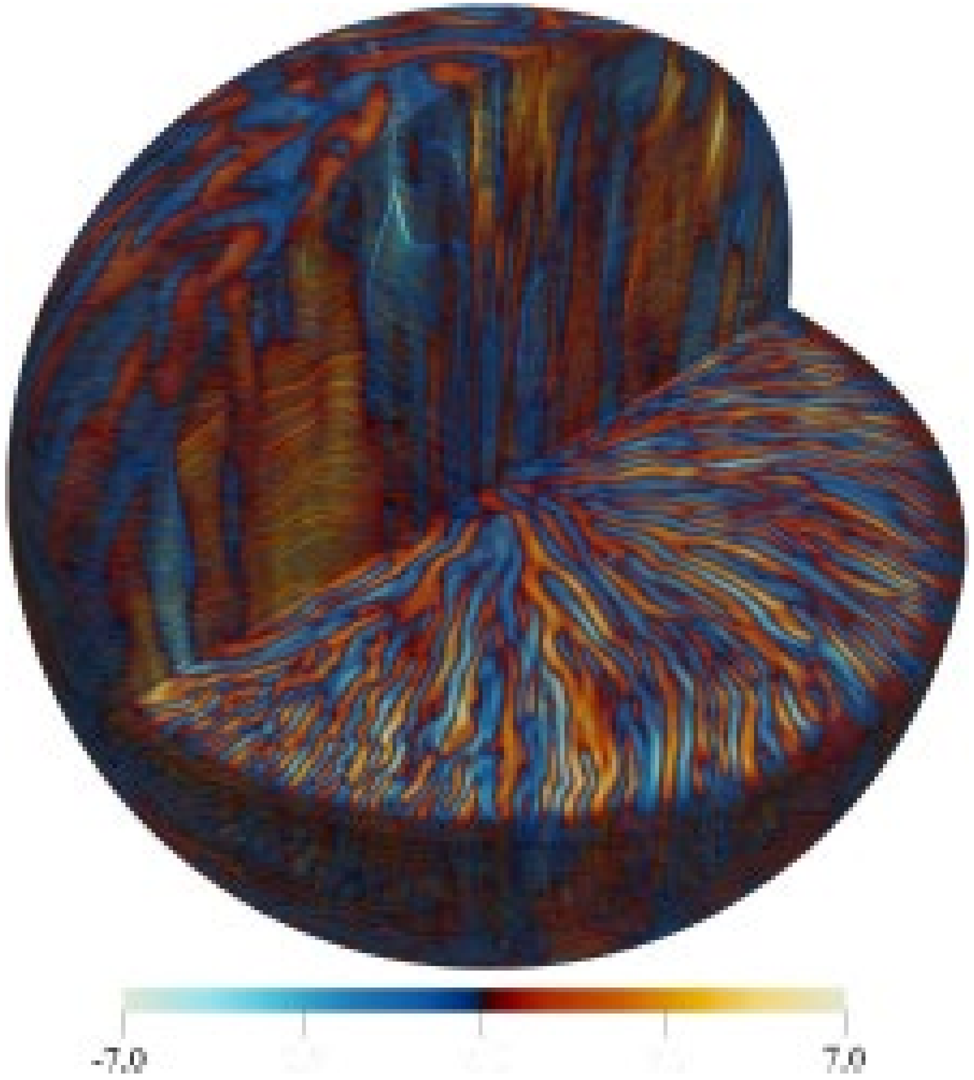
Today in Astronomy 241: polytropes II

- Luminosity in polytropes
- Convection in polytropes
- **Reading**: no new reading!



v, m sec^{-1}

Fluid speeds in a fully-convective stellar interior, from Kapyla 2021. (Not a polytrope though.)

Polytropes and the rest of the stellar-structure equations

- Just like the Cheat you employed in the last two sets of in-class problems, the solution to the Lane-Emden equation provides you the density structure of the star, only this time with a polytrope EOS.
- And just as was the case there, one can use the density to derive the pressure, temperature, and luminosity of the star, by integrating the structure equations that have hitherto been neglected in our polytrope discussions.
- Most of these require numerical integration, even most of the cases in which the LE equation yields analytical solutions.
- The rewards for doing so:
  - Can obtain luminosity, and thus the "theorist's" HR diagram of polytropes.
  - Can investigate the degree to which these models indeed are unstable to convection, or not.
- That's what we will explore, today and in Homework #6, using the LE-solving tool you developed in Homework #5.
  - Or hopefully will develop by the end of the day.

Today's in-class problems

1. (10.17) Solve the Lane-Emden equation analytically for n = 0, obtaining

$$D_0(\xi) = 1 - \frac{\xi^2}{6} \quad , \quad \xi_{\max} = \sqrt{6} \quad .$$

2. (10.18) Describe – i.e. sketch, or plot – the density distribution $\rho(r)$ for an n = 0 polytrope.

3. Derive an expression for the mass M_r enclosed by radius r, and the central density ρ_C, for the n = 0 polytrope. Express your answer in terms of the total mass M and radius R of the star.

4. Derive an expression for the pressure $P(r)$ in the n = 0 polytrope.

5. Derive an expression for the temperature $T(r)$ in the n = 0 polytrope. Note that $2 - 5x^2 + 4x^4 - x^6 = \left(2 - x^2\right)\left(1 - x^2\right)^2$.

6. Is the n = 0 polytrope convective?

Complete hints for the last two sets of in-class problems

Answers, in convenient units:

A. $\rho_C = 5.64\left(\frac{M}{1M_\odot}\right)\left(\frac{R_\odot}{R}\right)^3$ gm cm^{-3}

B. $P = \frac{\pi}{36}G\rho_C^2R^2\left[5-24\left(\frac{r}{R}\right)^2+28\left(\frac{r}{R}\right)^3-9\left(\frac{r}{R}\right)^4\right]$

$P_C = 4.44\times10^{15}\left(\frac{M}{1M_\odot}\right)^2\left(\frac{R_\odot}{R}\right)^4$ dyne cm^{-2}

C. $T = \frac{\pi}{36}\frac{G\mu m_\mathrm{H}}{k}\rho_C R^2\left(1-\frac{r}{R}\right)\left[5+10\frac{r}{R}-9\left(\frac{r}{R}\right)^2\right]$

$T_C = 9.62\times10^6\left(\frac{M}{1M_\odot}\right)\left(\frac{R_\odot}{R}\right)$K

D. $\left.\frac{dT}{dr}\right|_{r=R/2} = -\frac{3}{16\sigma}\frac{\kappa_0\rho_C^2}{4\pi R^2}\left(\frac{31\pi}{288}\frac{G\mu m_\mathrm{H}}{k}\rho_C R^2\right)^{-6.5}L_{R/2}$

E. $\left.\frac{dT}{dr}\right|_{r=R/2} = -\frac{29\pi}{144}\frac{G\mu m_\mathrm{H}}{k}\rho_C R$

F. $L_{R/2} = \frac{116\pi^3}{81}\left(\frac{31}{96}\right)^{6.5}\frac{\sigma}{\kappa_0}\left(\frac{Gm_\mathrm{H}}{k}\right)^{7.5}\frac{\mu^{7.5}M^{5.5}}{R^{0.5}} = L$

$L = 1.2L_\odot\left(\frac{M}{M_\odot}\right)^{5.5}\left(\frac{R_\odot}{R}\right)^{0.5}$