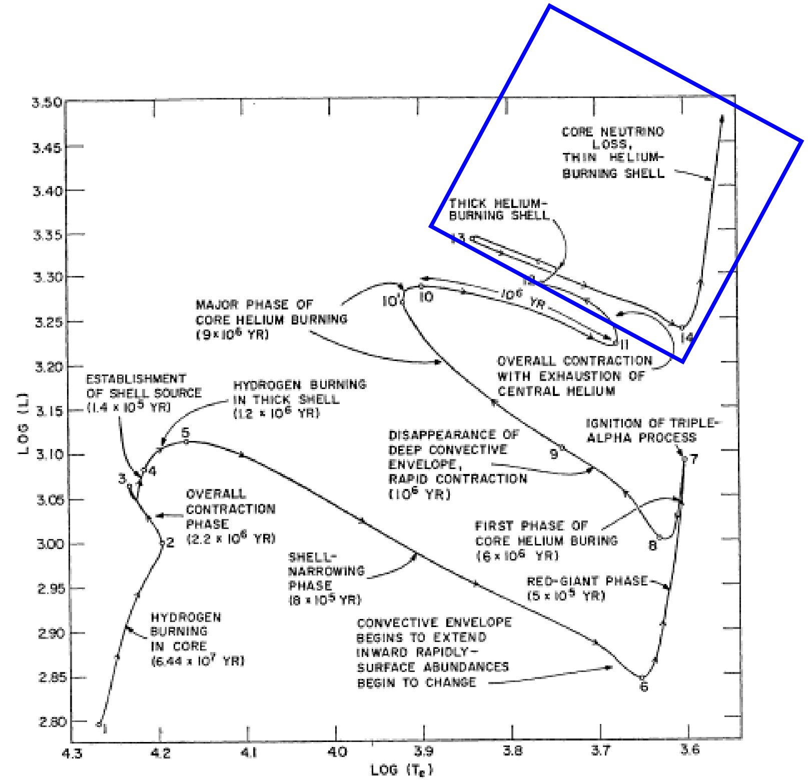


AGB

Iben 1967

After the horizontal branch (continued)

- Each of the successive fuel exhaustions is faster than the last. For a $20M_\odot$ star,
  - hydrogen burning (main sequence) lasts 10^7 years
  - helium burning (horizontal branch) lasts 10^6 years
  - carbon burning lasts 300 years
  - oxygen burning lasts 200 days
  - silicon burning lasts 2 days



AGB

[Iben 1967](#)

What happens when all the nuclear fuel is gone?

Most $M > 2M_\odot$ stars:

- During the burning of heavier elements, and radiative support of the stellar envelope, stars tend to be hydrodynamically unstable, leading to the loss of large fractions of stars' masses:
  - Oscillations: note that evolution takes stars across the instability strip (lecture, 3 February), which is nearly vertical at effective temperature ~ 7,000K.
  - **Stellar winds** (see this week's Workshop, and HW#4).
- This can keep a star's core mass below the SAC limit, and the final states of the star are just like that of less massive ones: planetary nebula phase and white dwarf remnant.

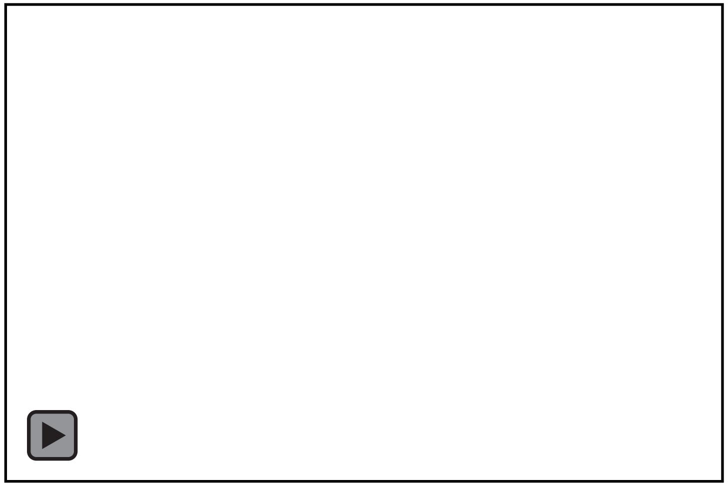
What happens when all the nuclear fuel is gone? (continued)

The most massive stars $\left(M \gtrsim 8M_{\odot}\right)$:

- Mass loss insufficient to keep core in white dwarf range: further collapse and neutronization.
  - Maximum mass of iron white dwarf $= 1.26M_{\odot}$.
- When the collapsing core reaches tens-of-km dimensions, neutron degeneracy pressure sets in, and this can stop or slow the collapse.
- However, since the collapse has been from white-dwarf dimensions to neutron-star dimensions, infalling material from the star's envelope is going *very* fast: a large fraction of *c*.
- The infalling material bounces off the stiffened neutron-degenerate material and blows up the rest of the star.

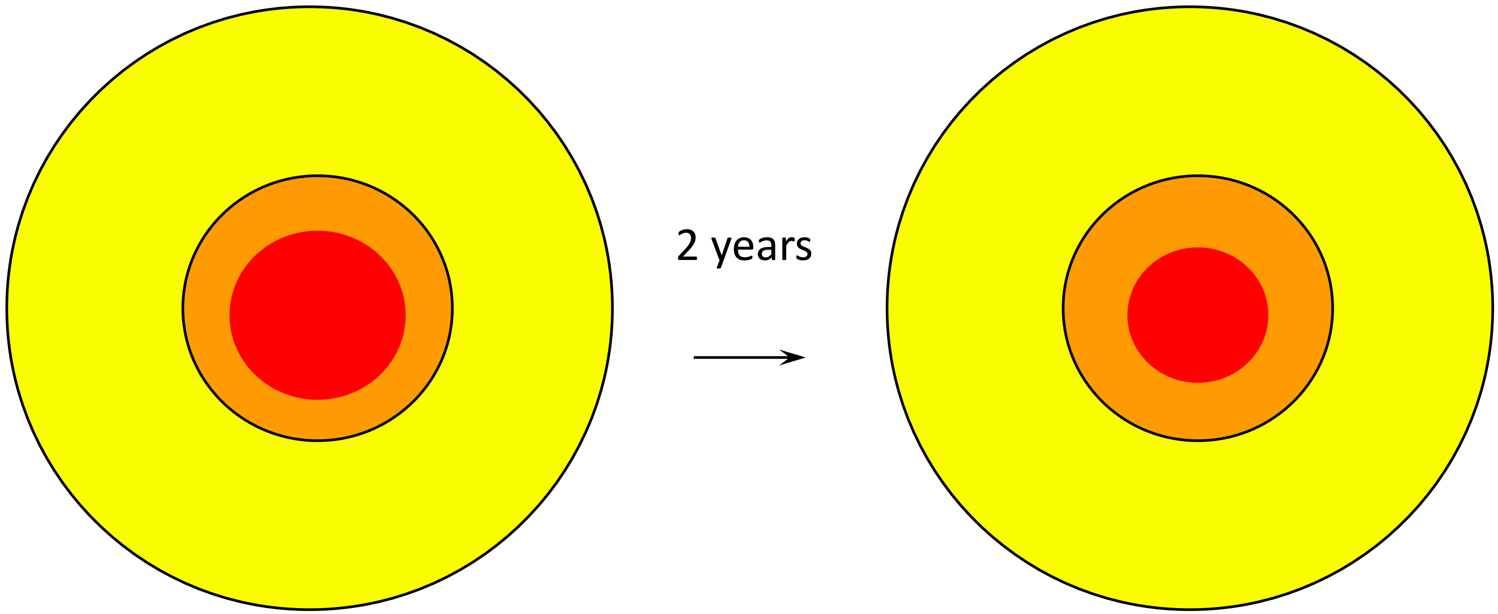
Core collapse and Type II supernovae

- This event is called a **core collapse**, or **type II, supernova.**
  - How did we get to type II before type I? We'll get to that later in the course, when we talk about the extragalactic distance scale.
- Remnant: a neutron star $(M = 1.3 - 2.2 M_{\odot})$ or, more rarely, a black hole, depending upon core mass.



Artist's conception of a SNII and the formation of a supernova remnant (Chandra/CfA/NASA).

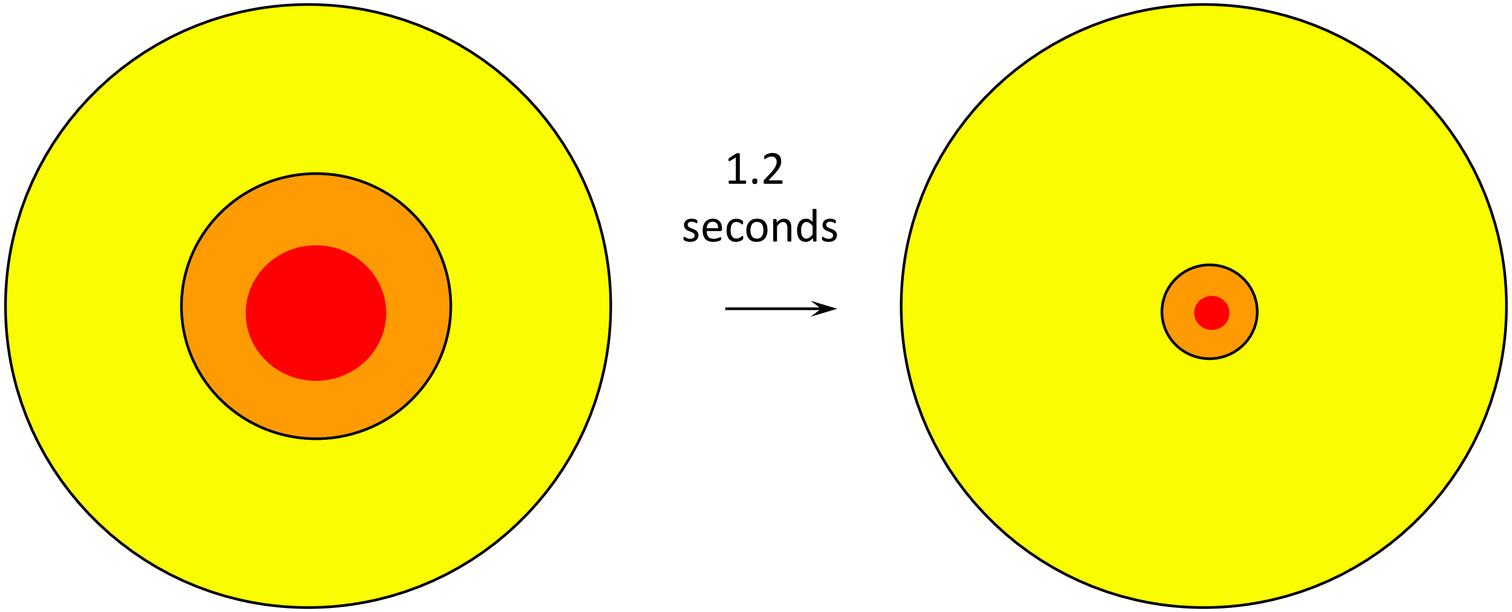
A SNII forms from a dead, massive star (not drawn to scale)



Star: 6 $M_\odot$, 10^7 km circumference
Core: 1.4 $M_\odot$, 10^5 km circumference

Core: 10^4 km circumference. Electrons and protons begin combining to form neutrons.

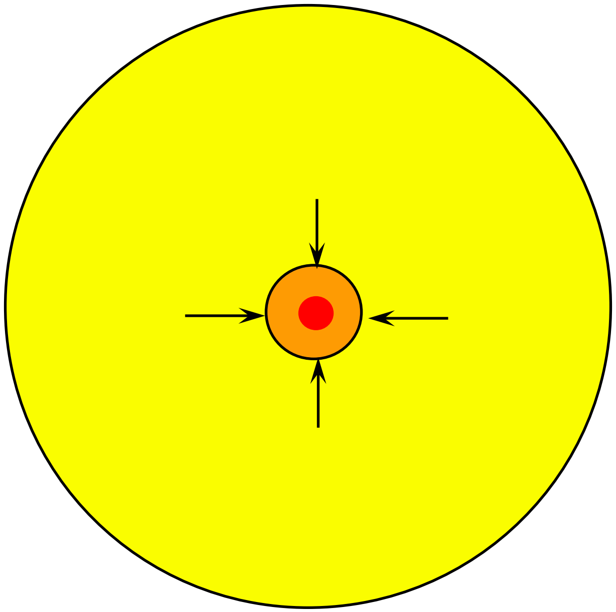
A SNII forms from a dead, massive star (continued)



Core: 10^4 km circumference. Electrons and protons begin combining to form neutrons.

Core: 75 km circumference, neutron degeneracy pressure sets in.

A SNII forms from a dead, massive star (continued)

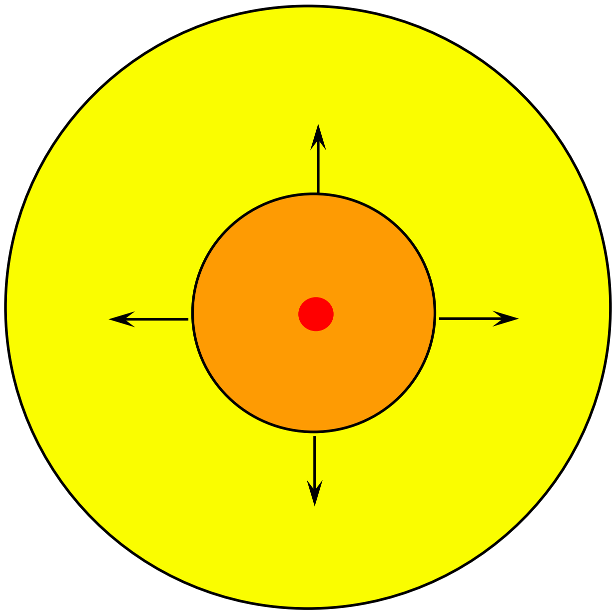


Core: 75 km circumference, neutron degeneracy pressure sets in. This makes the core very stiff.

Outside of core: still collapsing, moving inwards at about 10^{10} cm/s. Bounces off stiff core.

A SNII forms from a dead, massive star (continued)

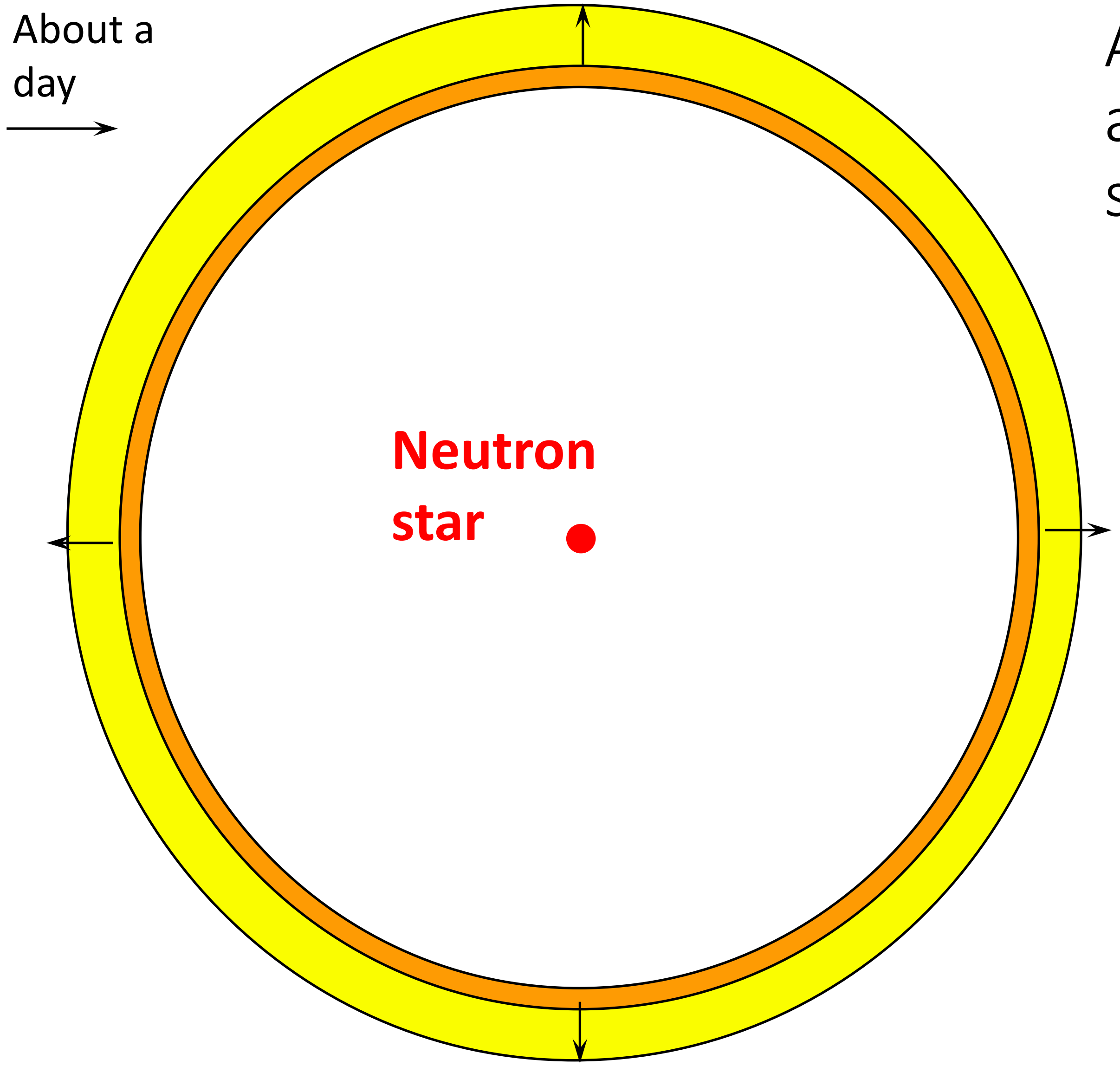
A few seconds



Core: Still 75 km circumference, it is now stable.

Outside of core: the rebounding outer-star material explodes the rest of the star. Energy comes from bounce, and from gravitational energy of core.

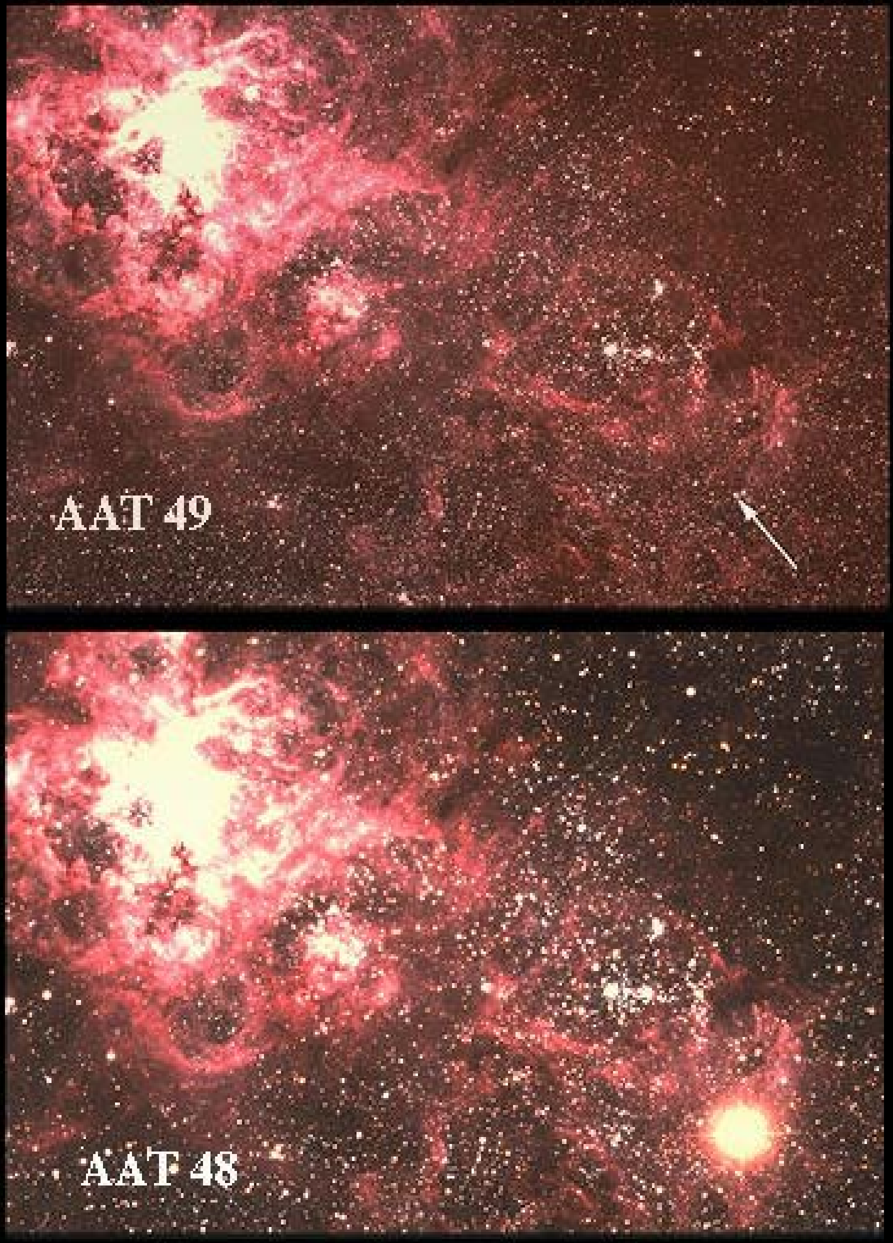
A SNII forms from a dead, massive star (continued)



Expanding supernova shell.

- Very, very bright for about a month after explosion: can outshine the rest of its galaxy.

Supernova 1987A in the Large Magellanic Cloud



Before (top; follow the arrow) and after (bottom; guess where) the explosion.

- SN1987A was the first supernova for which we knew the progenitor star, and was the most recent SN that could be seen with the naked eye. It was a SNII.

David Malin, Anglo-Australian Observatory