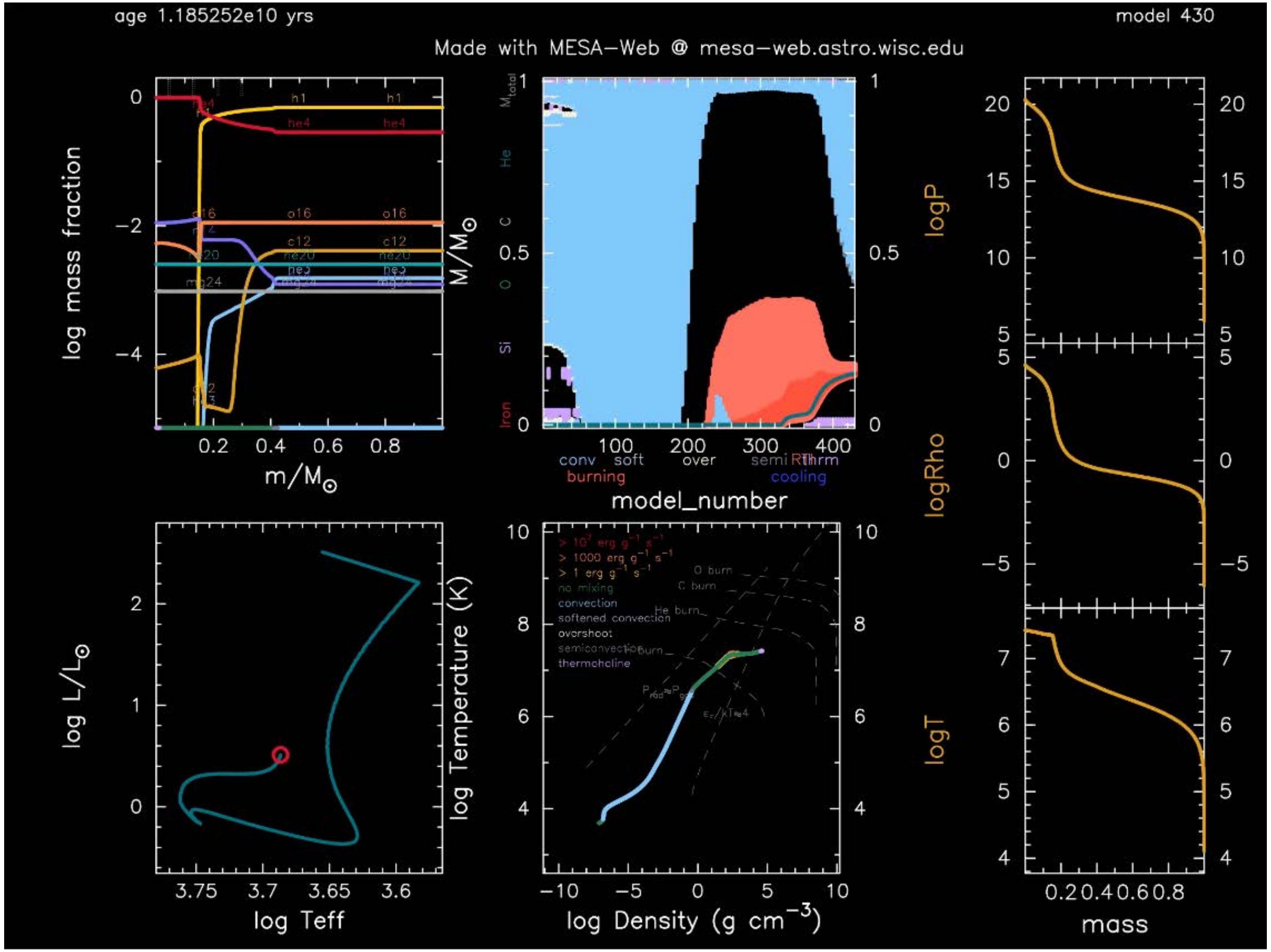
# Today in Astronomy 241: stellar evolution on, and slightly off, the main sequence

- Changes in the stellar core composition
- Buildup of an isothermal core
- The maximum in weight which can be supported by an isothermal core
- **Reading**: C&O chapter 13, pp. 446-457




MESA model of the Sun, through age 12 Gyr.

# Stellar evolution on the main sequence

- As hydrogen burns in the **core**, the mean molecular weight slowly increases. At a given temperature the ideal gas law says this would result in a lower pressure. Therefore
  - the core slowly contracts and heats up.
  - the radius and effective temperature of the star's **envelope** slowly increase, in response to the new internal temperature and density distribution.
  - the luminosity slowly increases, in response to the increase in radius and effective temperature.
- This all takes place much faster for higher-mass stars.

# Evolution on the main sequence (continued)

- Most of these trends are evident in the results of the simplified stellar models you have recently worked out. Consider the result from classes on 26 and 28 February:

$$L = \frac{\pi^3}{64^2}\left(\frac{7}{4}\right)^{6.5}\left(\frac{3}{4}\right)^{1.5}\frac{\sigma}{5\kappa_0}\left(\frac{G\mu m_H}{k}\right)^{15/2} M^{11/2}R^{-1/2}$$

$$T_e = \left(\left(\frac{\pi}{64}\right)^2\left(\frac{7}{4}\right)^{6.5}\left(\frac{3}{4}\right)^{1.5}\frac{1}{20\kappa_0}\right)^{1/4}\left(\frac{G\mu m_H}{k}\right)^{15/8} M^{11/8}R^{-5/8}$$

$$t \propto \frac{M}{L} \propto M^{-9/2}$$

# Shell hydrogen burning and the subgiant phase

- Eventually hydrogen is exhausted in the very center, and the temperature is insufficient to ignite helium burning, but is high enough just outside the center for a **shell** of hydrogen fusion to keep the star hydrostatic. Thus
  - ***T* is nearly constant in the core** (isothermal helium core, or *ic*), which keeps increasing in mass owing to hydrogen depletion;
  - luminosity increases and the envelope expands further;
  - effective temperature decreases further;
  - This is called the **subgiant phase**: the star moves off the main sequence, upwards and to the right in the HR diagram.
- Pressure exerted by core on envelope, C&O equation 13.7:

$$P_{ic} = \frac{3}{4\pi R_{ic}^3}\left(\frac{M_{ic}kT_{ic}}{\mu_{ic}m_{\mathrm{H}}} - \frac{1}{5}\frac{GM_{ic}^2}{R_{ic}}\right)$$

# Critical point for the isothermal core

- This pressure has a maximum, given by C&O equation 13.9:

$$P_{ic,\,\mathrm{max}} = \frac{375}{64\pi G^3 M_{ic}^2}\left(\frac{kT_{ic}}{\mu_{ic} m_{\mathrm{H}}}\right)^4$$

and thus there is a maximum to the mass of the core for hydrostatic stars (Schoenberg and Chandrasekhar 1942):

$$\frac{M_{ic}}{M} \approx 0.54\left(\frac{\mu_{env}}{\mu_{ic}}\right)^2 = 0.54\left(\frac{0.62}{1.34}\right)^2 = 0.12 \text{ for the Sun}$$

- The subgiant phase ends when this mass exceeded, and core pressure can no longer support the envelope’s weight; big changes begin to happen much faster thereafter.

# Electron degeneracy in the isothermal core

- Degeneracy pressure from electrons can increase the support of the core against its weight and that of the envelope; this leads to a modest increase (to about 0.13) over the Schoenberg-Chandra limit.

- Degenerate electron gases are polytropes, of order $n = 1.5$ for nonrelativistic electrons and $n = 3$ for ultrarelativistic ones.

# Today's in-class problems

1. Run MESA via MESA-Web.

   a. Go to MESA-Web and run a model of the Sun, with the default parameters except for a maximum age of 12 Gyr (under Custom Stopping Condition), and a profile output frequency of every 5 models (under General).

   While waiting:

2. Suppose that the pressure $P$ in a stellar interior consists of a fraction β of radiation pressure $P_{\mathrm{rad}}$ and a remaining fraction 1 – β of gas pressure $P_{\mathrm{gas}}$. Suppose further that β is uniform in the star's interior. Show that the resulting equation of state is a polytrope of order $n = 3$, i.e. $\gamma = C_P/C_V = 4/3$.

3. C&O 13.3.

1. b. In the description of MESA-Web's output, familiarize yourself with profile output. Find where MESA lists the important stellar parameters – age, total mass, outer radius, radius, density, pressure, temperature, mean molecular weight, adiabatic gradient – can be found. You will be using MESA regularly from now on.

# Today's in-class problems (continued)

Hints for the previous set of in-class problems:

1. I presume everybody has a smoothly running Lane-Emden solver now.

2. I get $\rho_C = 8.44\ \mathrm{gm\ cm^{-3}}$, $P_C = 8.66 \times 10^{15}\ \mathrm{dyne\ cm^{-2}}$, and $T_C = 7.72 \times 10^{6}\ \mathrm{K}$.

3. (C&O 10.19) Put the $n$ = 5 solution for $D_n$ into the expression for total mass of the star:

$$M = -4\pi\alpha^3\rho_C\xi_1^2 D_n'\left(\xi_1\right) = -4\pi\alpha^3\rho_C\xi_1^2\left[\frac{d}{d\xi}\left(1+\frac{\xi^2}{3}\right)^{-1/2}\right]_{\xi_1}$$

$$= -4\pi\alpha^3\rho_C\xi_1^2\left(-\frac{1}{2}\right)\left(1+\frac{\xi_1^2}{3}\right)^{-3/2}\left(\frac{2\xi_1}{3}\right) = \frac{4\pi}{3}\alpha^3\rho_C\left(\frac{1}{\xi_1^2}+\frac{1}{3}\right)^{-3/2} \xrightarrow[\xi_1\to\infty]{} 3^{3/2}\frac{4\pi}{3}\alpha^3\rho_C \quad .$$