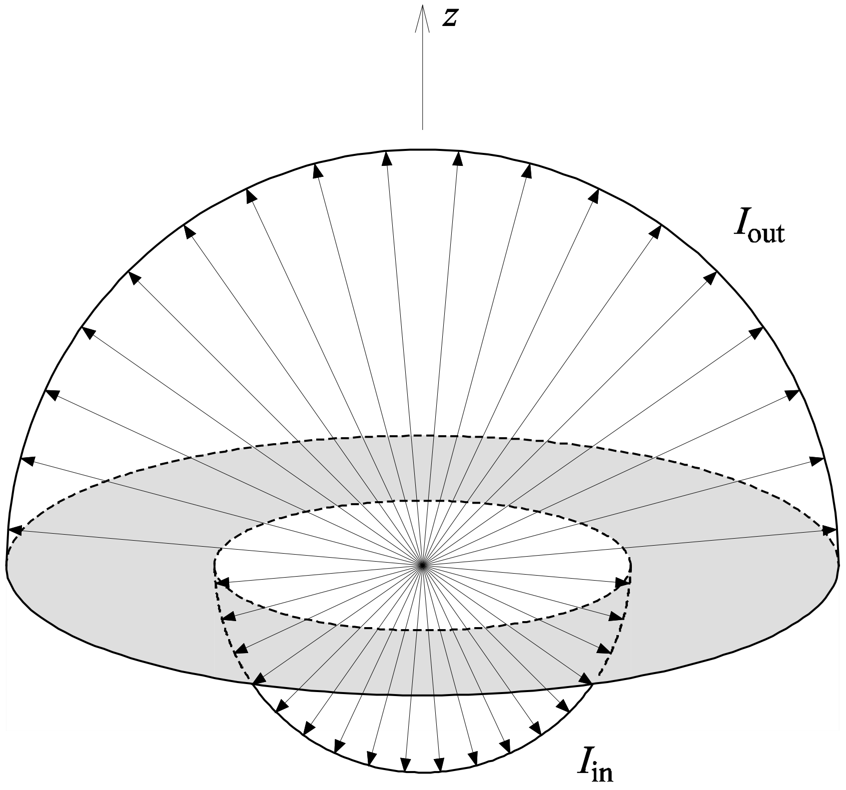
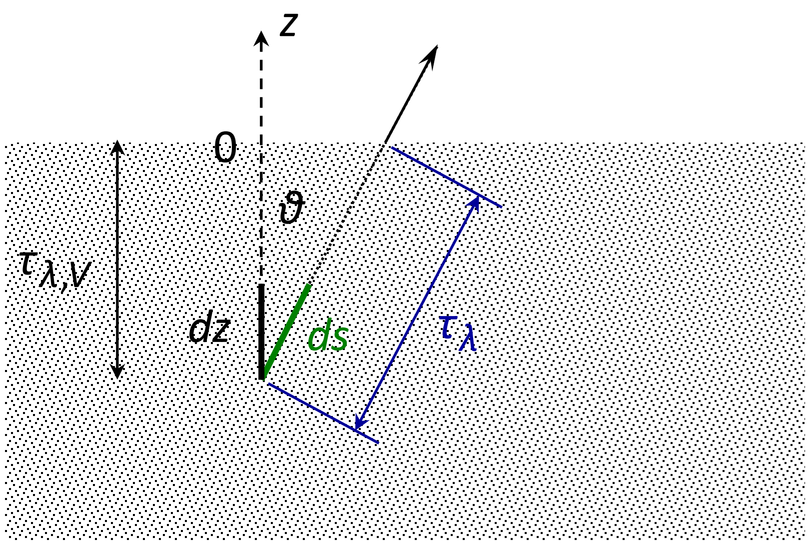
Today in Astronomy 241: My First Stellar Atmosphere Model

- Plane-parallel atmospheres
- Gray atmospheres
- The Eddington approximation
- Solution for the temperature structure of a gray, plane-parallel atmosphere in thermal equilibrium
- **Reading**: C&O chapter 9, pp. 244-258.



The Eddington approximation illustrated: C&O Figure 9.15

Plane-parallel and gray atmospheres



- Plane-parallel atmosphere:

$$-\cos\vartheta \frac{dI_\lambda}{d\tau_{\lambda,v}} = I_\lambda - S_\lambda$$

where $\tau_{\lambda,V}$ is called the vertical optical depth between the point at which light is emitted, and the surface at z = 0.

- The visible photosphere in ordinary stars is plane parallel, to good approximation.

- If opacity is **independent of wavelength** – e.g. if electron scattering or H^- photoionization dominates – this becomes

$$-\cos\vartheta \frac{dI}{d\tau_V} = I - S$$

Radiative transfer for **gray** atmospheres

Some useful relations for gray atmospheres

- From the first two moments of the plane-parallel, gray transfer equation: integrate over solid angle and one gets

$$\frac{dF_{\mathrm{rad}}}{d\tau_V} = 4\pi\left(\langle I \rangle - S\right)$$

- Multiply through by $\cos\vartheta$ and integrate over solid angle, and one gets

$$\frac{dP_{\mathrm{rad}}}{d\tau_V} = \frac{1}{c} F_{\mathrm{rad}}$$

- Furthermore, if one assumes thermal equilibrium everywhere, one obtains

$$F_{\mathrm{rad}} = \mathrm{constant} = \sigma T_e^4 \quad ,$$

$$\langle I \rangle = S \quad ,$$

and $P_{\mathrm{rad}} = \frac{1}{c} F_{\mathrm{rad}} \tau_V + \text{ constant} \quad .$

The Eddington approximation for the gray atmosphere

- The intensity of the radiation field at any depth z within the plane-parallel atmosphere can be broken into outward and inward parts, I_{out} and I_{in}, travelling in the $+z$ and $-z$ directions.

- But $I_{in} = 0$ at the top of the atmosphere, where the vertical optical depth is zero. Thus

$$\langle I \rangle = \frac{1}{2}\left(I_{out} + I_{in}\right)$$

$$F_{\mathrm{rad}} = \pi\left(I_{out} - I_{in}\right)$$

$$P_{\mathrm{rad}} = \frac{2\pi}{3c}\left(I_{out} + I_{in}\right) = \frac{4\pi}{3c}\langle I \rangle$$

$$\langle I \rangle = \frac{3\sigma}{4\pi} T_e^4 \left(\tau_V + \frac{2}{3}\right)$$

$$T^4 = \frac{3}{4} T_e^4 \left(\tau_V + \frac{2}{3}\right)$$

The Eddington approximation for the gray atmosphere (continued)

- That is, **the effective temperature is equal to the physical temperature at an optical depth of 2/3**.
- But the effective temperature determines the flux we see.
- Thus **our view is determined by the value of the source function at an optical depth of 2/3**.

Today's in-class problems

9.17; i.e. starting with the definition of the radiation moments and the Eddington approximation, derive the first three equations on slide 4.

You will derive the next two in Homework #2.

Hints for the last set of in-class problems

1. (B)(i.) Viewed at angle ϑ, the area appears to be $a\cos\vartheta$. So the intensity it radiates is peaked toward the vertical; its maximum is at $\vartheta = 0$ and it drops to zero at $\vartheta = \pi/2$. The necessity of a $\cos\vartheta$ factor is why flux is related to the first moment of the radiation field:

$$F_\lambda = \int I_\lambda \cos\vartheta d\Omega = 2\pi I_\lambda \int_0^{\pi/2} \cos\vartheta \sin\vartheta d\vartheta = 2\pi I_\lambda \int_0^1 u du = \pi I_\lambda \quad .$$

   (B)(ii.) That is, a **planar opaque surface radiates into π steradians**. This is built into Stefan's law: as we saw in last Thursday's problem A,

$$I = \int_\infty^0 B_\lambda d\lambda = \int_0^\infty B_\nu d\nu = \frac{2\pi^4 k^4}{15h^3c^2} T^4 = \frac{\sigma}{\pi} T^4 \quad \Rightarrow \quad F = \int I d\Omega = \pi I = \sigma T^4 .$$

2. (C) The Planck function is of course independent of optical depth; thus the source function is uniform at given wavelength, and the differential equation can be solved by direct integration:

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - B_\lambda \quad \Rightarrow \quad \int_{I_{\lambda,0}}^{I_\lambda} \frac{dI'_\lambda}{I'_\lambda - B_\lambda} = \int_{\tau_\lambda}^{0} d\tau'_\lambda \quad \Rightarrow \quad \ln\left(\frac{I_\lambda - B_\lambda}{I_{\lambda,0} - B_\lambda}\right) = -\tau_\lambda \quad \Rightarrow \quad I_\lambda = I_{\lambda,0} e^{-\tau_\lambda} + B_\lambda \left(1 - e^{-\tau_\lambda}\right) \quad \text{, q.e.d.}$$

Hints for the last set of in-class problems (continued)

If $\tau_\lambda \gg 1$, then $e^{-\tau_\lambda} \to 0$, and $I_\lambda = B_\lambda(T)$.

If, on the other hand, then $\tau_\lambda \ll 1$, then $e^{-\tau_\lambda} \approx 1 - \tau_\lambda$, and

$$I_\lambda \approx (1-\tau_\lambda) I_{\lambda,0} + \tau_\lambda B_\lambda$$

$$\to I_{\lambda,0} \quad \text{if } \tau_\lambda \to 0 \text{ and } I_{\lambda,0} \neq 0;$$

$$\to \tau_\lambda B_\lambda(T) \quad \text{if } I_{\lambda,0} \to 0 \text{ and } \tau_\lambda \neq 0.$$

The latter represents optically-thin emission from the cloud, a common case for bright nebulae as well as the lowest-density upper layers of a stellar atmosphere.