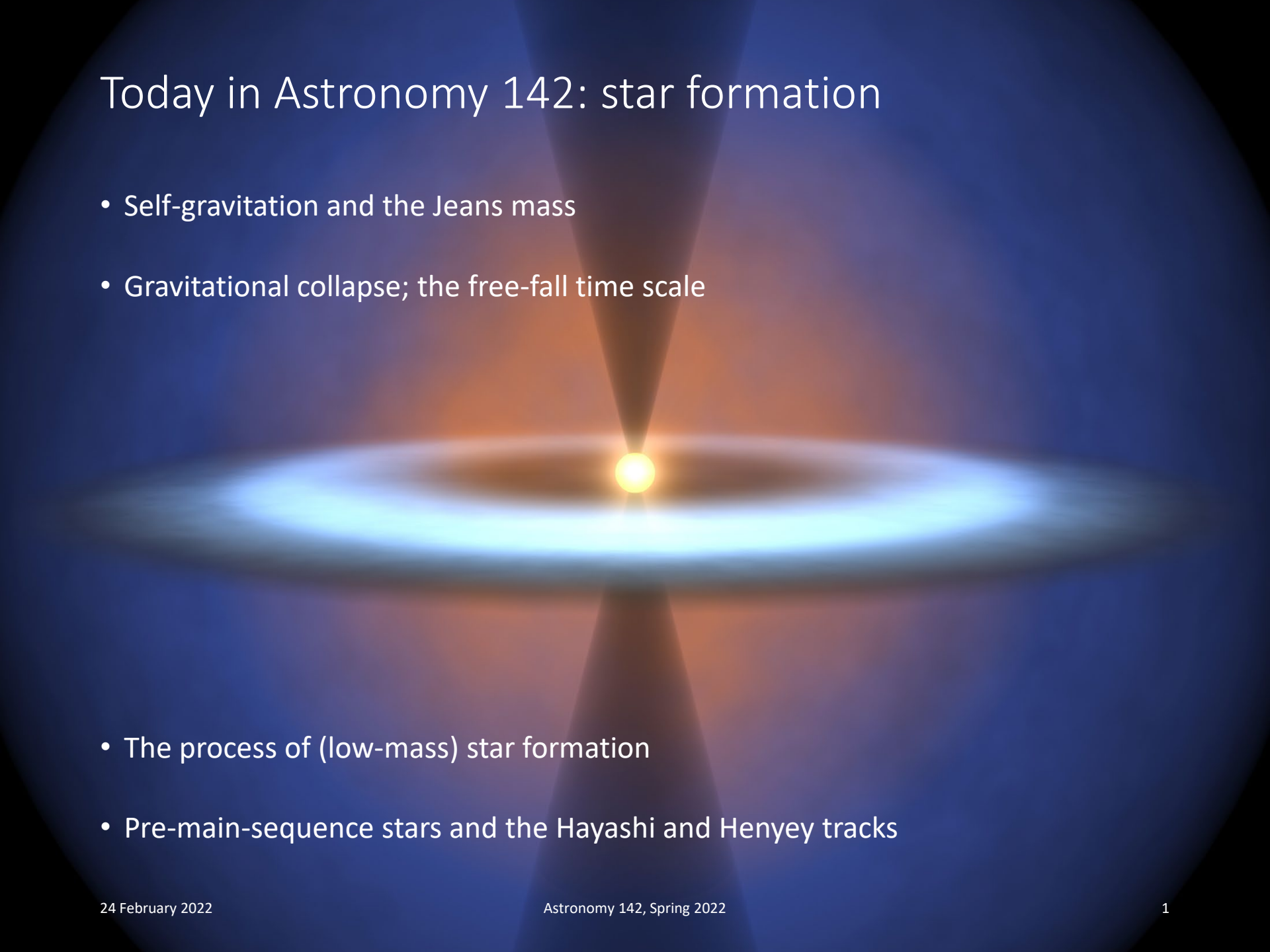
Today in Astronomy 142: star formation



- Self-gravitation and the Jeans mass
- Gravitational collapse; the free-fall time scale
- The process of (low-mass) star formation
- Pre-main-sequence stars and the Hayashi and Henyey tracks

Self-gravitation and the Jeans mass

Consider a uniform-density spherical clump and its central pressure:

$$P_C \approx \frac{GM^2}{R^4}$$ from weight,

$$= \frac{\rho kT}{\mu}$$ from the ideal gas law.

- Force balance: $$\frac{GM^2}{R^4} = \rho\frac{GM}{R} = \frac{\rho kT}{\mu} \quad \Rightarrow \quad \frac{M}{R} = \frac{kT}{\mu G}$$

- Uniformity: $$\rho = \frac{3M}{4\pi R^3} \quad \Rightarrow \quad R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

- Thus: $$M = \frac{kT}{\mu G}\left(\frac{3}{4\pi\rho}\right)^{1/3} M^{1/3} \quad \Rightarrow \quad M = \left(\frac{kT}{\mu G}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2}$$

Gravitational stability: the Jeans mass (continued)

- Define the **Jeans mass**: $$M_J = \left(\frac{kT}{\mu G}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2}$$

- If a clump's mass exceeds the Jeans mass for its density, temperature and composition, **it will collapse under its weight**.

- **Example**: this spherical, nonrotating, uniform, pure molecular hydrogen clump.

$$\left.\begin{array}{l}\mu = 3.3\times10^{-24}\ \text{gm}\\ n_{H_2} = 1.4\times10^{5}\ \text{cm}^{-3}\\ T = 20\ \text{K}\end{array}\right\} \Rightarrow \left.\rho = 4.6\times10^{-19}\ \text{gm cm}^{-3}\right\} \Rightarrow M_J = 1M_\odot$$

- Add to this a trace of CO, say, 1 part in 10^4. Then:
  - Collisionally-excited CO line emission would soon reduce the temperature below 20 K, reduce the Jeans mass below $1M_\odot$, and the cloud collapses.

Spherical free-fall time: derivation

How long does the collapse take?

- Follow a test particle that starts from rest, a distance r_0 from cloud center, at t = 0. and find the time at which it reaches the center, i.e. when the clump collapses to a point.

$$ma = F$$

$$m\frac{d^2r}{dt^2} = -\frac{GMm}{r^2}$$

1D, nonlinear, 2nd-order differential equation.

***M*: mass interior to *r*. (Constant!)**

- Multiply both sides by dr/dt, integrate over time, RHS first:

$$\int_0^t dt' \frac{dr}{dt'}\frac{d^2r}{dt'^2} = -GM\int_0^t \frac{1}{r^2}\frac{dr}{dt'}dt'$$

$$= -GM\int_{r_0}^{r}\frac{dr'}{r'^2} = \frac{GM}{r} - \frac{GM}{r_0}$$

Spherical free-fall time: derivation (continued)

- On LHS, substitute $v=dr/dt$, $dv=(d^2r/dt^2)dt$:

$$\int_0^t dt' \frac{dr}{dt'} \frac{d^2r}{dt'^2} = \int_0^v v' dv' = \frac{1}{2}v^2 = \frac{GM}{r} - \frac{GM}{r_0}$$

- Substitute back $v = dr/dt$:

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{GM}{r} - \frac{GM}{r_0}$$

$$\frac{dr}{dt} = -\sqrt{\frac{2GM}{r} - \frac{2GM}{r_0}}$$

Choose negative root, so that the particle moves to smaller *r*.

Spherical free-fall time: derivation (continued)

- Insert $M = \frac{4\pi}{3} r_0^3 \rho_0$:

$$\frac{dr}{dt} = -r_0 \sqrt{\frac{8\pi G \rho_0}{3}\left(\frac{r_0}{r} - 1\right)}$$

1D, nonlinear, 1st-order differential equation.

- Change variables: $r = r_0 \cos^2 u$

$$\frac{dr}{dt} = r_0 \frac{d}{dt}\cos^2 u$$

$$= -2 r_0 \cos u \sin u \frac{du}{dt}$$

Spherical free-fall time: derivation (continued)

- Then the differential equation becomes

$$\frac{du}{dt} = \frac{1}{2\cos u \sin u}\sqrt{\frac{8\pi G\rho_0}{3}\left(\frac{1}{\cos^2 u} - 1\right)}$$

$$= \frac{1}{2\cos^2 u \sin u}\sqrt{\frac{8\pi G\rho_0}{3}\left(1-\cos^2 u\right)} = \frac{1}{2\cos^2 u}\sqrt{\frac{8\pi G\rho_0}{3}}$$

- Separate and integrate over time, RHS first:

$$\int_{u_0}^{u} \cos^2 u' du' = \frac{1}{2}\sqrt{\frac{8\pi G\rho_0}{3}}\int_0^{t_{ff}} dt = \frac{t_{ff}}{2}\sqrt{\frac{8\pi G\rho_0}{3}}$$

Spherical free-fall time: derivation (continued)

- Now, the boundary conditions on the LHS for u and u_0:

$$t = 0: \quad r = r_0,\ r_0 = r_0 \cos^2 u_0 \quad \Rightarrow u_0 = 0$$

$$t = t_{ff}: \quad r = 0,\ 0 = r_0 \cos^2 u \quad \Rightarrow u = \frac{\pi}{2}$$

- Thus, integrating the LHS,

$$\frac{t_{ff}}{2}\sqrt{\frac{8\pi G\rho_0}{3}} = \int_0^{\pi/2} \cos^2 u\, du = \left[\frac{u}{2} + \frac{\sin 2u}{4}\right]_0^{\pi/2} = \frac{\pi}{4}$$

$$\boxed{t_{ff} = \sqrt{\frac{3\pi}{32 G\rho_0}}}$$

Free-fall time for a uniform, nonrotating, spherical cloud.

Time scale of spherical gravitational collapse

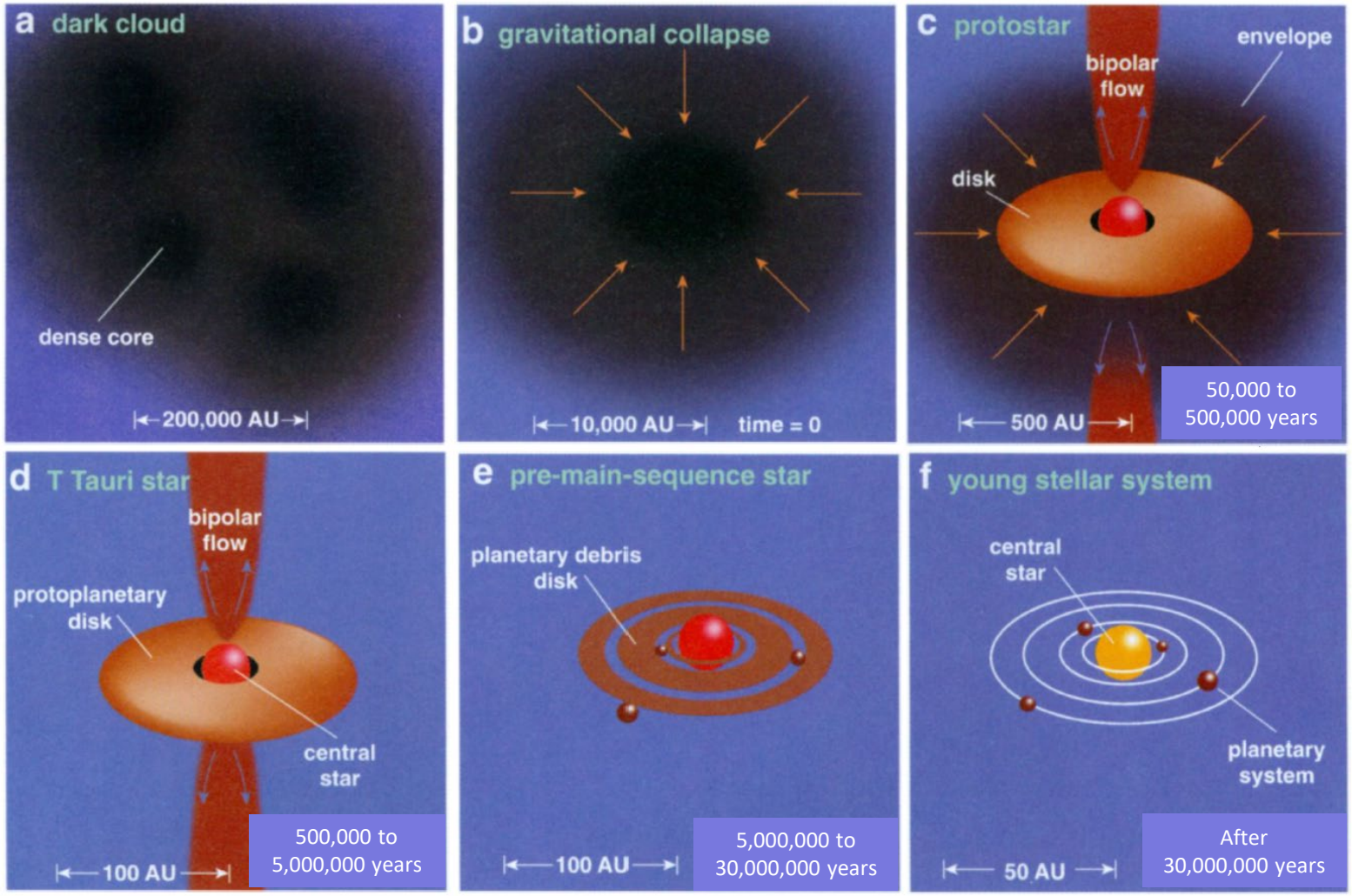
- So, starting at $\rho_0 = 4.6 \times 10^{-19}$ gm cm^{-3}, we get

$$t_{ff} = \sqrt{\frac{3\pi}{32 G \rho_0}} = 10^{12} \text{ sec} = 3 \times 10^4 \text{ years.}$$

  - This is fast by astrophysical standards. We should therefore have to be lucky to catch a star in the act of formation.
  - The result applies to a spherically-symmetric collapse, and we know the collapse can't really be spherical – since clumps tumble, and angular momentum is conserved – so this is offered only as a rough estimate.

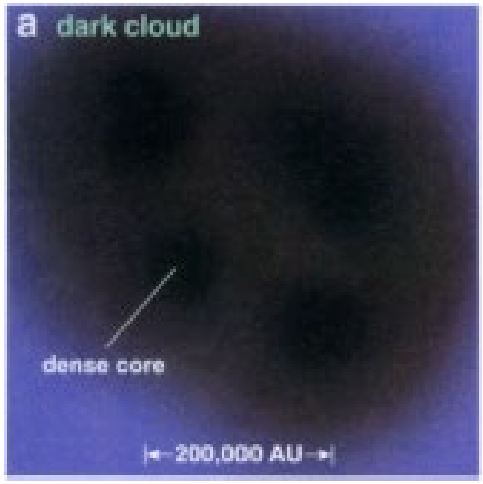
Star formation synopsis

Greene 2001

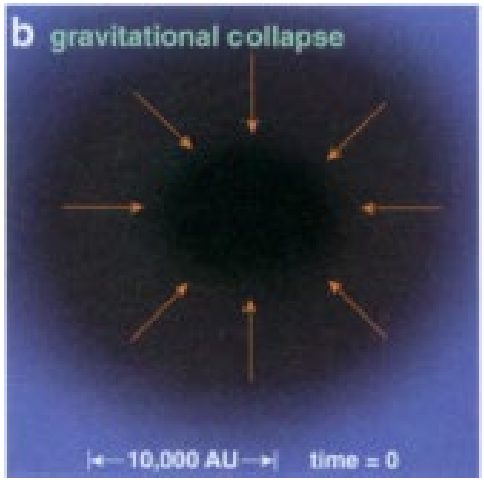


Star formation synopsis (continued)

A molecular clump (a) that gets too cold inside to support its weight with pressure, collapses gravitationally (b).



- Flattening as it does, in obedience to angular-momentum conservation:
  - Collapse parallel to the rotation axis proceeds quickly: essentially in free fall.
  - Collapse along the two dimensions is resisted by centrifugal forces.
  - Material proceeds in this direction til it is in orbit about the center.



- Also heating up as it does, but not yet to *pp*-chain fusion temperatures.

Star formation synopsis (continued)

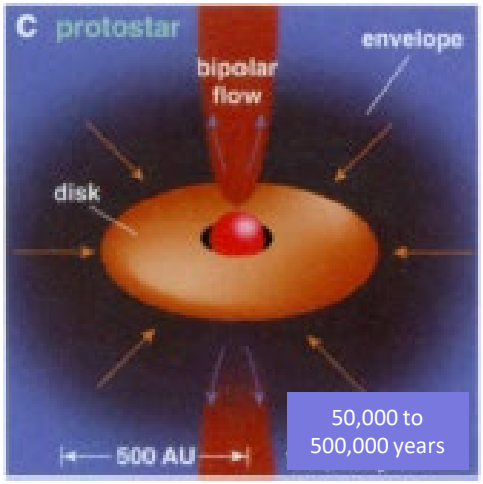
- Smaller, denser scales of the clump collapse faster ("inside-out collapse"), producing a **protoplanetary disk** in the center surrounding a round, dense, hot **core**.

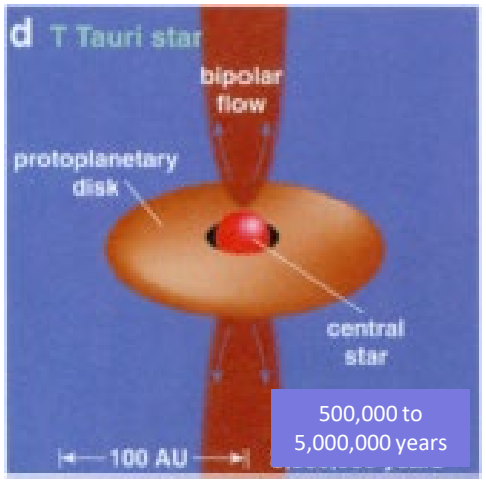
- You can call this whole assembly, or the core in the middle, a **protostar**.

  - Protostars have larger luminosity than the stars they will become, but their heat does not come from fusion.

  - Instead it is largely **accretion power**: gas falling from large radii onto the protostar's surface.

Example of accretion power:





Star formation synopsis (continued)

- The core of a protostar is about ten times larger in radius than the star it will become, and accretes gas from the disk at the rate

$$dm/dt = 10^{-5} - 10^{-7} M_\odot \text{ year}^{-1}$$

decreasing with core mass and as time goes on (Watson et al. 2016).

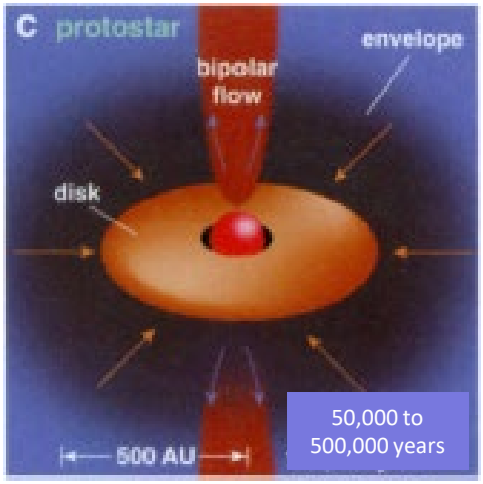
- By energy conservation, for a mass m falling in,

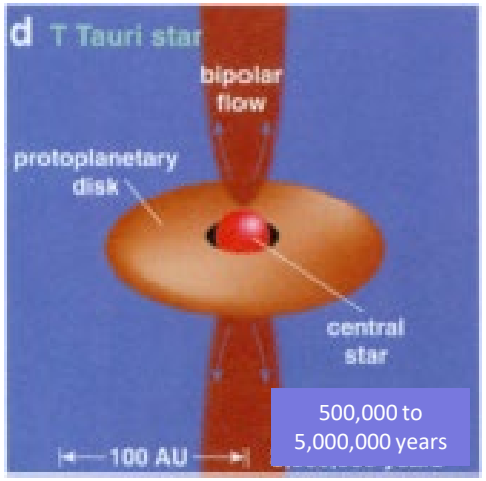
$$\Delta U = -m\frac{GM}{r} - m\frac{GM}{R} \xrightarrow{r\to\infty} -m\frac{GM}{R}$$

$$\Delta E = \Delta KE = -\frac{1}{2}\Delta U$$

$$L = \frac{dE}{dt} = \frac{GM}{2R}\frac{dm}{dt} = \frac{GM_\odot}{20R_\odot}\frac{dm}{dt} \approx 16L_\odot \gg L_\odot$$

for a $1M_\odot$ core and $dm/dt = 10^{-5} M_\odot$ year^{-1}.

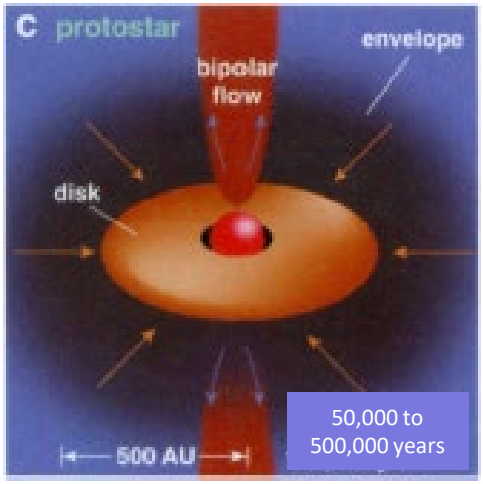


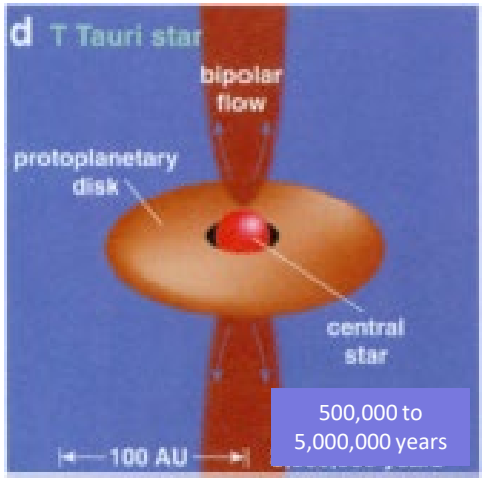


Star formation synopsis (continued)

- The disk accretes more, later-arriving, material from the clump (c).
- The core gradually accretes more material from the disk (d).
- Much of the angular momentum in the accreting material is carried back into the surrounding cloud by a bipolar outflow (c, less in d).
- By the time the clump outside the disk is used up or driven away by the outflow, the accretion rate is much less, and most of the luminosity is from **slow contraction of the core**.

  For low-mass stars the core is called a **T Tauri star** when in this phase.





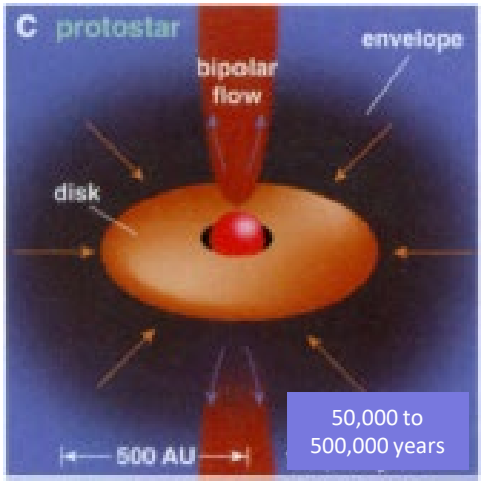
Star formation synopsis (continued)

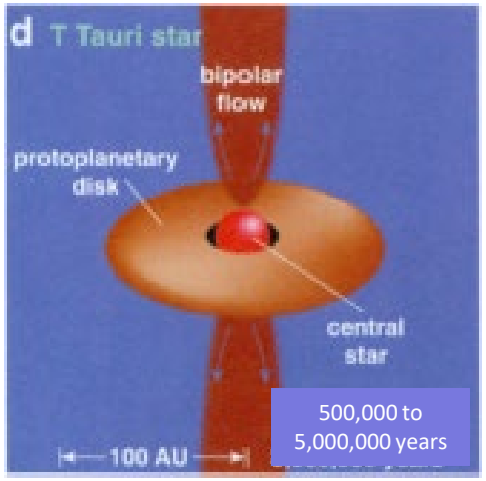
- During this slow gravitational contraction, the conversion of gravitational potential energy dominates the luminosity of young stars. For it is written (Homework #2):

$$L = -\frac{3}{5}\frac{GM^2}{R^2}\frac{dR}{dt}$$

- The gravitational-collapse luminosity decreases with time, and star's effective temperature changes little while this is going on.

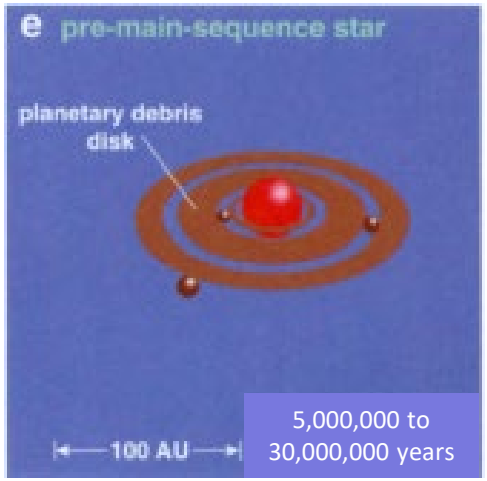
- So the young star descends almost **vertically** at first, through the H-R diagram. This path is called the **Hayashi track**.

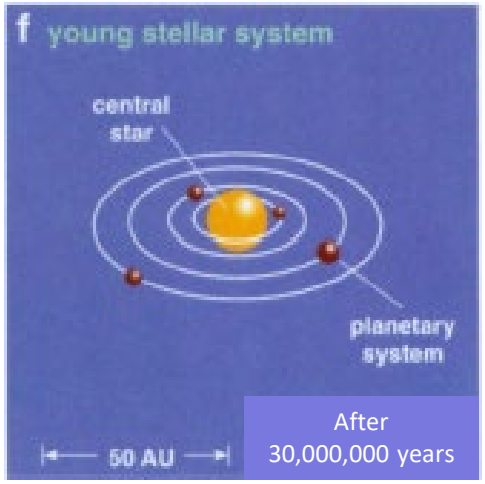




Star formation synopsis (continued)

- After 5 Myr or so, the core no longer accretes mass, and the gas in the disk is used up or driven away.
- It still lies well above its eventual resting place on the main sequence, in which condition we call it a **pre-main-sequence star**.
- Finally, the core reaches *pp*-chain temperatures, fusion begins, and the star heats up, moving **horizontally** in the HR diagram to bluer colors and the MS.
  - This part of its HR path is called the **Henyey track**.





Pre-main-sequence stars in the HR diagram

- The Henyey and Hayashi tracks happen to be the reverse of the path each star will follow at the end of its life, as it becomes a subgiant and a red giant.
- For a given age the redder (lower mass, later spectral type) stars lie further above the main sequence than the bluer ones.

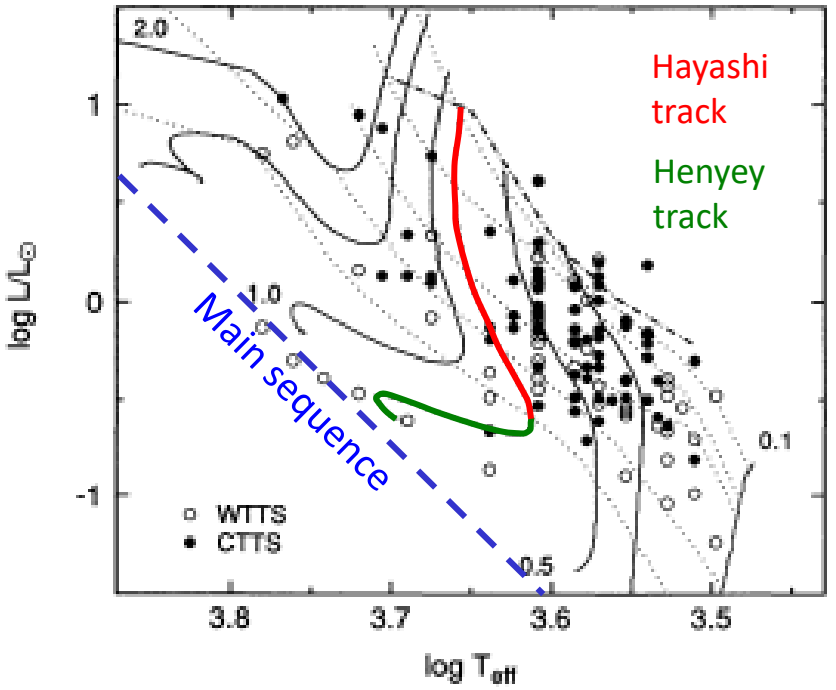


FIG. 15.—H-R diagram for Taurus-Auriga TTSs. Solid lines indicate CMA model tracks for $M_* = 0.1$, 0.3, 0.5, 0.8, 1.0, 1.5, and 2.0 $M_\odot$ (D'Antona & Mazzitelli 1994). Several tracks are labeled with their mass. Dotted lines denote isochrones for 10^5, 3×10^5, 10^6, 3×10^6, and 10^7 yr. The heavy dot-dashed line indicates the stellar birth line (Fletcher & Stahler 1994a, Table 1).

Kenyon & Hartmann 1995