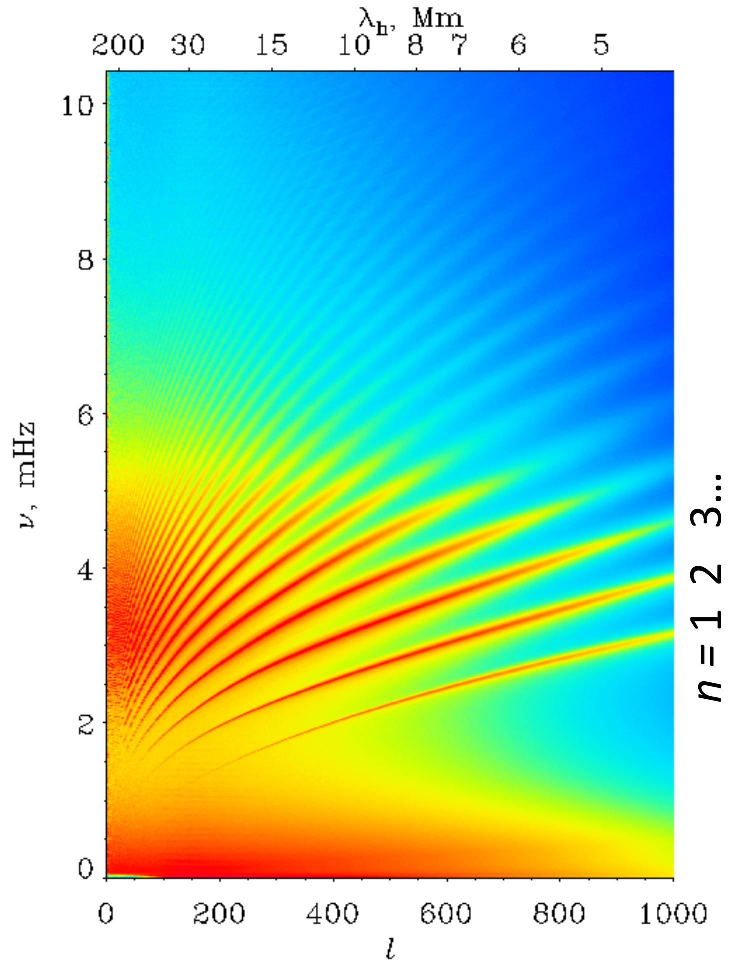
Today in Astronomy 241: the Sun

- The standard solar model
- **Reading**: C&O chapter 11, pp. 349-394

Because most of the basic facts about the Sun were covered adequately in ASTR 142 (e.g. classes 5 and 7), most of today's reading is a reminder, and can be done at your leisure.



The solar five-minute oscillations: normal modes of nonradial pulsation. Each dot represents a sound speed measurement along a unique path through the solar interior. See MDI/SOHO/NASA.

The Sun's interior

- Standard model: Bahcall+2006, Vinoles+2017

- Central conditions, according to models fit to helioseismological observations and neutrino emission as well as observations with plain old ordinary light:

$$T = 1.58 \times 10^{7} \text{ K}$$

$$P = 2.50 \times 10^{17} \text{ dyne cm}^{-2}$$

$$\rho = 1.62 \times 10^{2} \text{ gm cm}^{-3}$$

$$X = 0.336$$

$$Y = 0.643$$

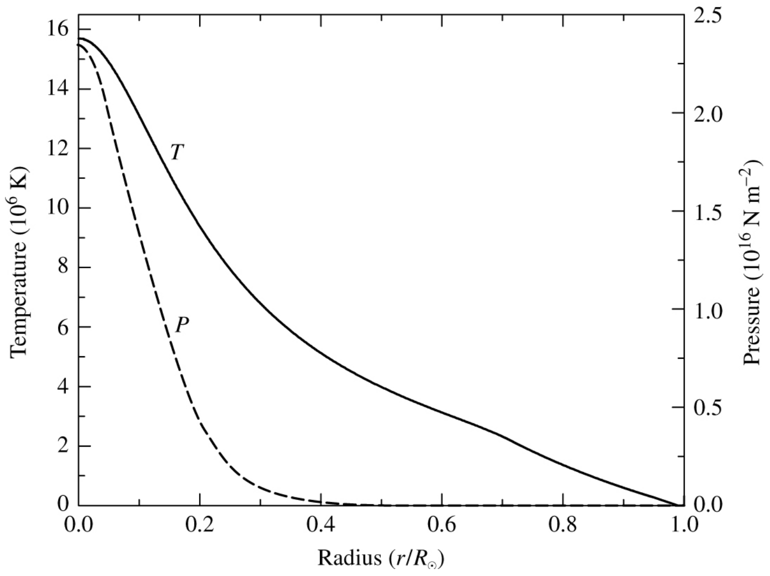
$$Z = 0.021$$

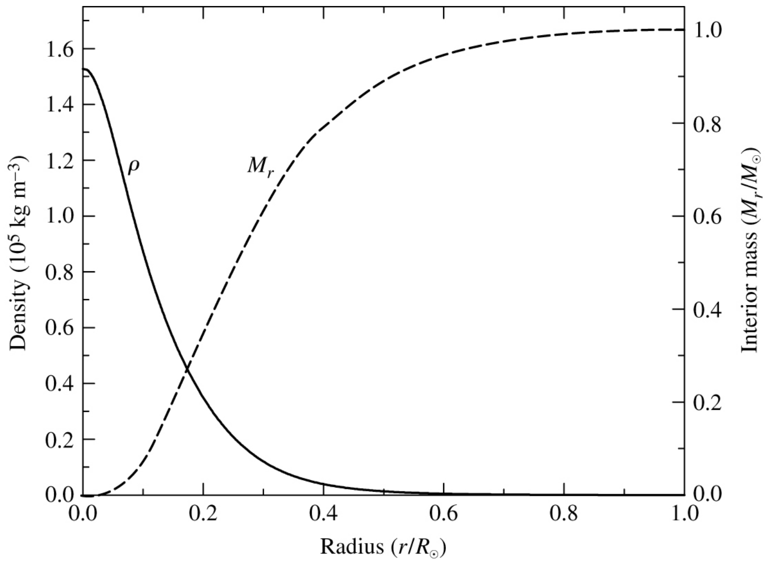
Hydrogen already substantially depleted.

- Outer 29% by radius is convective; convection zone extends all the way to the base of the photosphere. This is shown very directly in the helioseismological observations.

- For good reason, though, the conversion of helioseismological data to constraints on the solar structure is beyond the scope of this course.

The standard solar model

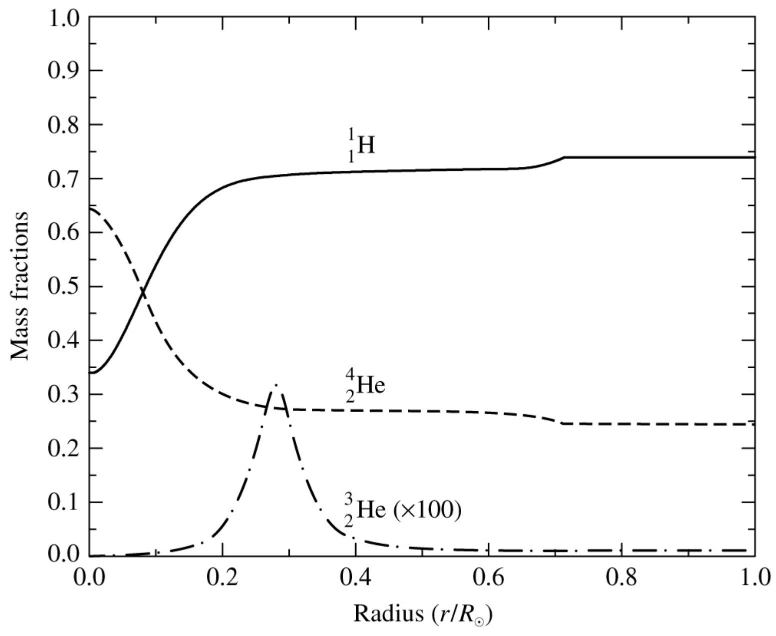


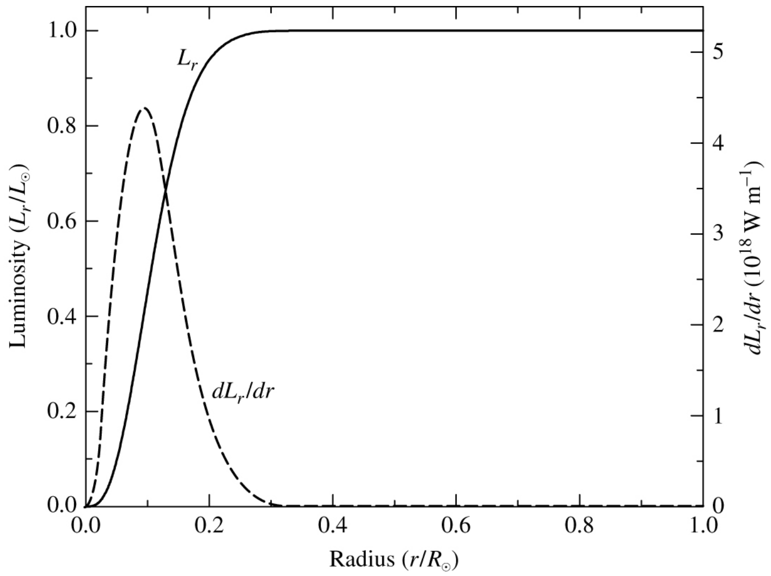


$$P(r), T(r), \rho(r), M(r)$$

Figure from Carroll and Ostlie, *Modern Astrophysics, 2e*.

The standard solar model (continued)

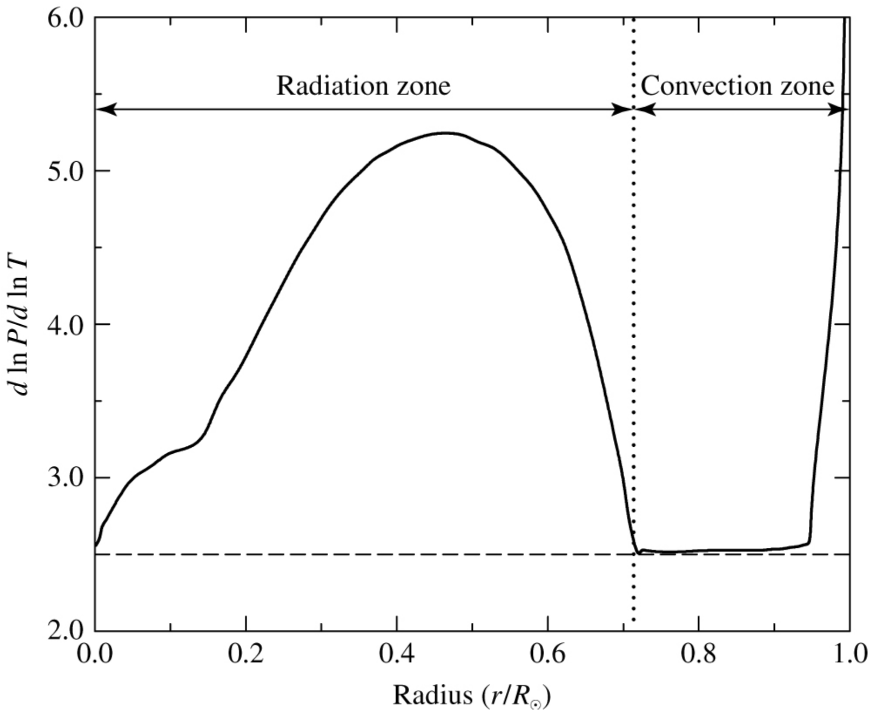




Abundances, fusion-power generation

Figure from Carroll and Ostlie, [*Modern Astrophysics, 2e*].

The standard solar model (continued)



Radiation and convection zones

Figure from Carroll and Ostlie, [*Modern Astrophysics, 2e*]().

The Sun is pretty close to static, in the standard model

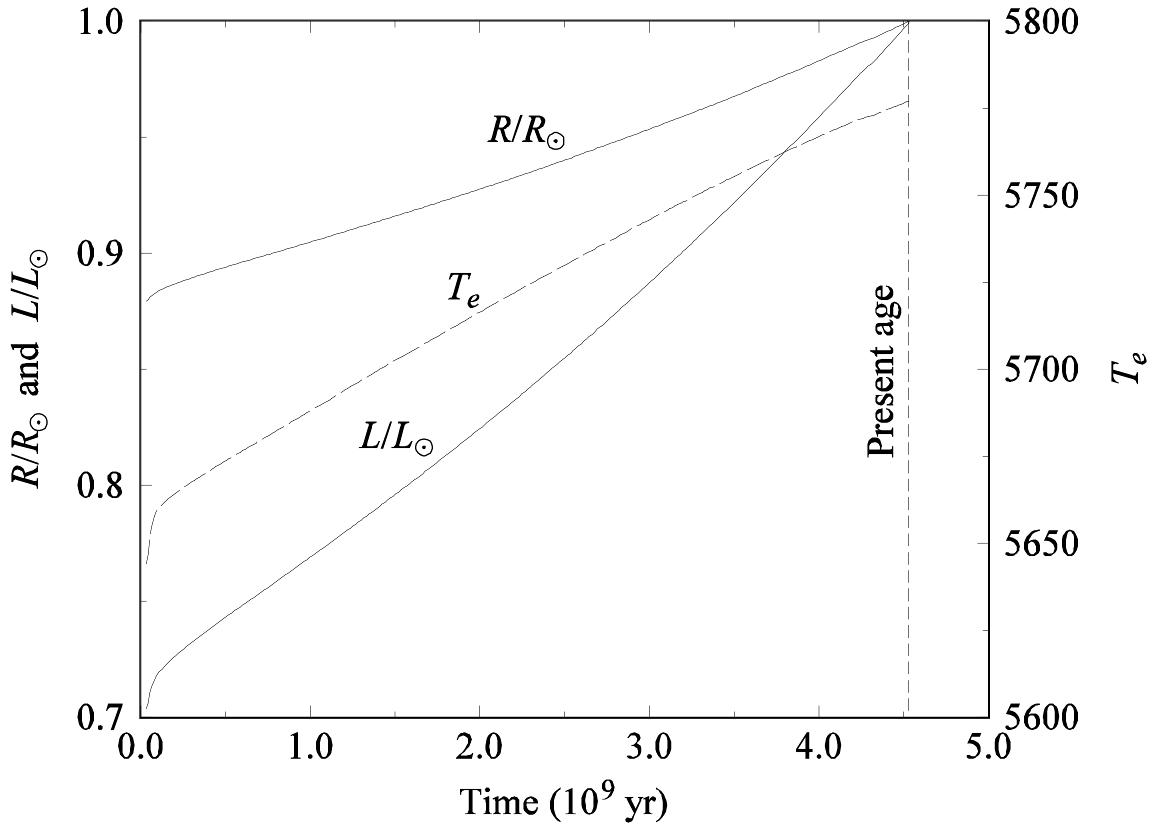


Figure from Carroll and Ostlie, *Modern Astrophysics,* 1e.

Today's in-class problems

1. In your Homework #5 teams, run your new Lane-Emden solver, and improve the numerical precision of the solution til you can replicate these results:

n	ξ_1	$\xi_1^2 D_n'(\xi_1)$
1.5	3.65375	-2.71405
3	6.89685	-2.01824

2. Then return to the solo problems, and see what your answers are for the central density, pressure, and temperature are for n = 3/2.

3. (10.19) Derive an expression for the total mass of an n = 5 polytrope, and show that, although $\xi_1 \to \infty$, the total mass is finite.

Complete hints for the last two sets of in-class problems

Answers, in convenient units:

A. $$\rho_C = 5.64\left(\frac{M}{1M_\odot}\right)\left(\frac{R_\odot}{R}\right)^3 \text{ gm cm}^{-3}$$

B. $$P = \frac{\pi}{36}G\rho_C^2R^2\left[5-24\left(\frac{r}{R}\right)^2+28\left(\frac{r}{R}\right)^3-9\left(\frac{r}{R}\right)^4\right]$$

$$P_C = 4.44\times10^{15}\left(\frac{M}{1M_\odot}\right)^2\left(\frac{R_\odot}{R}\right)^4 \text{ dyne cm}^{-2}$$

C. $$T = \frac{\pi}{36}\frac{G\mu m_\mathrm{H}}{k}\rho_C R^2\left(1-\frac{r}{R}\right)\left[5+10\frac{r}{R}-9\left(\frac{r}{R}\right)^2\right]$$

$$T_C = 9.62\times10^6\left(\frac{M}{1M_\odot}\right)\left(\frac{R_\odot}{R}\right)\text{K}$$

D. $$\left.\frac{dT}{dr}\right|_{r=R/2} = -\frac{3}{16\sigma}\frac{\kappa_0\rho_C^2}{4\pi R^2}\left(\frac{31\pi}{288}\frac{G\mu m_\mathrm{H}}{k}\rho_C R^2\right)^{-6.5}L_{R/2}$$

E. $$\left.\frac{dT}{dr}\right|_{r=R/2} = -\frac{29\pi}{144}\frac{G\mu m_\mathrm{H}}{k}\rho_C R$$

F. $$L_{R/2} = \frac{116\pi^3}{81}\left(\frac{31}{96}\right)^{6.5}\frac{\sigma}{\kappa_0}\left(\frac{Gm_\mathrm{H}}{k}\right)^{7.5}\frac{\mu^{7.5}M^{5.5}}{R^{0.5}} = L$$

$$L = 1.2L_\odot\left(\frac{M}{M_\odot}\right)^{5.5}\left(\frac{R_\odot}{R}\right)^{0.5}$$