Today in Astronomy 241: stellar pulsation I

- Brief review of types of pulsating stars
- Brief review of the ε, κ, and γ mechanisms
- Linear one-zone model of radial pulsation
- **Reading**: C&O chapter 14, pp. 503-512

On zoom 1-4 PM EST today.
Office hours after 4 PM unless unneeded. Thursday, too.

Cepheid pulsation, illustrating expansion+cooling and contraction+heating phases. From ESA.

All stars pulsate.

- Even the Sun, for which the strongest modes comprise the famous 5-minute oscillations.
  - Even Polaris, as some of you learned in ASTR 142.
- And that's a good thing. As you have learned elsewhere, stellar pulsations have two crucial uses in astrophysics:
  - **Probes of stellar structure.** As described before, here and in ASTR 142, the frequencies of oscillation modes are very sensitive to the internal structure and state of stars.
  - **Standard candles**. For some stars the oscillation frequency is related to the star's luminosity. If one knows the relationship, one can use a measurement of the frequency and average apparent flux to calculate the star's distance.

Loud sound waves in stars: pulsators, especially radial pulsators

Beginning with the discovery around 1600 of the pulsation of Mira (o Ceti), and in 1784 of that of δ Cephei, eight major types of pulsating stars have been found.

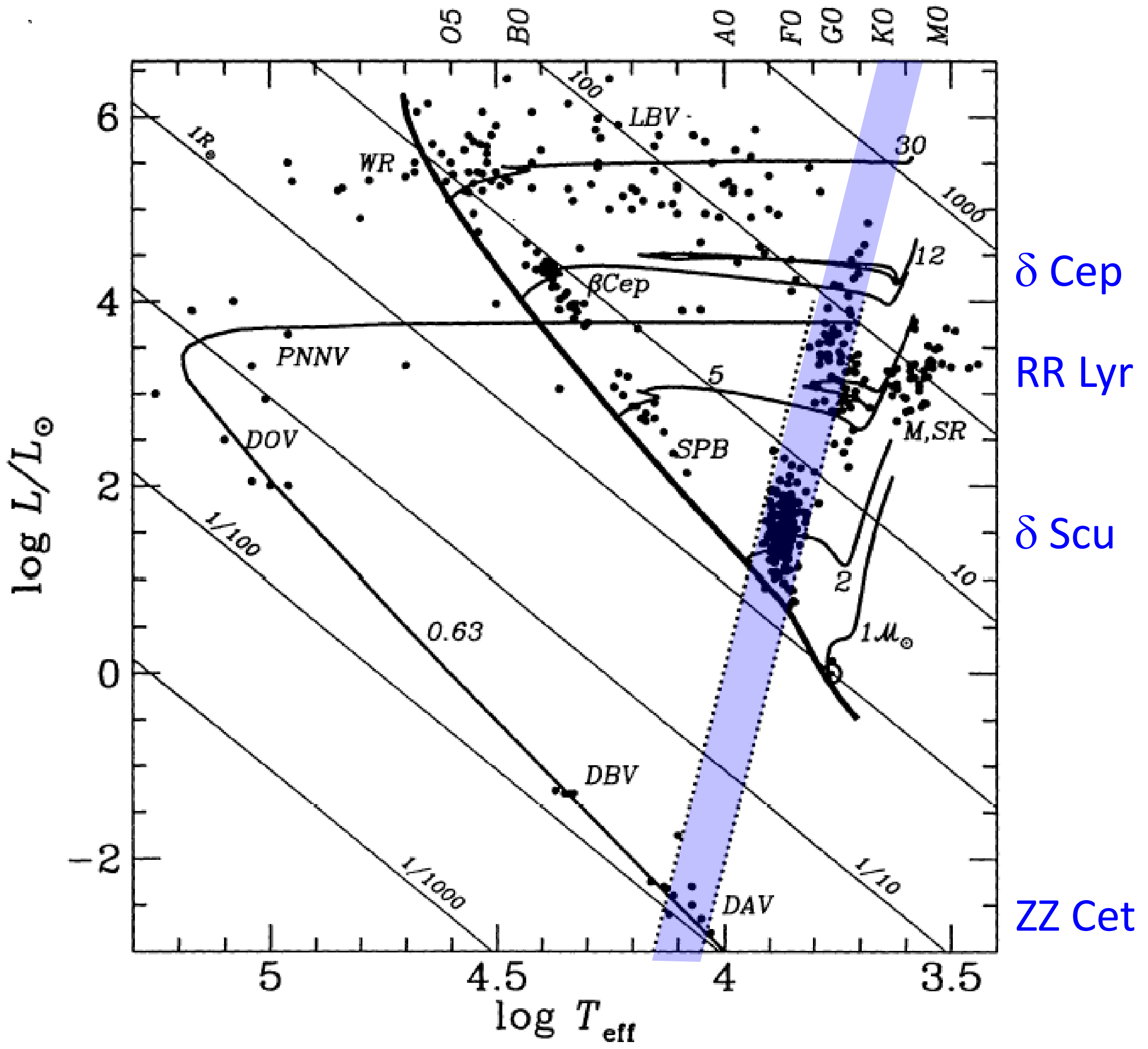
Type		**Period**	**Amplitude, Δm**
Long-period (Mira) variables		100-700 days	2-7 mag
Classical (Pop I) Cepheids		1-50 days	0.5-1.5 mag
W Virginis stars (Pop II Cepheids)		2-45 days	0.5-1.5 mag
RR Lyrae stars		1-48 hours	1-1.5 mag
δ Scuti stars		1-3 hours	0.1-0.6 mag
β Cephei stars	near MS	3-7 hours	0.05-0.2 mag
SX Phoenicis stars (Pop II δ Scus)		1-12 hours	0.01-0.15 mag
ZZ Ceti stars	white dwarfs	100-1000 sec	0.1-0.3 mag

The rest are giant or supergiant stars.

General Catalogue of Variable Stars

Loud pulsations in the HR diagram

The most numerous loud pulsators follow a distinct pattern in the HR diagram, called the **instability strip**.



Gautschy+1995

Physics of stellar pulsation

- Rough estimate of pulsation period: sound crossing time

$$v_S = \sqrt{\frac{\gamma P}{\rho}}$$

Adiabatic sound speed

$$P = \frac{2}{3}\pi G \rho^2 (R^2 - r^2)$$

for uniform density

$$\Pi = 4\int_0^R \frac{dr}{v_S} = \sqrt{\frac{6\pi}{\gamma G \rho}}$$

Standing wave period for tube open at one end, closed at the other

$$= 3 \text{ hours for } M = 1M_\odot, R = 1R_\odot$$

$$= 40 \text{ days for } M = 5M_\odot, R = 50R_\odot$$

Physics of stellar pulsation (continued)

- Driving: heat enters gas near maximum compression
- Damping: heat dissipates (leaves the gas)
- Types of pulsation driving and damping:
    - ε-mechanism: in core of star, driven by temperature and energy-generation rate rise toward center.
    - **κ**-mechanism: **opacity** increases with increasing density; thus soaks up radiative energy near maximum compression. Usually this mechanism dominates.
        - Mostly the opacity increase is from helium ionization.
        - This occurs deep enough in the star to drive oscillations for stars with $T_e \approx 7 - 10 \times 10^4$ K, hence the position of the instability strip in the HR diagram.
    - γ-mechanism: same as κ, plus thermal conduction.

One-zone pulsation model

Equation of motion of surface shell (the zone) with mass *m*, subject to radial forces by interior pressure and gravity:

$$m\frac{d^2R}{dt^2} = -\frac{GMm}{R^2} + 4\pi R^2 P$$

Linearize: that is, write $P = P_0 + \delta P$ and $R = R_0 + \delta R$; keep only first order terms in δP or δR :

$$m\frac{d^2}{dt^2}(\delta R) = \frac{2GMm}{R_0^3}\delta R + 8\pi R_0 P_0 \delta R + 4\pi R_0^2 \delta P$$

Link δP and δR by assuming adiabatic compression and expansion, i.e. polytrope equation of state:

$$PV^\gamma = K \quad \Rightarrow \quad V^\gamma dP + P dR \frac{d}{dR}\left(\frac{4\pi}{3}R^3\right)^\gamma = 0 \quad \xrightarrow[\text{first in-class problem}]{} \quad \frac{dP}{P_0} = -3\gamma \frac{dR}{R_0}$$

Physics of stellar pulsation (continued)

Use this last result to eliminate *δP.* Resulting equation of motion for *δR* is one of simple harmonic motion:

$$\frac{d^2}{dt^2}\left(\delta R\right) = -(3\gamma - 4)\frac{GM}{R_0^3}\delta R$$

so, as you have seen many times before, the angular frequency and period come straight from the *δR* term:

$$\omega^2 = (3\gamma - 4)\frac{GM}{R_0^3} \quad , \quad \Pi = \frac{2\pi}{\sqrt{\frac{4}{3}\pi G\rho_0 (3\gamma - 4)}} \quad .$$

Today’s in-class problems

- C&O problem 14.7.
- C&O problem 14.8.

Hints for the last set of in-class problems

A. Hydrostatic equilibrium and FTOC: $$\int_{\infty}^{R}\frac{dP}{dr}dr = P(R) = \int_{\infty}^{R}\frac{GM_r\rho}{r^2}dr \quad .$$

We have previously treated the stellar atmosphere as plane parallel and thin, in which the gravitational acceleration changes little over the range in which pressure and density change by very much. So using the same approximation for gravitational acceleration as already used in the optical depth integral in the next step.

$$P(R) = \frac{GM}{R^2\overline{\kappa}}\int_{\infty}^{R}\overline{\kappa}\rho dr = \frac{GM}{R^2\overline{\kappa}}\int_{\infty}^{R}d\tau = \frac{2}{3}\frac{GM}{R^2\overline{\kappa}} \quad .$$

B. Pressure balance at photosphere: $$P(R) = \frac{2}{3}\frac{GM}{R^2}\frac{1}{\kappa_0 P(R) T_e^3} \quad \Rightarrow \quad P(R) = \left[\frac{2}{3}\frac{GM}{R^2\kappa_0}\right]^{1/2} T_e^{-3/2}$$

Interior is a polytrope: $$K' = PT^{-5/3} = P(R)T_e^{-5/3} = \left[\frac{2}{3}\frac{GM}{R^2\kappa_0}\right]^{1/2} T_e^{-19/6}$$

Hints for the last set of in-class problems (continued)

B. (continued)

$$K'^2 = \frac{2GM}{3\kappa_0}\frac{4\pi\sigma T_e^4}{4\pi\sigma T_e^4 R^2}T_e^{-19/3} = \frac{8\pi\sigma GM}{3\kappa_0}\frac{1}{L}T_e^{-7/3}$$

$$L = \frac{8\pi\sigma GM}{3\kappa_0 K'^2}T_e^{-7/3} \propto T_e^{-7/3}$$

C. So in this phase, L **decreases** with increasing T_e , opposite to the behavior on the main sequence, and as depicted at right.