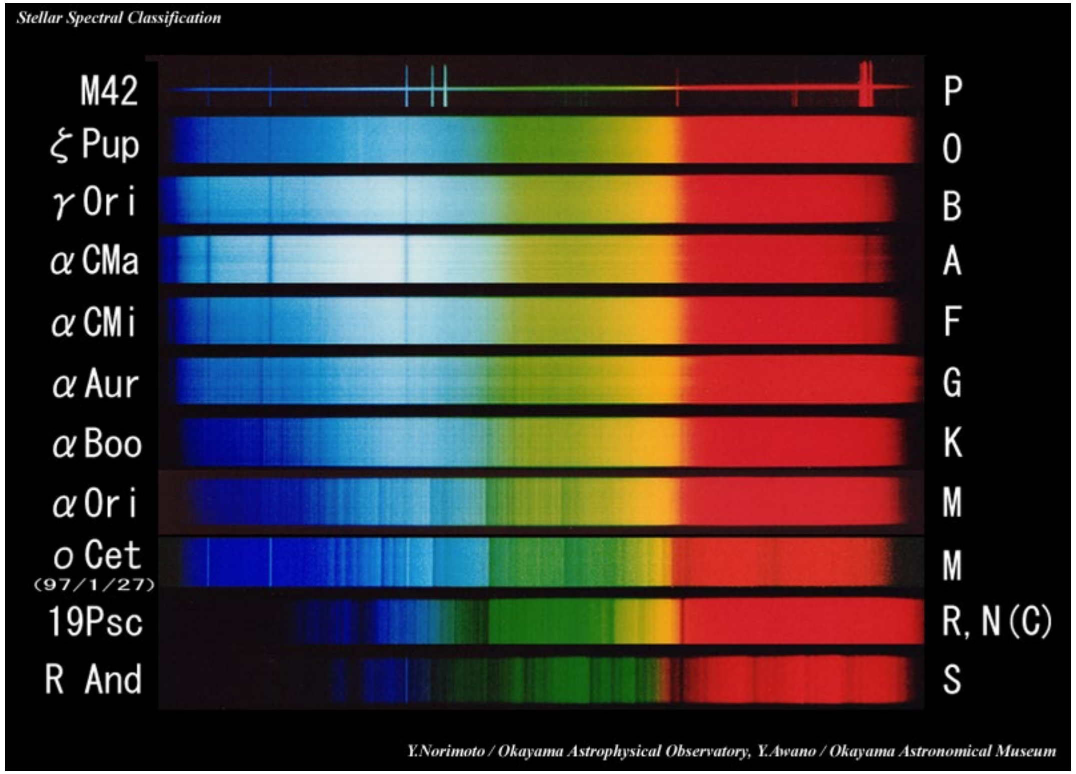
Today in Astronomy 241: stellar spectra and the temperatures of stellar atmospheres

- Stellar spectral types and luminosity types
- Excitation of atomic and molecular states in thermal equilibrium: the Maxwell-Boltzmann distribution
- Ionization state of atoms in thermal equilibrium: the Saha equation

Reading: C&O chapter 8.



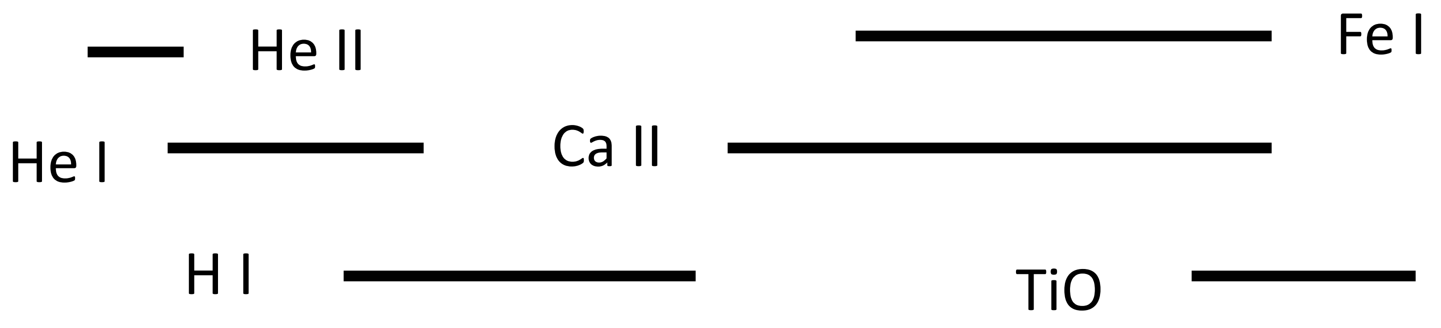
Spectra of main-sequence stars, and a few peculiar ones. From Y. Norimoto and Y. Awano, Okayama Astrophysical Observatory.

Classification of stellar spectra

The Annie J. Cannon – "Harvard" – classification system ([1901](#)), in the order shown later by Cecelia Payne ([1925](#)) to be a sequence of decreasing effective temperature:

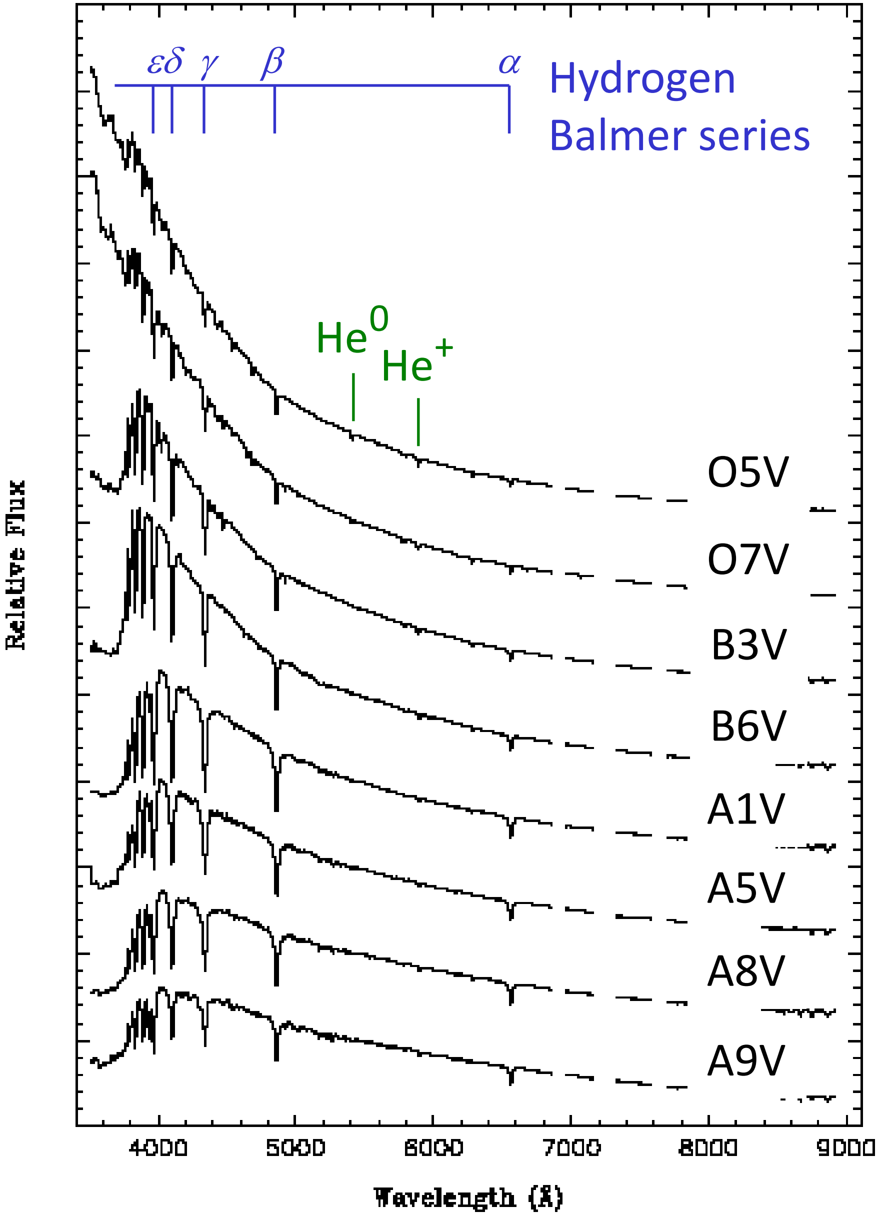
O B A F G K M

Strong absorption lines:



Effective temperature:

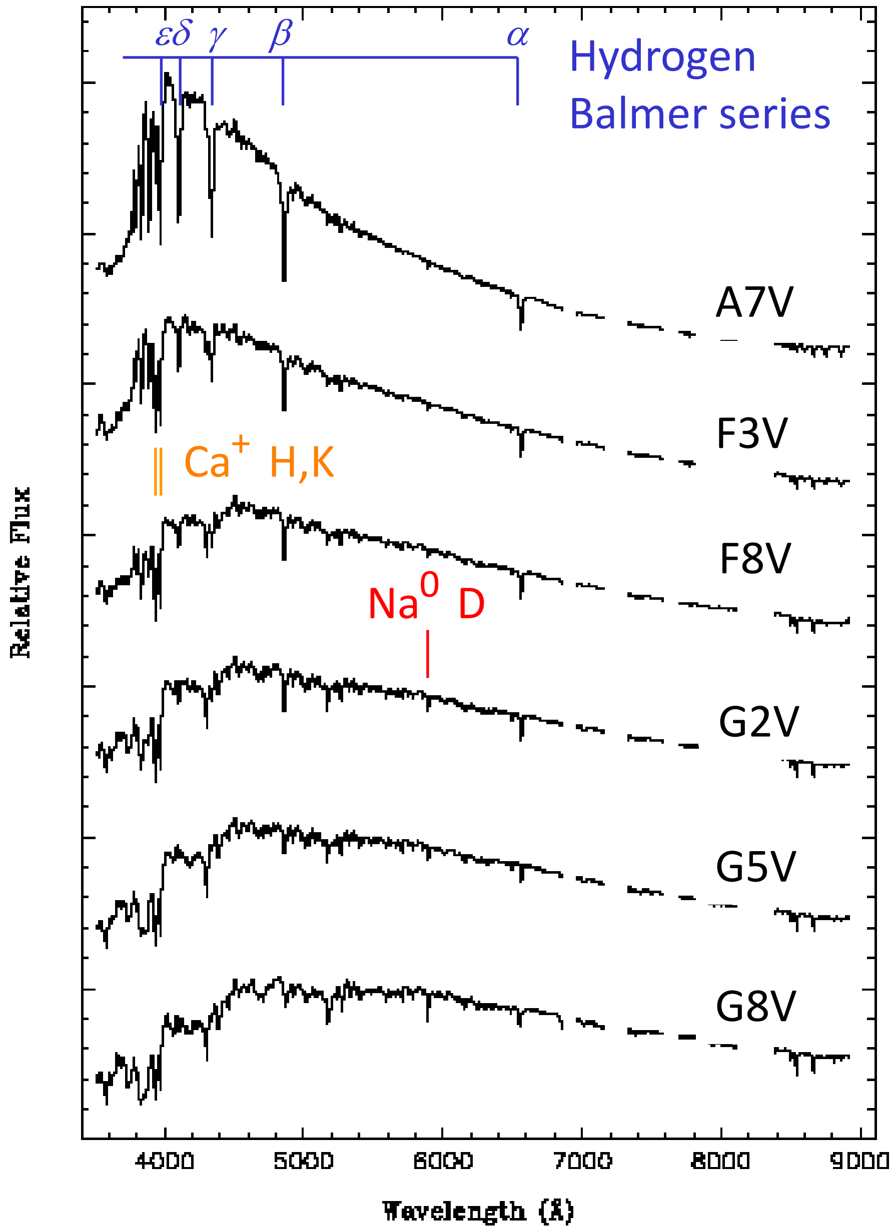
50000 20000 8000 6500 5500 4500 3300



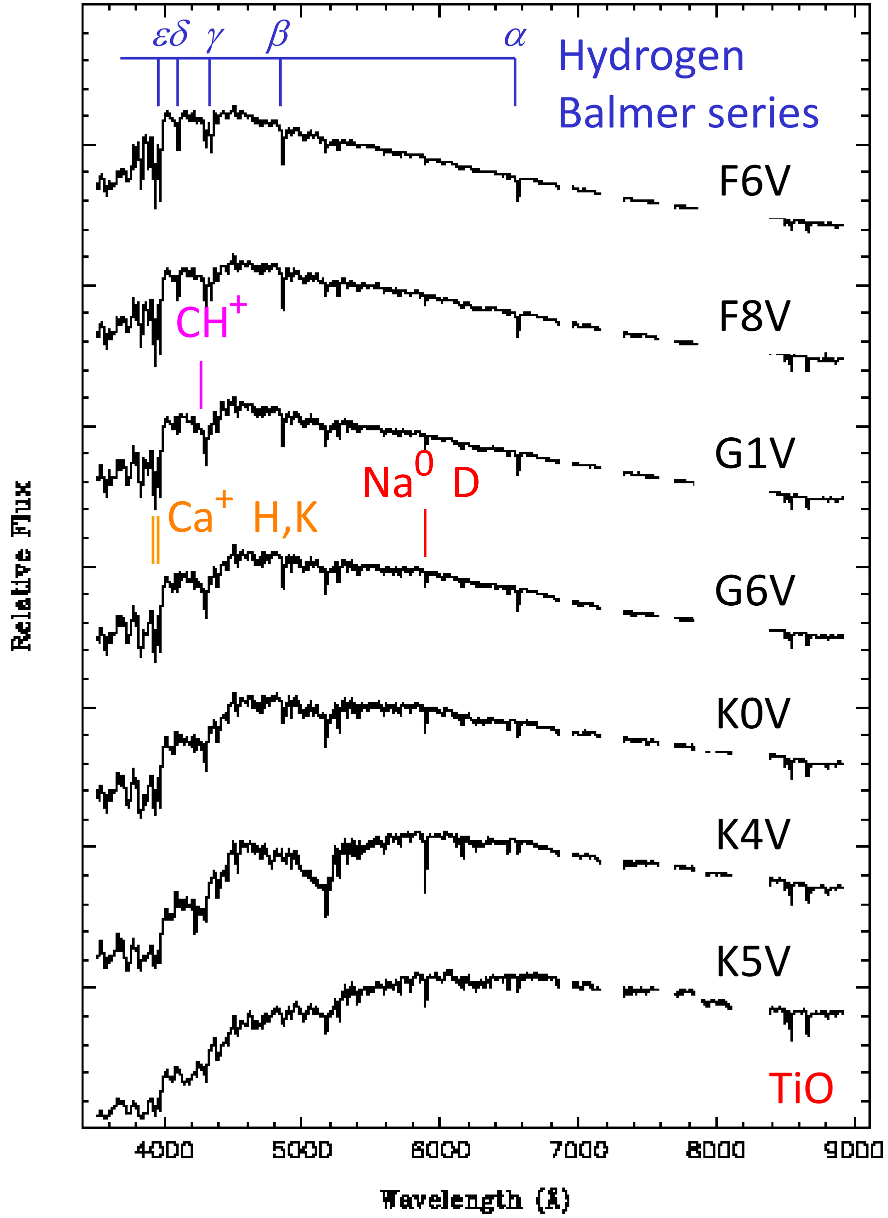
Main-sequence stellar spectra

- From O5 to A9, top to bottom, from David Silva's Ph.D. thesis.
- Effective temperatures for this sequence of stars are T_e = 44500 K – 7400 K, top to bottom.
    - Recall stellar effective temperature is related to luminosity by $L = 4\pi R^2 \sigma T_e^4$.

Main-sequence stellar spectra (continued)



- Same as before, but from A7 to G8, top to bottom.
- T_e = 7850 K – 5570 K, top to bottom.

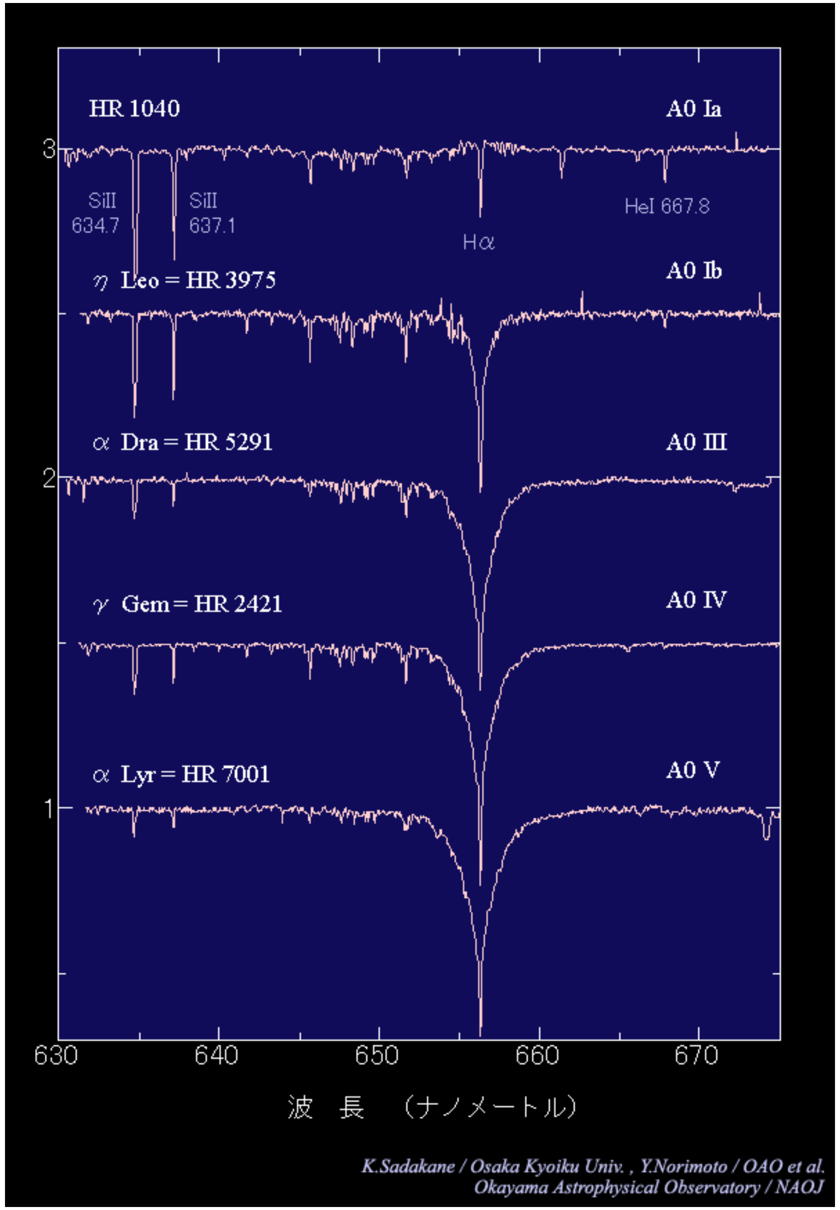


Main-sequence stellar spectra (continued)

- And again, for F6-K5.
- T_e = 6340 K – 4350 K, top to bottom.

Luminosity classification

- Stars with the same color exhibit a wide range of absorption-line **widths**, which turns out to be an effect of **surface gravity**: narrower lines go with weaker gravity, and thus larger – and more luminous – stars.
- This is the basis of the **MKK luminosity classification**: I for the narrowest lines (supergiants), V for the broadest (dwarfs, main sequence), II-IV for the ones in between (giants)
  - And VI, for white dwarfs.



From Y. Norimoto and Y. Awano, [Okayama Astrophysical Observatory](#).

Boltzmann and Saha equations

- Assumption of thermal equilibrium not a bad place to start for stellar atmospheres.

- Temperature at the surface mostly determines the appearance of the star's spectrum.

  - By "surface", we mean the **photosphere**: the opacity surface of the star, a photon mean-free-path away from outer space.

- Excitation and ionization state of species in **local thermodynamic equilibrium** (LTE) are described respectively by the **Maxwell-Boltzmann distribution** and the **Saha equation**.

Maxwell-Boltzmann distribution

For a gas of particles with mass *m*, density *n*, temperature *T*:

$$p(v)dv = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) 4\pi v^2 dv$$

Probability $p(v)dv$ of speed between v and $v+dv$. $p(v)$ itself is a probability **density.**

$$n_v dv = n\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) 4\pi v^2 dv$$

Number density n_v of particles with speeds between v and v+dv

$$v_{mp} = \sqrt{\frac{2kT}{m}} \qquad v_{rms} = \sqrt{\frac{3kT}{m}}$$

Most probable speed, and RMS speed

Maxwell-Boltzmann distribution (continued)

For two states *a* and *b* of any given atomic or molecular species,

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

where *N*, *g* and *E* are number of atoms or molecules; the degeneracy (statistical weight); and the energy of each state, and *T* is the temperature.

- In C&O this expression is called the "Boltzmann equation." Note, however, that the term is usually reserved for a much different equation in nonequilibrium statistical mechanics.

Saha equation

For a species with two ionization stages *i* and *i*+1, where an energy χ_i is required to liberate one electron from stage *i* to produce stage *i*+1, the ratio of numbers of atoms or molecules in the two states is

$$\frac{N_{i+1}}{N_i} = \frac{2}{n_e} \frac{Z_{i+1}}{Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i / kT}$$

where m_e, n_e are mass and number density of electrons, and

$$Z = g_1 + \sum_{j=2}^{\infty} g_j \exp\left(-\frac{E_j - E_1}{kT} \right)$$

is the partition function: the number of states occupied on the average, in the population of atoms/molecules.

Normal stars are ideal gases.

- We will deal most frequently with number density n: number of particles per unit volume, with units cm^{-3}.
- Stars are almost always made of ideal gases, so the form of the ideal gas law we will normally use is

$$P_e = \frac{N_e}{V} kT = n_e kT$$

for electrons.

Today's in-class problems

- Carroll & Ostlie problems 8.4, 8.6, and 8.12. For the latter, consult Example 8.1.4 to get started.

Hints for the last set of in-class problems

1. Since the stars are always on opposite sides of the center of mass, their separation is $r = r_1 + r_2$. We also have $r_2/r_1 = a_2/a_1$, and $r = a\left(1-\varepsilon^2\right)\Big/\left(1+\varepsilon\cos\eta\right)$. Combine all these expressions to eliminate the subscripted variables and get the result we seek.

2. $\int_0^{\pi/2} \sin i\, di = 1$;

$$\left\langle \sin^3 i \right\rangle = \int_0^{\pi/2} \sin^4 i\, di$$

Integrate by parts, with $u = \sin^3 i, du = 3\sin^2 i \cos i\, di, dv = \sin i\, di, v = -\cos i$:

$$\int_0^{\pi/2} \sin^4 i\, di = uv - \int v du = \left[-\sin^3 i \cos i\right]_0^{\pi/2} + 3\int_0^{\pi/2} \sin^2 i \cos^2 i\, di = 0 + 3\int_0^{\pi/2} \sin^2 i \left(1-\sin^2 i\right) di$$

$$= 3\int_0^{\pi/2} \sin^2 i\, di - 3\int_0^{\pi/2} \sin^4 i\, di \quad .$$

Note the $\sin^4 i$ integrals on both sides:

$$\left\langle \sin^3 i \right\rangle = \int_0^{\pi/2} \sin^4 i\, di = \frac{3}{4}\int_0^{\pi/2} \sin^2 i\, di = \frac{3}{8}\int_0^{\pi/2} \left(1-\cos 2i\right) di = \frac{3}{16}\int_0^{\pi} \left(1-\cos w\right) dw$$

$$= \frac{3\pi}{16} + \left[\frac{3}{16}\sin w\right]_0^{\pi} = \frac{3\pi}{16} \quad .$$