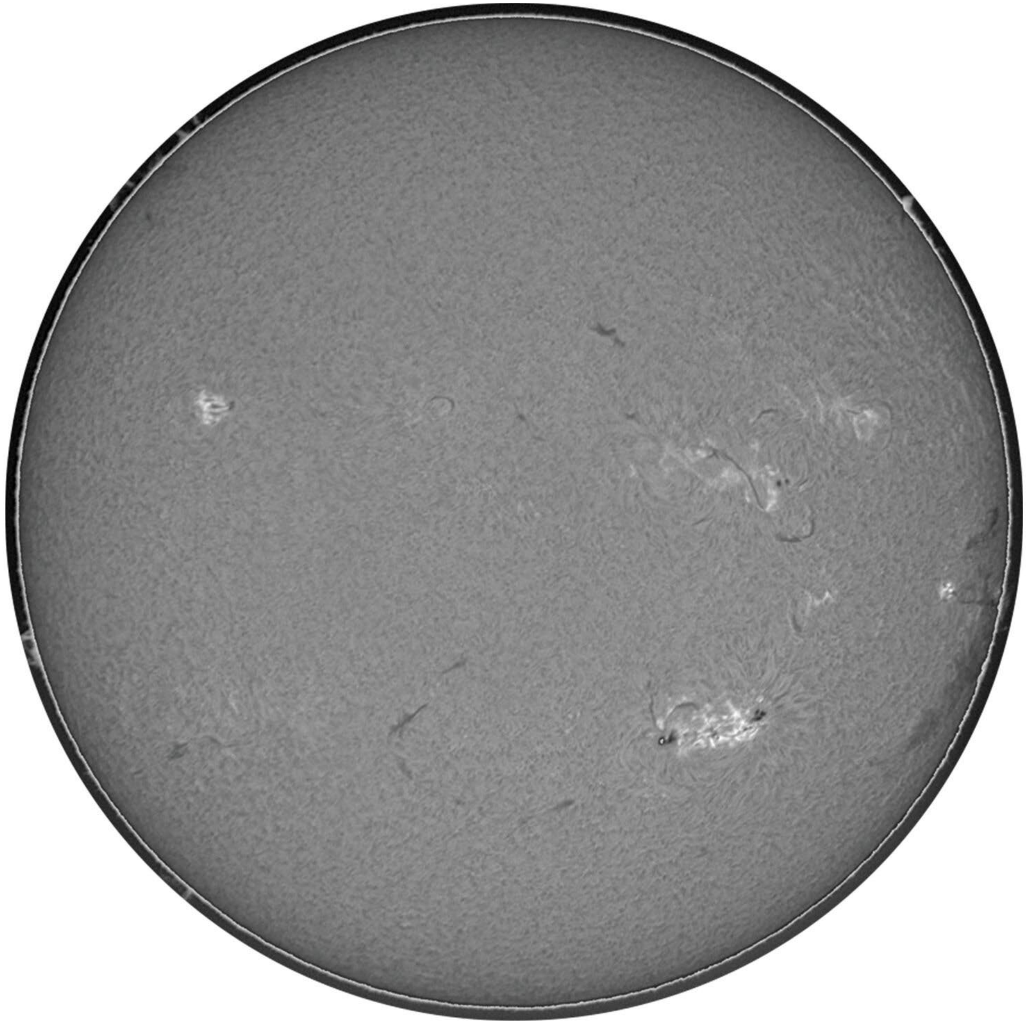
Today in Astronomy 241: spectral lines in stars

- Widths and profiles of spectral lines in stellar atmospheres
- Equivalent width
- The curve of growth
- **Reading**: C&O pp. 267-278

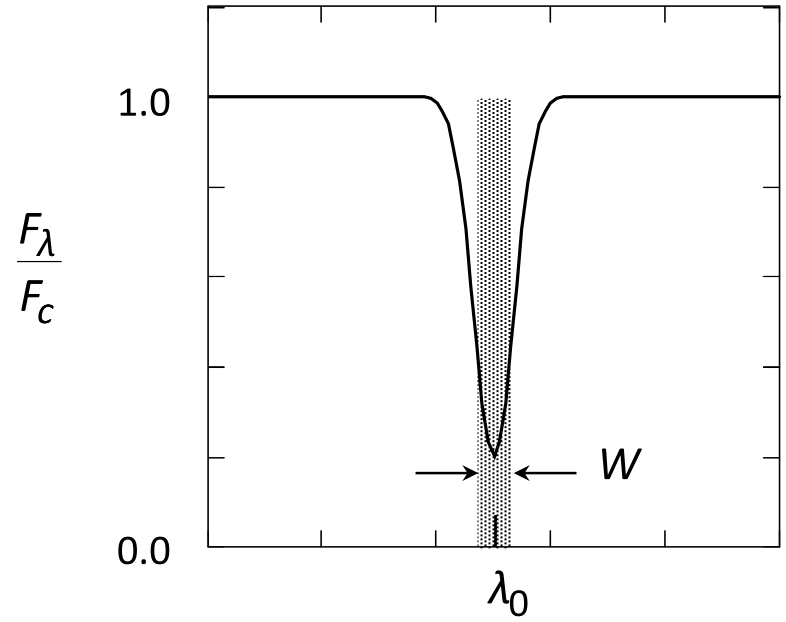


Solar image at λ = 656.3 nm (Hα). Note limb darkening, Hα absorption features, Hα emission features. From the US National Solar Observatory.

Equivalent width of an absorption line

- As discussed in ASTR 142 (e.g. here, page 8), most lines in most normal stars appear in absorption.
- Observers tend to characterize absorption lines by their equivalent width.
- The equivalent width W is the width in wavelength units of the rectangle reaching from zero flux up to the continuum (off-line) flux that has the same area as that under the spectral line's profile:

$$W = \int \frac{F_c - F_\lambda}{F_c} d\lambda$$

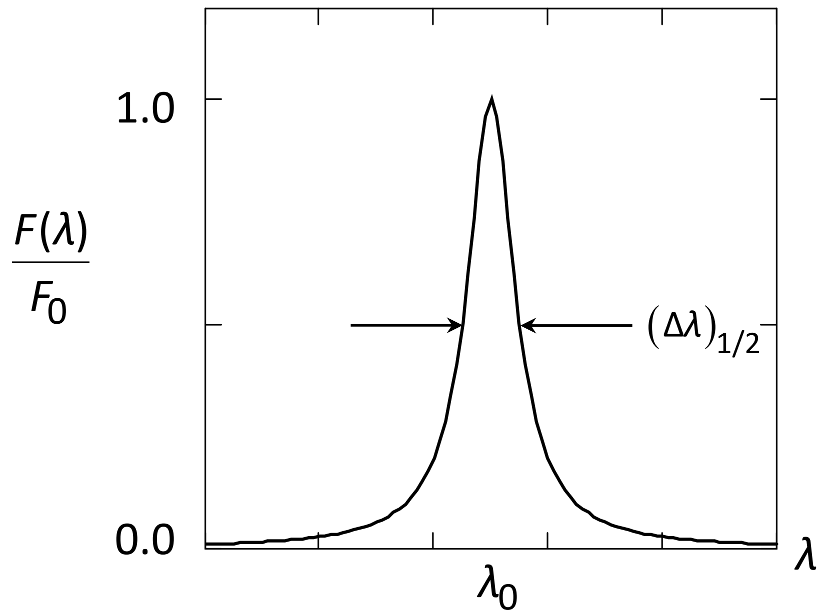


Widths and profiles of spectral lines

- **Natural broadening**: width due to lifetime of atomic or molecular states involved in the transition:

$$(\Delta\lambda)_{1/2} = \frac{\lambda^2}{\pi c}\frac{1}{\Delta t_0}$$

$$F(\lambda) = \frac{F_0}{1+\left(\dfrac{2(\lambda-\lambda_0)}{(\Delta\lambda)_{1/2}}\right)^2}$$



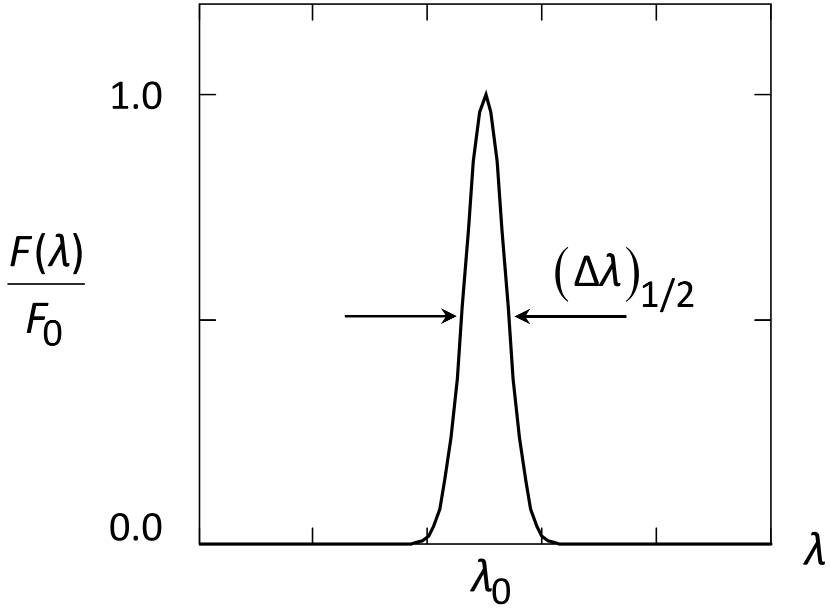
- This is a **Lorentzian** line shape.

Widths and profiles of spectral lines

- **Thermal Doppler broadening**: width due to thermal distribution of atomic or molecular velocities.

$$(\Delta\lambda)_{1/2} = \frac{2\lambda}{c}\sqrt{\frac{2kT\ln 2}{m}}$$

$$F(\lambda) = F_0 e^{-\ln 2(\lambda-\lambda_0)^2/(\Delta\lambda)_{1/2}^2}$$



- This is a **Gaussian** line shape.

Widths and profiles of spectral lines

- **Pressure broadening**: width due to perturbation of atomic or molecular states by collisions with others.

$$(\Delta\lambda)_{1/2} \approx \frac{\lambda^2}{c}\frac{n\sigma}{\pi}\sqrt{\frac{2kT}{m}}$$

σ: collision cross section
n: number density of colliders

Leads again to a Lorentzian line shape.

- Total line profile represents the **convolution** of all these effects, called the **Voigt profile**. Not easily expressed analytically:
  - Line core most strongly influenced by Gaussian parts.
  - Line wings most strongly influenced by Lorentzian parts.

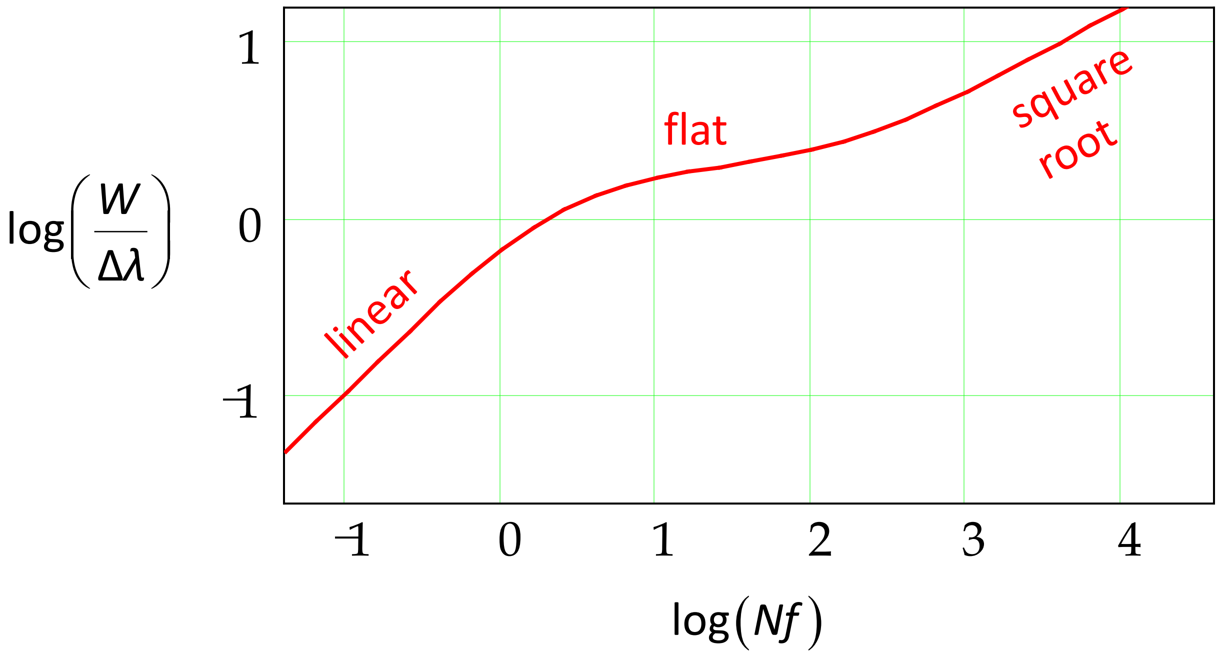
Curve of growth: dependence of line width on column density

- Column density = integral of number density along the line of sight. Units: cm^{-2}.
  - For small column density, the equivalent width *W* is proportional directly to column density *N* of absorbing atoms or molecules (**linear** part of the curve of growth).
  - For larger column density, after the Gaussian core bottoms out, the *W* is proportional to the square root of the logarithm of *N* (**flat** part of the curve of growth).
  - For even larger column density, after the absorption by the Lorentzian wings becomes strong, *W* is proportional to $\sqrt{N}$ (**square root** part of the curve of growth).
- In general there's a different curve of growth, *W*(*N*), for each spectral line.

Curve of growth (continued)

- "Universal" curve of growth: the ratio of W to Doppler line width $\Delta\lambda$ depends upon the product of N and a line's **oscillator strength** f in the same way for every spectral line.

- See also "general" curve of growth, in C&O.



Curve of growth (continued)

- Main use of the curve of growth, before computer models of stellar atmospheres became "easy:" abundance measurement.
  - Measure *W* for a lot of lines (each with known *f*) of a bunch of atomic or ionic species.
  - Plot $W/\Delta\lambda$ against $\chi N f$, where
    - *N* is the column density of one species,
    - χ is the **relative abundance** of the atomic species that gives rise to the line: the ratio of number density of that species to the number density of the first species.
  - Adjust χ, *N*, and $\Delta\lambda$ until the points fit the universal curve of growth.
  - Then one knows these three quantities for each species.

Curve of growth (continued)

- Results for observations of Fe I, Ti I, Fe II and Ti II lines in the Sun, for:

$$T = 5100 \text{ K},$$

$$V_{\text{turbulent}} = 1.5 \text{ km sec}^{-1},$$

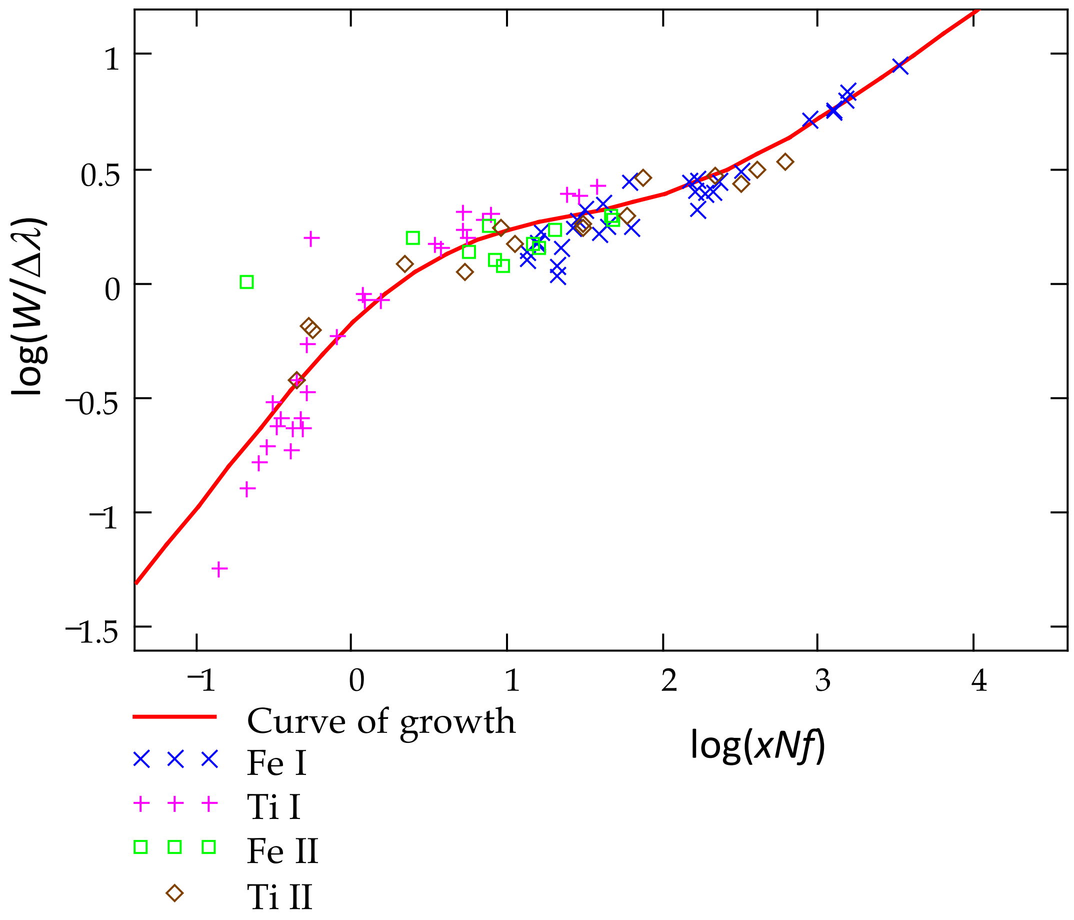
$$\frac{\text{Ti}^0}{\text{Fe}^0} = 10^{-4},$$

$$\frac{\text{Fe}^+}{\text{Fe}^0} = 15.8, \quad \frac{\text{Ti}^+}{\text{Ti}^0} = 631.$$

Thus, from the Saha equation,

$$T_{\text{ionization}} = 3400 \text{ K},$$

$$n_e = 1.7 \times 10^8 \text{ cm}^{-3}$$



Today's in-class problems

9.24 and **9.25**.

Hints for the last set of in-class problems

D. $$I_\lambda(0,\boldsymbol{s}) = -\int_\infty^0 d\tau_{\lambda,V} \sec\vartheta S_\lambda(\boldsymbol{s}') e^{-\tau_{\lambda,V}\sec\vartheta} = \int_0^\infty d\tau_{\lambda,V} \sec\vartheta I_\lambda(0,V) \sum_{n=0}^{N} a_{\lambda,n} \tau_{\lambda,V}^n e^{-\tau_{\lambda,V}\sec\vartheta}$$

$$\frac{I_\lambda(0,\boldsymbol{s})}{I_\lambda(0,V)} = \sum_{n=0}^{N} a_{\lambda,n} \int_0^\infty d\tau_{\lambda,V} \sec\vartheta \tau_{\lambda,V}^n e^{-\tau_{\lambda,V}\sec\vartheta}$$

$$= \sum_{n=0}^{N} a_{\lambda,n} \int_0^\infty du u^n \cos^n\vartheta e^{-u}$$

Recall:

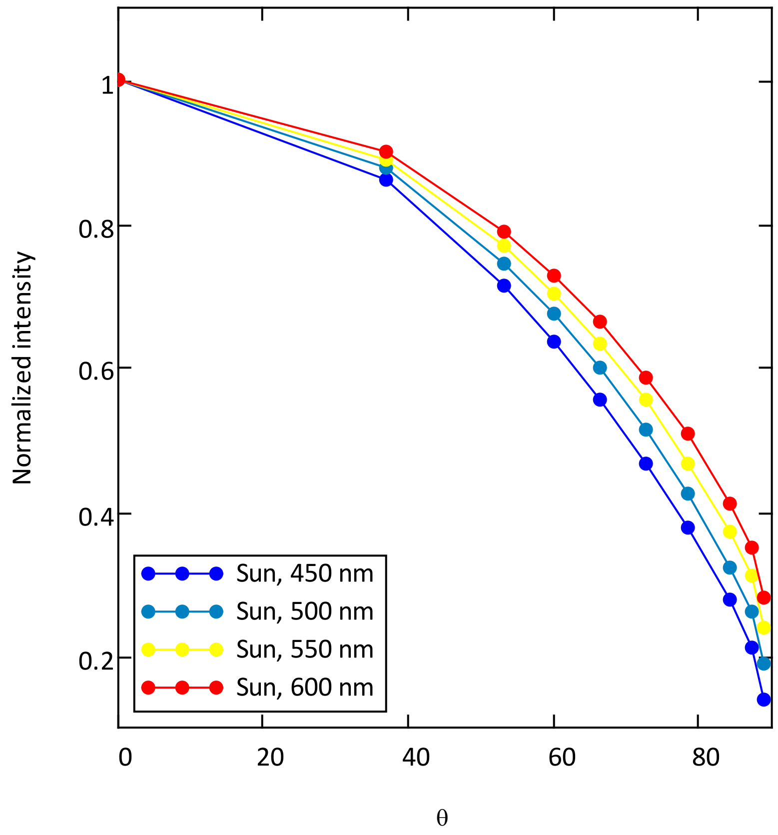
$$\int_0^\infty du u^n e^{-u} = n!$$ which one obtains by integrating by parts *n* times; so

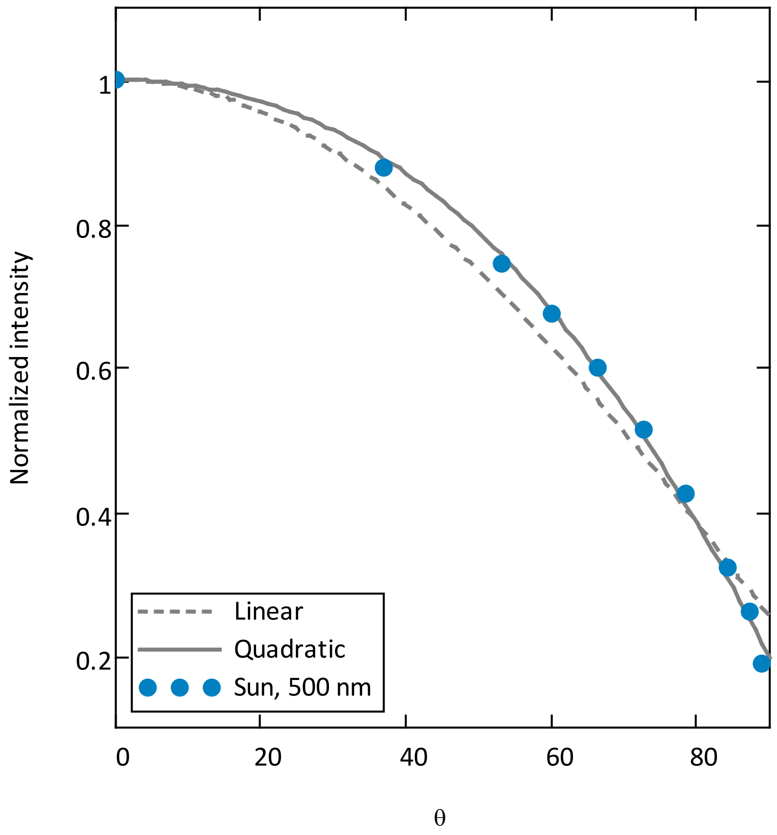
$$\frac{I_\lambda(0,\boldsymbol{s})}{I_\lambda(0,V)} = \sum_{n=0}^{N} a_{\lambda,n} n! \cos^n\vartheta$$

E. This is done in C&O on page 266.

Hints for the last set of in-class problems

F. The first part is also done in C&O on page 266. I used data from Allen's Astrophysical Quantities, 3rd edition, and found the coefficients by a least-squares fit: *b* = 0.745 for linear limb darkening, *b* = 1.119, *c* = -0.318 for quadratic. In both cases *a* = 1 – (*b* or *b* + *c*). Note that quadratic limb darkening works significantly better than linear.





Normalized intensity $= I_\lambda(0,\vartheta)/I_\lambda(0,V)$